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A DYNAMICAL ORIGIN FOR THE ELECTROMAGNETIC FIELD

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A B S T R A C T

An attempt to manufacture the electromagnetic field from a zero range vector interaction between fermions is discussed. The mathematical method is similar to that used by Nambu and others in their discussion of "superconductor" solutions in field theory. A solution is found for which the ground state is degenerate with respect to Lorentz transformation and which is not TCP invariant. The gauge invariance of the theory masks this lack of symmetry from experimental detection. The photon appears as "collective excitation" and, to all orders of perturbation theory, the theory is equivalent to quantum electrodynamics. It is thus independent of the strength of the original four-fermion coupling, provided the vacuum charge asymmetry is infinite.

I. Introduction

It is not necessary to dwell upon the desirability of manufacturing the electromagnetic field from other fields ¹⁾. A new possibility for accomplishing this is suggested by the "superconductor" solutions of the Bardeen, Cooper, Schreffer (BCS) theory and the transcription by Nambu and others ²⁾ into the language of relativistic field theory. These authors exhibit a connection between the existence of an infinitely degenerate vacuum caused by dynamical instability and the existence of zero mass bosons; the "collective excitations" of the BCS theory. It seems to be worth asking whether the photon might be understood in a similar way.

We shall describe a model which starts from a non-linear Heisenberg-type Lagrangian of a single fermion self-coupled with a vector interaction. If one admits a vacuum which is neither Lorentz invariant nor TCP invariant, we show, within serious liberties taken with respect to badly divergent integrals, that a degenerate solution exists. The degeneracy is with respect to a choice of Lorentz frame: however, the gauge invariance of the theory protects the non-Lorentz invariant character of the states from experimental detection. The photon appears as the "collective excitation" and within the above-mentioned ambiguity with regard to divergences, the model is renormalizable and equivalent to quantum electrodynamics to all orders of perturbation theory. There is no dependence of S-matrix elements upon the strength of the original non-linear coupling.

In Section II the model is defined. In Sections III - VI we calculate graphs. Section VII is devoted to an attempt to interpret what is happening, and Section VIII consists of conclusions.

II. The model

Our model consists of a single fermion field $\psi(x)$ of bare mass m self-coupled via a vector interaction; the Lagrangian density is

$$\mathcal{L} = \bar{\psi}(x)(i\not{\partial} - m)\psi(x) - \frac{G}{2} [\bar{\psi}(x)\gamma_\mu\psi(x)] [\bar{\psi}(x)\gamma^\mu\psi(x)]. \quad (1)$$

Such an interaction will be represented in Feynman graphs as if a photon were present (Fig. 1) in order to keep spinor factors under control; then there is a one-to-one correspondence between the Feynman graphs in quantum electrodynamics and in the present model, the only change being the replacement of the bare photon propagator $ie_o^2 g_{\mu\nu} [q^2]^{-1}$ by $-iGg_{\mu\nu}$. One may then adopt all the topological jargon of quantum electrodynamics³⁾. In particular, the wiggly line of Fig.1 will be called a "photon" line, although it has at this stage only formal significance.

The interaction term has been symmetrized; in ordinary theories TCP invariance of the vacuum assures that

$$\langle 0 | j_\mu(x) | 0 \rangle = \frac{1}{2} \langle 0 | [\bar{\psi}(x)\gamma_\mu\psi(x)] | 0 \rangle = 0. \quad (2)$$

Here, however, we shall assume that the vacuum current density is not zero. A picture of how this happens is that the customary neutral, charge-symmetric vacuum is unstable, the instability being perhaps caused in some way by the structure of the interaction.

Fig. 1

We do not understand the mechanism behind the creation of this instability (if it exists); it involves the most divergent quantities in the theory and therefore the structure of the "cut-off".

We touch off the assumed instability by adding a small term to the Lagrangian density of the form ⁴⁾

$$-\frac{\epsilon}{2} [\bar{\Psi}(x), \gamma_{\mu} \Psi(x)] \eta^{\mu}, \quad (3)$$

represented by the graph of Fig. 2, η_{μ} is a unit time-like vector ⁵⁾.

Fig. 2

The response, illustrated in Fig. 3, is then assumed to be non-vanishing, even as $\epsilon \rightarrow 0$.

Fig. 3

4.

We now proceed to compute graphs, which will be done in the following order; all graphs are included in the procedure:

1. all "Hartree bubbles" (Figs. 2, 3) exactly;
2. the proper "photon" self-energy part at zero momentum transfer, $\Pi_{\mu\nu}(0)$, exactly;
3. the proper "photon" self-energy part $\Pi_{\mu\nu}(q)$ at finite momentum transfer, to lowest order;
4. the proper vertex part and fermion propagator to first order;
5. $\Pi_{\mu\nu}(q)$ to first order, and the procedure iterated (i.e., the next step is to compute the vertex to second order);
6. in higher orders, closed loop contributions involving more than two photon lines.

In the above outline and in what shall follow, we define the order of a graph as the number of "photon" lines remaining after all proper "photon" self-energy parts have been removed.

III. The Hartree energy

We first evaluate the totality of graphs of the class shown in Figs. 2-3. To first order in G a fermion line in momentum space is modified by the replacement

$$\frac{i}{\not{p} - m} \rightarrow \frac{i}{\not{p} - m} i \not{A}^{(1)} \frac{i}{\not{p} - m}.$$

with

$$Q_\mu^{(1)} = -\epsilon \eta_\mu + G \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \gamma_\mu \frac{i}{\not{k} - m + i\epsilon} = -\epsilon \eta_\mu + G \text{Tr} \gamma_\mu S_F(x) \Big|_{x=0}. \quad (4)$$

If one computes to all orders, the bubble may be inserted an arbitrary number of times into a fermion line, and any internal line of a graph is modified according to

$$S_F(p) = \frac{i}{\not{p} - m + i\epsilon} \rightarrow \frac{i}{\not{p} + \cancel{Q} - m + i\epsilon}. \quad (5)$$

Q_μ may be computed exactly if one replaces the bare propagator in (4) by the complete propagator $S_F'(k+Q)$; $S_F'(k)$ is the propagator computed with all Hartree bubbles of Fig. 3 omitted from the charged line associated with k . Instead of the approximate expression (4), one obtains

$$\begin{aligned} Q_\mu &= -\epsilon \eta_\mu + G \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \gamma_\mu S_F'(k+Q) \\ &= -\epsilon \eta_\mu + G \text{Tr} \gamma_\mu S_F'(x) \Big|_{x=0} \\ &= \eta_\mu \left[-\epsilon + \frac{G}{2} \langle 0 | [\psi_\alpha(0), \psi_\alpha^\dagger(0)] | 0 \rangle \right]. \end{aligned} \quad (6)$$

6.

with the commutator evaluated in the frame in which η_μ is pure time-like. What we have done is to make the linear Hartree approximation to the original non-linear Dirac equation; i.e., to write

$$\begin{aligned} (i\not{\partial} - m)\psi(x) &\cong (\epsilon\not{\eta} + \frac{G}{2} \langle 0 | [\bar{\psi}(x), \gamma_\mu \psi(x)] | 0 \rangle \gamma^\mu) \psi(x) \\ &= -\not{Q} \psi(x). \end{aligned} \quad (7)$$

Our assumption is that Q_μ is non-vanishing as $\epsilon \rightarrow 0$; this means a non-vanishing vacuum charge density. The question of whether this charge asymmetry is of dynamical origin, inherent in the geometry of the unperturbed vacuum at high momenta, or simply impossible, we leave here as an open question. It will be discussed again in Section VII.

If one cuts off (in some gauge-invariant way) the momentum-space version of (6), one obtains the self-consistency equation (as $\epsilon \rightarrow 0$)

$$Q_\mu \equiv \eta_\mu Q = G \eta_\mu F(Q^2), \quad (8)$$

with

$$\eta_\mu F(Q^2) = \text{Tr} \int \frac{d^4 k}{(2\pi)^4} \gamma_\mu S'_F(k + \eta Q). \quad (9)$$

Since

$$F(Q^2) = \frac{1}{2} \langle 0 | \psi(0) \psi^\dagger(0) | 0 \rangle - \frac{1}{2} \langle 0 | \psi^\dagger(0) \psi(0) | 0 \rangle \quad (10)$$

is the difference of two cubically divergent terms, we shall hereafter assume

$$F(Q^2) \sim \Lambda^3, \quad (11)$$

where Λ is the cut-off in momentum space.

IV. Vacuum polarization; the photon mass

We next consider the graphs illustrated in Fig. 4 which contribute vacuum polarization corrections to the "photon" propagator. The proper photon self-energy part is given by the equation

$$\Pi_{\mu\nu}(q) = i \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \gamma_\mu S'_F(k+Q) \Gamma_\nu(k+Q, k+q+Q) S'_F(k+q+Q). \quad (12)$$

The complete propagator $D_{\mu\nu}$ which connects two vertices is then given by the iteration of the lowest order interaction with $\Pi_{\mu\nu}$ as kernel.

$$D_{\mu\nu}(q) = -iG g_{\mu\nu} + G \Pi_\mu^\lambda(q) D_{\lambda\nu}(q). \quad (13)$$

$\Pi_{\mu\nu}(0)$ is the analogue of the "photon mass" of electrodynamics, and is there set to zero by gauge invariance arguments. Here, however, because of the vacuum asymmetry and degeneracy the gauge arguments lead to a different conclusion. Because the Lagrangian is invariant with respect to gauge transformations of the first kind we expect that the propagator of the fermion is related to the vertex function by the Ward identity ⁶⁾.

$$\frac{\partial}{\partial Q^\nu} S'_F(k+Q) = i S'_F(k+Q) \Gamma_\nu(k+Q, k+Q) S'_F(k+Q). \quad (14)$$

Fig. 4

8.

That the Ward identity holds will, in fact, be determined self-consistently. Then, from (12),

$$\Pi_{\mu\nu}(0) = \frac{\partial}{\partial Q^\nu} \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \gamma_\mu S'_F(k+Q). \quad (15)$$

While carrying out the differentiation, Q_μ is to be considered an arbitrary vector; after the differentiation Q_μ is set equal to the value determined by the self-consistency conditions (8) and (9). Since the integral (15) is that encountered in (9), we find

$$\Pi_{\mu\nu}(0) = \frac{\partial}{\partial Q^\nu} \frac{Q_\mu}{\sqrt{Q^2}} F(Q^2) = \frac{1}{G} (g_{\mu\nu} - \eta_\mu \eta_\nu f), \quad (16)$$

where

$$f = 1 - 2Q^2 \frac{\partial}{\partial Q^2} \log F(Q^2). \quad (17)$$

Equation (16) is an exact relation and we can begin to see the photon emerge. For instance, from (13) and (16)

$$D_{ii}(0) = iG + D_{ii}(0) \quad (i=1,2,3)$$

in the frame in which Q_μ is pure time-like.

Thus $D_{ii}(0)$ is infinite, i.e., has a singularity at $q = 0$.

A little more information about $\Pi_{\mu\nu}(q)$ may be obtained by the use of the generalized Ward identity ⁷⁾

$$(p-p')_{\mu} \Gamma^{\mu}(p, p') = i S_F'^{-1}(p) - i S_F'^{-1}(p') \quad (18)$$

which will be verified self-consistently, after Γ_{μ} and S_F' are computed. Using (18) we find

$$\begin{aligned} q^{\nu} \Pi_{\mu\nu}(q) &= \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \gamma_{\mu} [S_F'(k+q+Q) - S_F'(k+Q)] \\ &= \frac{Q_{\mu} + q_{\mu}}{\sqrt{(Q+q)^2}} F\{(Q+q)^2\} - \frac{Q_{\mu}}{\sqrt{Q^2}} F(Q^2) \\ &= q^{\nu} \Pi_{\mu\nu}(0) \left[1 + \mathcal{O}\left(\frac{q}{Q}\right) \right]. \end{aligned} \quad (19)$$

If, as assumed in (11), $Q \sim G\Lambda^3$ and $|q| \lesssim \Lambda$, we see that, ignoring terms of order $[G\Lambda^2]^{-1}$

$$q^{\nu} [\Pi_{\mu\nu}(q) - \Pi_{\mu\nu}(0)] = 0. \quad (20)$$

It is not as easy to compute $\Pi_{\mu\nu}(q)$ for finite q ; however, in lowest order $\Pi_{\mu\nu}(q) - \Pi_{\mu\nu}(0)$ is identical to the vacuum polarization integral encountered in lowest order quantum electrodynamics (Fig. 5); and has the structure

Fig. 5

$$\Pi_{\mu\nu}(q) - \Pi_{\mu\nu}(0) = (q^2 g_{\mu\nu} - q_\mu q_\nu) \Pi(q^2). \quad (21)$$

$\Pi(q^2)$ is a logarithmically divergent integral which, for $q^2 \gg m^2$, is given by

$$\Pi(q^2) \approx \frac{1}{12\pi^2} \log \frac{\Lambda^2}{q^2} \quad (22)$$

The general structure (21) will be preserved in higher orders, as we shall verify self-consistently. Then, putting (16) and (21) together and inserting into Equation (13), we may compute $D_{\mu\nu}(q)$

$$D_{\mu\nu}(q) = \frac{i}{q^2 \Pi(q^2)} \left[g_{\mu\nu} - \frac{q_\mu \eta_\nu}{(\eta \cdot q)} - \frac{q_\nu \eta_\mu}{(\eta \cdot q)^2} + \frac{q_\mu q_\nu}{(\eta \cdot q)^2} \right] - \frac{i G q_\mu q_\nu}{f(q \cdot \eta)^2} \quad (23)$$

The $\eta_\mu \eta_\nu$ term has disappeared. Since $D_{\mu\nu}$ is inserted between conserved currents, terms with factors q_μ or q_ν can be ignored in computing S-matrix elements. This statement suffers some ambiguity if the "photon" line is coupled to a closed loop; this problem will be discussed

and disposed of in a later section. However, temporarily ignoring this question, we see that $D_{\mu\nu}(q)$ bears a striking resemblance to the unrenormalized photon propagator of electrodynamics, which is, to the same order,

$$D_{\mu\nu}^{\text{QED}}(q) = \frac{ie_0^2 g_{\mu\nu}}{q^2 [1 + e_0^2 \Pi(q^2)]} = \frac{ig_{\mu\nu}}{q^2 [\frac{1}{e_0^2} + \Pi(q^2)]} . \quad (24)$$

To this order, formal identification with unrenormalized quantum electrodynamics can be made by adding in (23) an infinitesimal quantity e_0^{-2} to $\Pi(q^2)$; at the end of the calculation one may let $e_0^2 \rightarrow \infty$.

However, one need never introduce e_0^2 ; the renormalized propagator to this order is identical to the corresponding expression in quantum electrodynamics. The fine-structure constant ⁸⁾ is the residue of the pole in (23), namely,

$$\alpha = \frac{1}{(4\pi)\Pi(0)} \sim \frac{3\pi}{\log \frac{\Lambda^2}{m^2}} . \quad (25)$$

V. Equivalence with quantum electrodynamics

We shall ignore all q_μ and q_ν terms in the photon propagator, and in the next section show that they contribute corrections $\sim 1/G\Lambda^2$. Then the next step is to compute the unrenormalized vertex function to first order (which includes graphs such as illustrated in Fig. 6. Because the photon propagator to this order is identical to the electrodynamic propagator, this vertex function will agree with the electrodynamic result. The fermion propagator may be computed via the generalized Ward identity (14), and armed with these results the first order corrections to $\Pi_{\mu\nu}(q)$ can be computed by means of (12). $\Pi_{\mu\nu}(q) - \Pi_{\mu\nu}(0)$ will still have the gauge invariant structure of (21) and will therefore be independent of η_μ and identical with that computed by the rules of quantum electrodynamics. Thus $D_{\mu\nu}(q)$ will have the structure of (23) and also agree with the electrodynamic expression. The whole procedure may then be iterated to any order, the only stumbling blocks being possible extraneous contributions from closed loops with more than 2 external photon lines or contributions from q_μ and q_ν terms in the photon propagator.

In external charged lines the "bare momentum" p_μ is not observable; rather, the kinetic momentum $p_\mu + Q_\mu$, whose square is m^2 , must be identified as the observable momentum. In this formalism, there have not appeared "in" and "out" states for the photon; there should, however, be no difficulty in describing, for instance, Compton scattering provided the source and sink of the photon is included in the description of the scattering process.

Fig. 6

VI. The three-photon vertex and other ambiguous contributions

In the same way that we derived the 2-photon loop at zero momentum transfer by differentiating the Hartree energy, we may derive the n-photon loop at zero momentum transfer. Indeed, if one of the photons in an n-photon loop has zero four-momentum, the value of the contribution is independent of the momentum of the remaining photons. For instance, for the three-photon loop shown in Fig. 7

$$\Pi_{\mu\nu\lambda}(k_1, k_1, 0) = \frac{\partial}{\partial Q^\lambda} \Pi_{\mu\nu}(k_1, k_1) = \frac{\partial}{\partial Q^\lambda} \Pi_{\mu\nu}(0, 0) \quad (26)$$

$$= \frac{\partial^2}{\partial Q^\lambda \partial Q^\nu} \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \gamma_\mu S'_F(k+Q)$$

$$\sim \frac{1}{GQ} \sim \frac{1}{G(G\Lambda^3)}$$

Fig. 7

Provided the quantum electrodynamic description is valid in lower orders, the doubly subtracted quantity

$$\Pi_{\mu\nu\lambda}(k_1, k_2, k_1 - k_2) - \Pi_{\mu\nu\lambda}(k_1, k_1, 0) - \Pi_{\mu\nu\lambda}(k_1, 0, k_1) + \Pi_{\mu\nu\lambda}(0, 0, 0) \quad (27)$$

converges and is therefore gauge invariant and equal to the electrodynamic expression, namely zero. As a result

$$\Pi_{\mu\nu\lambda}(k_1, k_2, k_1 - k_2) = \Pi_{\mu\nu\lambda}^{\text{QED}}(k_1, k_2, k_1 - k_2) + \mathcal{O}\left(\frac{1}{G^2 \Lambda^3}\right) \quad (28)$$

where the last term is a constant independent of k_1 and k_2 . By induction, one may deduce in a similar way

$$\Pi_{\mu_1 \dots \mu_n}(k_1 \dots k_n) = \Pi_{\mu_1 \dots \mu_n}^{\text{QED}}(k_1 \dots k_n) + \mathcal{O}\left(\frac{1}{G(G\Lambda^3)^{n-2}}\right). \quad (29)$$

There are then two new elements which differ from the case of quantum electrodynamics. The first is the term

$$\frac{-iG g_\mu g_\nu}{f(q, \eta)^2}$$

in the photon propagator which we shall call the longitudinal photon propagator. The second is a point n -photon vertex of strength $\sim 1/G(G\Lambda^3)^{n-2}$. We now eliminate these two elements. We first observe that the longitudinal photon propagator can only couple point n -photon vertices, since these are the only non-gauge invariant elements in the graphs. We may then study the effect of "radiative corrections" of n -photon vertices due to longitudinal photons. If two of the photons in such a vertex are contracted as in Fig. 8 the resultant expression will be of order

$$\frac{G \int d^4 q}{G(G\Lambda^3)^{n-2}} \sim \frac{1}{G(G\Lambda^2)(G\Lambda^3)^{n-4}}$$

Fig. 8

which is a correction of order $1/G\Lambda^2$ to the $(n-2)$ -photon vertex. If a longitudinal photon connects an n_1 -photon and n_2 -photon vertex, as in Fig. 9.

Fig. 9

and the resultant combined object is treated as a single (n_1+n_2-2) -photon vertex, it will be of order

$$\frac{G}{G^2(G\Lambda^3)^{(n_1+n_2-2)-2}}$$

which is of the same order as the original vertex. Then by such contractions of various n-photon vertices, followed by removal of "radiative corrections", one may eliminate all longitudinal photon contributions without increasing the order of magnitude of the n-photon vertices. In order to eliminate the n-photon vertices from consideration, it is only necessary to verify that the insertion of an additional photon line into a graph does not increase the degree of divergence. There are several cases to consider, illustrated in Fig. 10:

- a) the added photon couples two fermion lines; two new fermion lines and one new photon line are created, providing a correction

$$\sim \int \frac{d^4 q}{q^2 \not{q} \not{q}} \sim \log \frac{\Lambda^2}{m^2}$$

This is as in standard quantum electrodynamics.

- b) the added photon couples to an n-photon vertex and an external line, providing a factor

$$\sim \frac{1}{G\Lambda^3} \int \frac{d^4 q}{q^2 \not{q}} \sim \frac{1}{G\Lambda^2}$$

- c) the photon creates a 3-photon vertex and couples to an external line; two new photon lines and one fermion line are created, providing the factor

$$\frac{1}{G(G\Lambda^3)} \int \frac{d^4 q}{q^4 \not{q}} \sim \frac{1}{(G\Lambda^2)^2}$$

Fig. 10a

Fig. 10b

Fig. 10c

Fig. 10d

d) the photon creates two 3-photon vertices; the correction is

$$\frac{1}{G^2(G\Lambda^3)^2} \int \frac{d^4 q}{q^6} \sim \frac{1}{(G\Lambda^2)^4}.$$

The case of the photon coupling only to n-photon loops has already been eliminated.

Thus any graph containing an n-photon vertex ($n \geq 3$) is of an order less than $1/G\Lambda^2$ and can be ignored in either the limit $G \rightarrow \infty$ or $\Lambda \rightarrow \infty$. It then follows that the other terms in $D_{\mu\nu}$ which depend upon q_μ but not upon G can also be dropped, since they must couple to an n-photon vertex. This justifies the claim made in Section IV that they may be ignored.

These arguments we have used must be understood in the sense of induction arguments. If to order n , the theory is equivalent to electrodynamics, the arguments we have used extend the equivalence to the $(n+1)$ th order.

VII. A possible physical interpretation

The solution we have found may have an interpretation along the lines discussed by Dyson⁹⁾. If G is such that particles repel antiparticles (but attract each other) it may be energetically favourable for the "bare" vacuum to dissociate, with many particles of the same charge escaping to infinity. This is a problem of macroscopic as well as microscopic physics which, out of ignorance, we shall not discuss. Instead, accepting this picture, we again discuss in somewhat different language the content of Section II-IV.

Suppose, having touched off the instability with the perturbation of (3) and thereby having determined the sign of the vacuum charge, that the resultant vacuum charge-current density $\langle j_\mu(x) \rangle$ is homogeneous and given by

$$Q_\mu = G \langle j_\mu(x) \rangle = G \mathcal{F}(\epsilon) \eta_\mu \quad (30)$$

where $\mathcal{F}(\epsilon)$ is non-vanishing as $\epsilon \rightarrow 0$. This, we must add, is the charge-current density as seen by an isolated external test particle according to the interaction $G j_\mu(x) \langle j^\mu(x) \rangle$.

Now suppose an extra external perturbation $\delta j_\mu^{\text{ext}}(x) \ll \epsilon$ is added onto ϵ . The additional vacuum current $\delta j_\mu(x)$ with which a test particle will interact will be determined, for sufficiently small $\delta j_\mu^{\text{ext}}(x)$, by

$$\langle \delta j_\mu(x) \rangle = \delta j_\mu^{\text{ext}}(x) + G \int d^4 x' \tilde{\Pi}_{\mu\nu}(x-x') \langle \delta j^\nu(x') \rangle \quad (31)$$

with $\tilde{\Pi}_{\mu\nu}(x-x')$ the co-ordinate space version of the polarizability tensor defined in (12). If we average (31) over a macroscopic region of space-time and define

$$\bar{\delta j}_\mu = \frac{\int d^4 x \delta j_\mu^{\text{ext}}(x)}{\int d^4 x} = \epsilon \delta \eta_\mu + \eta_\mu \delta \epsilon \quad (32)$$

with

$$(\eta \cdot \delta \eta = 0)$$

we find

$$\overline{\delta j_\mu} = \epsilon \delta \eta_\mu + \eta_\mu \delta \epsilon + G \Pi_{\mu\nu}(0) \overline{\delta j^\nu}. \quad (33)$$

However, the static properties of the vacuum and thus of $\Pi_{\mu\nu}(0)$ are already determined by the function $\mathcal{F}(\epsilon)$, and from (30),

$$\overline{\delta j_\mu} = \mathcal{F}(\epsilon) \delta \eta_\mu + \mathcal{F}'(\epsilon) \eta_\mu \delta \epsilon, \quad (34)$$

since the zero-frequency part of $\delta j_\mu^{\text{ext}}$ only modifies ϵ and the orientation of η_μ . Inserting (34) into (33), one finds consistency for arbitrary variations $\delta \eta_\mu$ and $\delta \epsilon$ only if

$$G \Pi_{\mu\nu}(0) = \left(1 - \frac{\epsilon}{\mathcal{F}(\epsilon)}\right) g_{\mu\nu} + \eta_\mu \eta_\nu \left(\frac{\epsilon}{\mathcal{F}(\epsilon)} - \frac{1}{\mathcal{F}'(\epsilon)}\right) \quad (35)$$

If $\mathcal{F}(\epsilon) = \epsilon$, i.e., there is no response to the original perturbation ϵ , $\Pi_{\mu\nu}(0)$ vanishes. This is consistent with the argument in standard quantum electrodynamics. If as $\epsilon \rightarrow 0$ $\mathcal{F}(\epsilon)$ remains finite, we reproduce (16), and thereby a singular behaviour of the vacuum current at zero frequency. That this singularity at zero frequency has as a consequence elastic waves with the properties of photons is a consequence of the detailed structure of $\Pi_{\mu\nu}(x-x')$ for the non-zero frequency components. This depends on the structure of the correlations of fluctuations in charge density of the negative energy "sea" of Dirac particles.

Since the interaction of a test particle with the vacuum perturbed by $\delta j_\mu^{\text{ext}}(x)$ is

$$G j_\mu(x) \langle \delta j_\mu(x) \rangle$$

we may make the identification

$$G \langle \delta j_\mu(x) \rangle = e A_\mu^{\text{ext}}(x) \quad (36)$$

i.e., $A_\mu^{\text{ext}}(x)$ is the electromagnetic potential due to the infinitesimal external current $\delta j_\mu^{\text{ext}}(x)$. The relation between $A_\mu^{\text{ext}}(x)$ and its source is found from the solution of (31), already given in (13) and (23). Thus, if $\Pi_{\mu\nu}$ has the structure given in (21),

$$e A_\mu^{\text{ext}}(x) = i \int d^4 x' D_{\mu\nu}(x-x') \delta j_\nu^{\text{ext}}(x') \quad (37)$$

$$\cong - \int \frac{d^4 q d^4 x'}{(2\pi)^4} e^{iq \cdot (x-x')} \frac{e^2}{q^2} \left[\delta j_\mu^{\text{ext}}(x') - \frac{q_\mu \eta^\nu}{(q \cdot \eta)} \delta j_\nu^{\text{ext}}(x') \right]$$

provided $\delta j_\mu^{\text{ext}}(x)$ is conserved and does not contain appreciable Fourier components above the electron Compton wavelength. From (37), we may extract - in the frame in which η_μ is pure time-like - the differential equations

$$\begin{aligned} A_0^{\text{ext}} &= 0 \\ \left(\frac{\partial^2}{\partial t^2} - \nabla^2 \right) \vec{A}_{\text{tr}}^{\text{ext}}(x) &= e \vec{\delta j}_{\text{tr}}^{\text{ext}}(x) \\ \vec{\nabla} \cdot \frac{\partial \vec{A}^{\text{ext}}(x)}{\partial t} &= -e \delta j_0^{\text{ext}}(x) \end{aligned} \quad (38)$$

$\vec{A}_{tr}^{ext}(x)$ is the transverse part of the vector potential. These are the Maxwell equations in the rather unfamiliar gauge in which the electrostatic potential vanishes identically.

VIII. Conclusions

We have viewed the electrodynamic interaction as originating from a short range interaction between currents

$$\vec{j}_\mu(x) \vec{j}^\mu(x)$$

instead of the long-range starting point of quantum electrodynamics

$$\int d^4x' \vec{j}_\mu(x) D_F^{\mu\nu}(x-x') \vec{j}_\nu(x')$$

The long range character of the effective interaction is obtained from the short range by setting the ambiguous quadratically divergent integral for the "photon mass" equal to 1 instead of zero. This possibility is associated, due to gauge invariance, with the existence of a charge asymmetry in the vacuum, the origin of which is not well understood. In itself this is not really progress; we have traded in an ambiguous quadratically divergent integral of quantum electrodynamics for an even more ambiguous cubically divergent integral, the Hartree energy. However, it is our hope that this new look at an old subject may raise questions whose answers have additional physical consequences.

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The article by B. Zumino in Werner Heisenberg und die Physik in unserer Zeit, Vieweg, (Braunschweig 1961), contains other references.
- 3) Our notation, except for the definition of the propagators S_F and D_F is that of Schweber, Bethe and de Hoffmann, Mesons and Fields, vol. 1, Row Peterson (Evanston, Ill.), 1956. In particular

$$\frac{1}{2} \{ \gamma_\mu, \gamma_\nu \} = g_{\mu\nu} = (1, -1, -1, -1) \text{ for } \mu, \nu = 0, 1, 2, 3.$$

- 4) This point has been emphasized in connection with superconductivity and ferro-magnetism by N.N. Bogoliubov. See Physica 26 (Supplement), S1 (1960).
- 5) We limit ourselves here to time-like η_μ , it might be that η_μ is space-like or null.
- 6) J. Ward, Phys. Rev. 78, 182 (1950).
- 7) Y. Takahashi, Nuovo Cimento 6, 371 (1957).
- 8) A similar expression has been obtained by P. Freund with a somewhat similar starting point. P. Freund, Acta Physica Austriaca 14, 445 (1961).
- 9) F. Dyson, Phys. Rev. 85, 631 (1952).