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NEUTRON ELECTROMAGNETIC FORM FACTORS

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In the previous paper ¹⁾ a new method of constructing the deuteron wave function was proposed. Now, we wish to apply the resulting function to examine the problem of the electromagnetic deuteron form factor. We will base our discussions on the experimental data as given in ²⁾ and ³⁾.

The main problem is to find the charged form factor of the neutron for small momentum transfers. As is well known, this form factor takes very small values in this region, and so it may turn out to be sensitive on how accurately our wave function fits all the deuteron data ^{*)}. The wave function, as given in ¹⁾, corresponded to the p_d value which was reasonable from the physical point of view, but nevertheless did not lead to an accurate value for the magnetic moment of the deuteron. It followed from the fact that the difference $\mu_d - (\mu_p + \mu_n)$ depends in fact on few factors, and only one of them (i.e., p_d value) is taken into account in the non-relativistic theory. Consequently, in order to get a good μ_d value we should use an "effective" p_d value rather than the "true" one; this "effective" p_d stands for all the factors which are unknown or difficult to calculate, influencing the μ_d value. As is well known, this "effective" value is about 0.04, whereas the "true" value is most probably about 0.06 or even more. To have a good description of all the deuteron properties in our non-relativistic approach we have introduced this correction to our theory. Some other corrections, though very small numerically, were also taken into account.

Now we wish to point out how these changes influenced our wave function. As before we tried to get a solution of our problem for r^2 values not too far from that we know from the analysis of other blocks of experimental data. It turned out, however, that if this assumption

*) The author is indebted to Dr. M. Gourdin for numerous useful comments and discussions on this point.

is to be maintained, we should not demand both the wave functions $\underline{u(r)}$ and $w(r)$ to have the same "poles" ξ_1 and ξ_2 . We were led even to a somewhat simpler solution than before, i.e., that the "pole" ξ_1 should not be present in the $w(r)$. It means that we looked for the solution of the following type:

$$\begin{aligned}
 u(r) &= A e^{-\kappa r} \left[1 + \int \frac{\sigma^+(\alpha) e^{-\alpha r}}{\alpha(\alpha+2\kappa)} d\alpha + H \int \frac{\sigma^-(\alpha) e^{-\alpha r}}{\alpha(\alpha+2\kappa)} d\alpha + \right. \\
 &\quad \left. + \gamma_1 e^{-\xi_1 r} + \gamma_2 e^{-\xi_2 r} \right] \\
 w(r) &= A e^{-\kappa r} \left[H + \int \frac{\sigma^+(\alpha) e^{-\alpha r}}{\alpha(\alpha+2\kappa)} d\alpha + H \int \frac{\sigma^-(\alpha) e^{-\alpha r}}{\alpha(\alpha+2\kappa)} d\alpha + \right. \\
 &\quad \left. + e^{-\xi_2 r} (\gamma_3 + \gamma_4 r) \right]. \quad (1)
 \end{aligned}$$

We tried to fulfil the following deuteron data, given $u(r)$ and $w(r)$:

- a) triplet effective range $r_{tr} = 1.2309$
- b) normalization $\int u^2(r) dr + \int w^2(r) dr = 1$
- c) quadrupole moment $Q = 0.1369$ and
- d) p_d of the order of 0.04.

We got the following two solutions, the first for $p_d = 0.039$ and the second for $p_d = 0.042$:

- A) $f^2 = 0.098$, $H = 0.02935$, $\gamma_1 = -2.1957$, $\gamma_2 = 1.7276$, $\gamma_3 = 0.4115$,
 $\gamma_4 = 1.3402$
- B) $f^2 = 0.095$, $H = 0.02914$, $\gamma_1 = -2.3317$, $\gamma_2 = 1.8440$, $\gamma_3 = 0.3433$,
 $\gamma_4 = 0.9953$

In both cases $\xi_1 = 4$ and $\xi_2 = M = 6.72$.

As in ¹⁾, some discussion was made to find how the solutions depend on ξ_1 and ξ_2 . It turned out, however, that the dependence on them is rather weak, and in general the bigger the ξ 's (in the

region of a few pion masses) the better agreement can be reached for f^2 close to the experimental value. Moreover, we found that conclusions concerning the deuteron electromagnetic structure do not depend much on which of the two solutions A) or B) is used.

In further calculations we have used some very well-known formulae (see e.g. ⁴⁾), which we list below for the sake of reference:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \cdot G^2, \quad (2)$$

where G^2 may be split into two parts A and B depending only on the four momentum transfer q^2 :

$$G^2 = A(q^2) + B(q^2) + q^2 \frac{\Theta}{2}. \quad (3)$$

On the other hand both the A and B functions are connected to the electromagnetic form factors of the deuteron:

$$A = F_{CH}^2 + \frac{1}{18} \left(\frac{q}{M_D} \right)^4 F_Q^2 + \frac{1}{6} \left(\frac{q}{M_D} \right)^2 \left(1 + \frac{q^2}{4M_D^2} \right) F_M^2 \quad (4a)$$

$$B = \frac{1}{3} \left(\frac{q}{M_D} \right)^2 \left(1 + \frac{q^2}{4M_D^2} \right)^2 F_M^2 \quad (4b)$$

where F_{CH} , F_Q and F_M are, respectively, charged, quadrupole electric and magnetic deuteron form factors. In the impulse approximation they are connected to the nucleon electromagnetic isoscalar form factors F_{CH}^S and F_M^S in the following way:

$$\left. \begin{aligned} F_{CH} &= F_{CH}^S \cdot C_E, \quad F_Q = F_{CH}^S \cdot C_Q, \\ F_M &= \frac{M_D}{M} \left[F_M^S \cdot C_S + \frac{1}{2} F_{CH}^S \cdot C_L \right] \end{aligned} \right\} \quad (5)$$

4.

where

$$\left. \begin{aligned} C_E &= U_0 + W_0, \quad C_Q = \frac{6\sqrt{2}}{q^2} M_D^2 \left(U_W - \frac{1}{2\sqrt{2}} W_2 \right), \\ C_S &= U_0 - \frac{1}{2} W_0 - \sqrt{2} U_W - W_2, \quad C_L = \frac{3}{2} (W_0 + W_2), \end{aligned} \right\} \quad (6)$$

and

$$\left. \begin{aligned} U_0 &= \int_0^\infty u^2(r) j_0\left(\frac{qr}{2}\right) dr \\ W_0 &= \int_0^\infty w^2(r) j_0\left(\frac{qr}{2}\right) dr \\ U_W &= \int_0^\infty u(r) w(r) j_2\left(\frac{qr}{2}\right) dr \\ W_2 &= \int_0^\infty w^2(r) j_2\left(\frac{qr}{2}\right) dr, \end{aligned} \right\} \quad (7)$$

where $j_0(x)$ and $j_2(x)$ are the spherical Bessel functions of the order 0 and 2, respectively. Notice that the isoscalar nucleon form factors are normalized as follows:

$$F_{CH}^S(0) = 1, \quad F_M^S(0) = 1 + \mu_p + \mu_n,$$

where μ_p and μ_n are the anomalous magnetic moments of the proton and neutron.

It is easy to express $F_{CH}^S(q^2)$ and $F_M^S(q^2)$ by the functions $A(q^2)$ and $B(q^2)$; one gets the following formulae:

$$F_{CH}^S(q^2) = \pm \sqrt{\frac{A - \frac{1}{2} B (1 + \frac{q^2}{4M_D^2})^{-1}}{(U_0 + W_0)^2 + 4(U_W - \frac{1}{2\sqrt{2}} W_2)^2}}, \quad (8)$$

$$F_M^S(q^2) = \left\{ -\frac{3}{2} F_{CH}^S(q^2) (W_0 + W_2) \pm \sqrt{\frac{3BM^2}{q^2 (1 + \frac{q^2}{4M_D^2})^2}} \right\} \cdot (U_0 - \frac{1}{2} W_0 - \sqrt{2} U_W - W_2)^{-1}.$$

In the above formulae M_D is the deuteron mass, and M is the nucleon mass.

We present our results in the Tables I and II. We omit here the results for solution B) since they do not differ significantly from those for solution A). In Table I we present the data for the isoscalar charged form factor F_{CH}^S and, then, the neutron charged form factor $F_{CH}^n = F_{CH}^S - F_{CH}^p$.

q^2	F_{CH}^S	F_{CH}^n
0.6	0.9738 ± 0.0062	$+0.0008 \pm 0.0116$
1.2	0.9231 ± 0.0050	-0.0168 ± 0.0111
2.0	0.8685 ± 0.0121	-0.0181 ± 0.0176
3.2	0.8217 ± 0.0058	-0.0283 ± 0.0158
4.4	0.7544 ± 0.0083	-0.0356 ± 0.0142

TABLE I

The second column of Table I was calculated on the basis of data for the proton charged form factor F_{CH}^p as given in ³⁾.

It is well known that the main contribution to the magnetic form factor comes from the second and not the first term in formula (8) in view of small values of W_0 and W_2 , and it holds at least for $q^2 \ll M^2$, which is true in our case. So, the corresponding calculations can be made only when B is measured. That was done for $q^2 = 2.0$ ²⁾ and $q^2 = 3.2$ ³⁾. The results are presented in Table II.

q^2	$F_M^S(q^2)$	$F_M^n(q^2)$
2.0	$1.2552 \pm \begin{matrix} 0.1952 \\ 0.2304 \end{matrix}$	$-1.2120 \pm \begin{matrix} 0.1952 \\ 0.2304 \end{matrix}$
3.2	$0.8456 \pm \begin{matrix} 0.4140 \\ 0.8866 \end{matrix}$	$-1.5393 \pm \begin{matrix} 0.4140 \\ 0.8866 \end{matrix}$

TABLE II

In fact, there are two solutions for the charged form factor, as indicated by the sign in the upper formula (8); one of them, that corresponded to the negative F_{CH}^S was, however, rejected. With this solution there are still two solutions for F_{Mag}^S (see lower formula (8)), but here also one of them was rejected since it led to much too high a value for $|F_M^n|$, so that F_M^n/μ_n was not of the order of $F_1^p(q^2)$, as would be expected.

Now, we would like to make some comments on the results obtained. First, as one can see from Table I the charged neutron form factor, as calculated here, seems not to be exactly zero, but is rather negative and even decreases as q^2 increases. It is not excluded, then, that for $q^2 < 0.6$ F_{CH}^n is very small and positive, though our data

cannot give any support to such a hypothesis. However, if this were true, then it could mean that the linear extrapolation of the results coming from neutron-electron scattering ⁵⁾ is valid only for extremely small q^2 values. Further experimental data are needed here, however, before any conclusion can be drawn.

The second point is that F_M^n/μ_n (which closely approximates Hofstadter's F_2^n) seems to be smaller than $F_M^p(1+\mu_p)^{-1}$; e.g., for $q^2 = 2$ this second quantity is 0.8866, whereas the first one is $0.6338 + 0.1205 - 0.1021$. When $q^2 = 3.2$ the data are not so precise, but in this case that effect seems to vanish. It can be seen by comparison of the two numbers: 0.857 for $F_M^p(1+\mu_p)^{-1}$ and $0.8049 + 0.4637 - 0.2164$ for F_M^n/μ_n . The possibility of non-monotonic dependence of F_2^n on q^2 was already mentioned by Hofstadter ⁶⁾ (see especially Figs. 6-9), but the picture presented by him differed quantitatively from ours; moreover no new support for such an "abnormal" effect has so far been given.

The third comment we wish to make is of more general character. As is well known we do not have at our disposal today a good relativistic theory of the deuteron. All our results may be subject to large changes when such a theory is constructed. If for example one takes into account the "effective" p_d value, which was assumed here to be lower than the "true" one, one sees that in the relativistic theory the other factors, contained now in the "apparent" p_d might not influence F_{CH}^n as strongly as p_d itself. It may happen that just p_d influences F_{CH}^n , for example, and the other corrections do not, and that they only give some contribution to μ_d . But it was checked by us that when p_d approaches higher values, such as 0.06, the picture we get changes significantly. In that case, one can even get positive F_1^n . We choose in this note the way which at the moment seems to be the most likely to be correct, but nevertheless we stress that our conclusions, as all the conclusions based on the non-relativistic theory, cannot be treated as sure.

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R E F E R E N C E S

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