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THE SELECTION RULES IN THE FEINBERG AND PAIS
FIELD THEORY OF WEAK INTERACTIONS

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A B S T R A C T

The peratization method of Feinberg and Pais is applied to semi-leptonic processes. A matrix formalism is presented which makes it possible to treat these processes in a way closely similar to what is done in the simpler case of the purely leptonic processes. It is shown that, when peratization is applied to contributions of the higher order graphs, a) the concept of universality and the CVC theory remain meaningful and b) the set of selection rules implied by a one-boson-exchange approximation remains unchanged to the order g^2 . In particular, if the fundamental Lagrangian involves no $\Delta S = -\Delta Q$ or $|\Delta S| = 2$ currents, such selection rules as $\Delta S/\Delta Q = 1$ and $\Delta S \neq 2$ are still valid.

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I. INTRODUCTION

The fact that weak interactions are not renormalizable has for a long time given some suspicion that perhaps higher order effects might, in spite of the weakness of the coupling constant, be important. It was not known, however, how to evaluate such effects and consequently it was generally assumed that, at least at low energy, these effects did not compete in any appreciable way with the lowest order ones.

In a recent paper, Feinberg and Pais ¹⁾ have shown that this assumption may not be correct. Working within the intermediate vector boson theory, they have treated in detail the case of pure leptonic interactions and have shown that the sum of a well-defined class of higher order graphs produces a term with the same power in the coupling constant as the contribution of the lowest order. More precisely, they have considered the infinite subset of uncrossed ladder graphs which can be generated by iteration of a Bethe-Salpeter equation. They start an approximation scheme in which the first term is obtained by summing in the ladder graphs series the most divergent part of the perturbation graphs. The corresponding approximate Bethe-Salpeter equation is then explicitly solved in co-ordinate space. The most striking feature of the result is that the term in g^2 in the amplitude of the allowed processes (associated with graphs with an odd number of exchanged mesons) gets reduced in the zero momentum transfer limit by a factor $\frac{3}{4}$ compared with the lowest order. However, the amplitude of the charge retention process $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$, generated by ladder graphs with an exchange of an even number of vector mesons, is found to be of the order of g^4 . One is tempted to say that, the lowest order result being represented by g^2 , the sum of odd rungs graphs with $n \geq 3$ rungs gives $-\frac{1}{4}g^2$, while the sum of even rungs graphs is of the order of g^4 .

Feinberg and Pais have pointed out the importance that the above remark may have for the problem of selection rules in baryon-lepton interaction. In baryon-lepton interaction they consider the same set of ladder graphs, one of the lepton lines being replaced by a baryon one. They give some arguments in favour of the following assumption concerning the treatment of the baryon line :

- 1) in the baryon propagator the mass of the baryon can be taken to be zero to leading order in g ;

and also they have considered, in order to illustrate what may happen in semi-leptonic processes, a model in which :

- 2) the influence of strong interactions on the baryon propagator and on the baryon-W vertices is neglected.

Then, for a W-baryon interaction of the type $\bar{\psi} i \gamma_{\mu} (a + b \gamma_5) \psi$, apart from the complication involving the different types of baryons, the theory developed for the pure leptonic processes can be extended to the semi-leptonic processes. Most of the selection rules which have been proposed in the theory of weak interactions of strange particles are only valid in the lowest order. For instance, if $\Delta S = -\Delta Q$ decays are forbidden in the lowest order, ladder graphs with an odd number of exchanged meson ≥ 3 can generate $\Delta S = -\Delta Q$ amplitude. Since in pure leptonic processes the sum of such ladder graphs gives a contribution comparable to lowest order, Feinberg and Pais, being aware of the complications involving the different kinds of baryons, conjectured however that the $\Delta S = -\Delta Q$ selection rule may be violated to the order g^2 although the Hamiltonian does not contain any $\Delta Q = -\Delta S$ current. In the present paper we show that this conjecture is not correct in the particular model of baryon-lepton reactions which we have just described. In order to take simultaneously into account the $\Delta S = 0$ and $|\Delta S| = 1$ interactions, we introduce a matrix that describes the full set of these interactions. On this matrix, put in diagonal form, we apply the peratization technique of Feinberg and Pais and show that if only one intermediate boson field is present, the matrix obtained by summing the ladder graphs is simply proportional to the lowest order matrix, neglecting contributions of the order of g^4 . Although there exists an infinite series of ladder graphs with $\Delta S = -\Delta Q$ or $|\Delta S| = 2$, the summation of the series amounts to a modification of the allowed amplitude but does not generate any forbidden amplitude of order g^2 . This result, somewhat paradoxical, is due to the fact that one has to deal with geometrical divergent series. In Appendix II we shall give a very simple mathematical example where the same kind of paradox occurs.

Unfortunately, we have not been able to take into account the strong interaction corrections to these processes and for that reason, among others, the result should still be considered more as a conjecture than anything definite : it shows, however, that the modification brought into the experimental predictions of the theory of weak interactions by the work of Feinberg and Pais might be somewhat less drastic than was first believed, and this is probably satisfactory.

II. A MATRIX FORMALISM FOR LEPTONIC PROCESSES

Let us briefly consider the purely leptonic processes and let us translate the main steps of the Feinberg and Pais procedure into a matrix language, which is more convenient for the generalization to baryon-lepton processes.

As an example, let us consider the reactions between the two lepton pairs $(\nu_\mu e^-)$, $(\nu_e \mu^-)$ having the following quantum numbers :

electric charge : $Q = -1$

leptonic charge : $L = 2$

leptonic spin : $L_3 = 0$.

[We recall that leptonic spin is defined so that :

$$L_3 = \frac{1}{2} \text{ for } e^-, \nu_e$$

$$L_3 = -\frac{1}{2} \text{ for } \mu^-, \nu_\mu]$$

The recurrence formula for the ladder graphs is given by

$$m_{ij}^{(n+1)} = \sum_{\ell} G_{i\ell} \int K m_{\ell j}^{(n)} \quad (1)$$

where $G_{i\ell}$ K is connected with the lowest order interaction term between the two leptons in the manner shown in Fig. 1.

Fig. 1

i, j and l label the different lepton pairs. In our particular example $i, j, l = 1, 2$ with $(1) = (\nu_\mu e^-)$, $(2) = (\nu_e \mu^-)$; G_{il} is just a product of coupling constants and

$$G_{11} = G_{22} = 0 \quad ; \quad G_{12} = G_{21} = g^2 \quad (2)$$

The exact form of the kernel K is easily derived from Fig. 1. Making the summation over n one then defines

$$m_{ij} = \lim_{N \rightarrow \infty} \sum_{n=1}^N m_{ij}^{(n)} \quad (3)$$

Summing the two terms of (1) one then gets the following set of coupled integral equations

$$m_{ij} = m_{ij}^{(1)} + \sum_l G_{il} \int K m_{lj} \quad (4)$$

with

$$m_{ij}^{(1)} = G_{ij} \Lambda_j^{(1)} \quad (5)$$

$M^{(1)}$ is just the lowest order contribution which is given by

$$M^{(1)}(p'_1, p'_2; p_1, p_2) = -i \gamma_\mu^{(1)} (1 + \gamma_5^{(1)}) \gamma_\nu^{(2)} (1 + \gamma_5^{(2)}) \left(\delta_{\mu\nu} + \frac{q_\mu q_\nu}{m^2} \right) \frac{1}{q^2 + m^2} \quad (6)$$

with $p'_1 - p_1 = q$.

Equation (4) is identical to the system of equations (4.14) of Ref. ¹⁾ where

$$\begin{aligned} M^{odd} &= m_{12} = m_{21} \\ M^{even} &= m_{11} = m_{22} \end{aligned} \quad (7)$$

Written in matrix form, Eq. (4) reads

$$m = m^{(1)} + G \int K m \quad (8)$$

where $G, m, m^{(1)}$ are two-by-two matrices with

$$m^{(1)} = G M^{(1)} \quad (9)$$

and

$$G = g^2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (10)$$

In order to solve Eq. (8), it is convenient to diagonalize the matrix G . We therefore define

$$G' = S G S^{-1}; \quad m^{(1)'} = S m^{(1)} S^{-1}; \quad m' = S m S^{-1} \quad (11)$$

$\Delta S = 0$ and $\Delta S = \pm 1$ with $\Delta S = \Delta Q$. For the sake of simplicity we first assume that the form of these currents is universal $\overline{\psi}_\mu (1 + \gamma_5)$ everywhere; this restriction is not essential and the more general case is treated in Appendix I. We also consider baryon-lepton pairs having a well-defined value for each of the quantum numbers B (baryonic), L (leptonic), L_3 (leptonic spin) and Q (electric charge), for instance, $B = Q = -L = 1$ and $L_3 = -\frac{1}{2}$. The lowest order term is given in matrix form by

$$\mathcal{M}^{(1)} = G M^{(1)} = -G (\gamma_\mu^{(1)} (1 + \gamma_5^{(1)}) \gamma_\nu^{(2)} (1 + \gamma_5^{(2)}) \Delta_{\mu\nu}(q) \quad (29)$$

where G is the matrix

$$\begin{matrix} & \begin{matrix} (ne^+) & (\Lambda e^+) & (\Sigma^0 e^+) & (\Xi^0 e^+) & (p \bar{\nu}_e) & (\Sigma^+ \bar{\nu}_e) \end{matrix} \\ \begin{matrix} (ne^+) \\ (\Lambda e^+) \\ (\Sigma^0 e^+) \\ (\Xi^0 e^+) \\ (p \bar{\nu}_e) \\ (\Sigma^+ \bar{\nu}_e) \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 & g^2 & 0 \\ 0 & 0 & 0 & 0 & gg' & gg'' \\ 0 & 0 & 0 & 0 & gg''' & \sqrt{2}g^2 \\ 0 & 0 & 0 & 0 & 0 & gg^{iv} \\ g^2 & gg' & gg''' & 0 & 0 & 0 \\ 0 & gg'' & \sqrt{2}g^2 & gg^{iv} & 0 & 0 \end{pmatrix} \end{matrix} = G \quad (30)$$

With the help of hypotheses 1) and 2) the derivation of the integral equation goes along the same line as in the lepton case. The scattering amplitude $\mathcal{M}$ is again given by Eqs. (8) and (9) where now $\mathcal{M}$ is a 6-by-6 matrix. As in the leptonic case, we diagonalize the matrix G by an orthogonal transformation. The orthogonal matrix S that makes the transformation could easily have been calculated, but this is not necessary for our purpose. In fact, what we need is just a proof of its existence which - G being symmetrical - follows indeed from general theorems²⁾. Equation (8) then takes the form (13) from which follows again that

$\mathcal{M}'$, given by (11), is also diagonal. The diagonal elements of $\mathcal{M}'$ are given in terms of the eigenvalues G'_{ii} of G by Eq. (16). We apply next the "peratization" prescription. The diagonal elements of $\mathcal{M}'$ are then given by Eqs. (17), (20) and (21). Comparing (20) to the transform of (29) one sees that here again $\mathcal{M}^{(1)'}_{\mu\nu}$ and $\mathcal{M}'_{\mu\nu}$ are proportional matrices ^{*}). Neglecting again contributions of the order of g^4 , we can transform back the matrix Eq. (23) equivalent to Eq. (20) to the original representation by means of S ; one gets, writing directly the result for $\mathcal{M}$:

$$\mathcal{M}(q) = G F_{\mu\nu}(q) \gamma_{\mu}^{(1)} (1 + \gamma_5^{(1)}) \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}) \quad (31)$$

which is to be compared with the lowest order result

$$\mathcal{M}^{(1)}(q) = G \Delta_{\mu\nu}(q) \gamma_{\mu}^{(1)} (1 + \gamma_5^{(1)}) \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}) \quad (32)$$

The "peratization" prescription in the low energy limit, ignoring the contribution of the order g^4 can be stated in this way :

Keep only the lowest order graphs but replace the usual vector boson propagator $\Delta_{\mu\nu}(q)$ by

$$F_{\mu\nu}(q) = \frac{-i}{q^2 + m^2} \left[\frac{3}{4} \delta_{\mu\nu} + \frac{1}{m^2} (q_{\mu} q_{\nu} - \frac{1}{4} \delta_{\mu\nu} q^2) \right]$$

^{*}) The determinant of the matrix G is zero, which means that at least one (in this case only one) of the eigenvalues G'_{ii} of this matrix is zero. This fact, however, as is easily seen from an equation analogous to (19), does not alter the proportionality of the matrices $\mathcal{M}'$ and $\mathcal{M}^{(1)'}_{\mu\nu}$.

The result implies, in particular, that the selection rules $|\Delta S| = 1$, $\Delta S = +\Delta Q$ valid in lowest order perturbation theory remain valid to order g^2 in the "peratization" theory, if one accepts hypotheses 1) and 2) stated in the introduction.

IV. A FEW REMARKS ABOUT THE BOSON-LEPTON CASE

This section is devoted to a few remarks concerning the boson-lepton reactions. This case seems to be more complicated unless one assumes a universal boson-W interaction of the type

$$i \left[\frac{\partial \phi'}{\partial x_\mu} \phi - \phi' \frac{\partial \phi}{\partial x_\mu} \right] w_\mu \quad (33)$$

for both $\Delta S = 0$ and $|\Delta S| = 1$ interactions, where ϕ and ϕ' are strongly interacting boson fields. With assumptions similar to hypotheses 1) and 2) of the introduction, one can easily extend the peratization method to the boson-lepton reactions. The prescription, the same as in the baryonic case, can be stated as follows : in order to obtain the g^2 contribution to the reaction amplitude in the low energy limit (g^4 terms, etc., are neglected), keep only the lowest order graph and replace the usual vector boson propagator $\Delta_{\mu\nu}(q)$ by $F_{\mu\nu}(q)$ given by Eq. (21). This result is very satisfactory from the point of view of the CVC theory because it shows that this theory can remain meaningful when higher order effects are treated by the peratization method of Feinberg and Pais. However, as far as the selection rules problem is concerned, the situation remains the same as in the baryon case, i.e., the selection rules valid in lowest order perturbation theory (example $\Delta S = \Delta Q$) remain valid to the g^2 order.

It could happen that the fundamental interaction of the W's with bosons is not the form (33) but is more general. In this case the generalization of the Pais-Feinberg formalism seems to us to give rise to some difficulties. We were unable to solve the general problem in a satisfactory manner and for such a general type of W-boson interactions we cannot say anything definite about the validity of the CVC theory and the problem of the selection rules.

V. CONCLUSIONS

In the present paper an attempt has been made to extend the basic ideas of the peratization technique to semi-leptonic processes, in what seems to us the most straightforward and simple way. This procedure leads to definite conclusions which can be stated as follows :

a) the processes which are "allowed" by an exchange of a single W are subject, in the limit of q^2 small, to a kind of renormalization of the corresponding coupling constants by a factor $\frac{3}{4}$. This result is quite general. It is true for transitions $\Delta S = \pm 1$ with $\Delta S = \Delta Q$ just as well as for transitions $\Delta S = 0$. Thus for such reactions the higher order graphs give a contribution already to the lowest order term.

b) the processes which cannot occur through the exchange of a single W remain "forbidden" after the peratization is performed; i.e., they are of the order g^4 and therefore much smaller.

This means, in particular, that if one starts with a Hamiltonian involving no $\Delta S = -\Delta Q$ current, the transitions of this type remain negligible. The same applies to the transitions $|\Delta S| = 2$ (no $\Xi \rightarrow N e \nu$ decays).

c) The possibility of applying the peratization technique to the non-leptonic processes is, of course, even more questionable. If one does, however, and if one simply neglects all the strong interaction corrections, then the above procedure can be repeated and leads to similar conclusions. If therefore the basic Hamiltonian of the weak interactions involves no $|\Delta S| = 2$ term, the $|\Delta S| = 2$ processes remain forbidden (no $\Xi \rightarrow N \pi$ decay; K_1^0, K_2^0 mass difference comparable with $\Gamma_{K_1^0}$). Similarly if the basic Hamiltonian is a component of an isospinor, the $|\Delta \vec{I}| = \frac{1}{2}$ remains valid.

Most of these conclusions look quite natural and also - we think in particular of the concept of universality and of the CVC theory - rather gratifying in view of the experimental data at hand. One of our conclusions, however, namely that no

appreciable $\Delta S = -\Delta Q$ interaction is induced by the peratization procedure, may look surprising for - as already pointed out by Feinberg and Pais - the fact that for $\Delta S = \Delta Q$ processes the graphs with several W 's contribute to the lowest order term would naturally lead to the suspicion that a very similar phenomenon might give some appreciable $\Delta S = -\Delta Q$ contribution, independently of whether the basic Hamiltonian contains such terms or not.

From the present analysis it seems, however, that this is not the case. Whether this is a disappointing or a gratifying aspect of the peratization method will of course depend on the results of experiments now in progress and, for the time being, it seems too early to make any definite statement.

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After completion of this work, Professor A. Pais has kindly informed us that he and Professor G. Feinberg have reached conclusions similar to ours.

A P P E N D I X I

In Section III we have considered the semi-leptonic processes under the assumption that all baryonic currents are of the type $\bar{\psi}_i \gamma_\mu (1 + \gamma_5) \psi_k$. Here we want to consider the more general case of currents

$$\bar{\psi}_i [g_{ik} \gamma_\mu + h_{ik} \gamma_\mu \gamma_5] \psi_k$$

for a transition from a state k to i . The Born term is given by the matrix

$$M_{ij}^{(1)} = [G_{ij} \gamma_\mu^{(1)} (1 + \gamma_5^{(1)}) \gamma_\nu^{(2)} (1 + \gamma_5^{(2)}) + H_{ij} \gamma_\mu^{(1)} (1 - \gamma_5^{(1)}) \gamma_\nu^{(2)} (1 + \gamma_5^{(2)})] M_{\mu\nu}^{(1)}(q) \quad (A.1)$$

where G_{ij} and H_{ij} are the matrices for left-handed and right-handed coupling constants analogous to the matrix G defined in Eq. (30). The system of integral equations is, of course, still of the form :

$$M_{ij} = M_{ij}^{(1)} + \sum_l \int (G_{il} K^L + H_{il} K^R) M_{lj} \quad (A.2)$$

The approximate kernels K^L and K^R are :

$$K^L = (1 + \gamma_5^{(1)}) (1 - \gamma_5^{(2)}) A \quad (A.3)$$

where A contains no Dirac matrices. The solution of the integral equations (A.2) with the kernels (A.3) is given in matrix form by :

$$M = M_{\mu\nu}^L \gamma_\mu^{(1)} (1 + \gamma_5^{(1)}) \gamma_\nu^{(2)} (1 + \gamma_5^{(2)}) + M_{\mu\nu}^R \gamma_\mu^{(1)} (1 - \gamma_5^{(1)}) \gamma_\nu^{(2)} (1 + \gamma_5^{(2)}) \quad (A.4)$$

where

$$\mathcal{M}_{\mu\nu}^L = G M_{\mu\nu}^{(1)} + 4G \int A \mathcal{M}_{\mu\nu}^L \quad (\text{A.5})$$

and

$$\mathcal{M}_{\mu\nu}^R = H M_{\mu\nu}^{(1)} + 4H \int A \mathcal{M}_{\mu\nu}^R \quad (\text{A.6})$$

satisfy two decoupled linear integral equations; this follows from the projection properties of $1 \pm \gamma_5$.

The matrices $\mathcal{M}^L$ and $\mathcal{M}^R$ can now be diagonalized separately by two different unitary matrices S^L and S^R . The explicit solutions in co-ordinate space are given by expressions similar to (18). The argument of Section III is then separately applied to $\mathcal{M}^L$ and $\mathcal{M}^R$, the result obviously being in momentum space :

$$\begin{aligned} \mathcal{M}^L &= G F_{\mu\nu}(q) \gamma_\mu^{(1)} (1 + \gamma_5^{(1)}) \gamma_\nu^{(2)} (1 + \gamma_5^{(2)}) \\ \mathcal{M}^R &= H F_{\mu\nu}(q) \gamma_\mu^{(1)} (1 - \gamma_5^{(1)}) \gamma_\nu^{(2)} (1 + \gamma_5^{(2)}) \end{aligned} \quad (\text{A.7})$$

which is the generalization of Eq. (31).

The discussion at the end of Section III and conclusions a) and b) [Section IV] remain of course unchanged.

A P P E N D I X II

We would like to give a simple mathematical example of the kind of paradox which occurs in connection with the summation of divergent ladder graphs with a number of exchange mesons $n \geq 3$.

Let us consider the series $M^{(1)} + M^{(2)} + \dots + M^{(n)} + \dots$ where $M^{(n)}$ for $n > 1$ is the divergent integral :

$$M^{(n)} = \int_{-1/2}^{1/2} \lambda^n (1 + \delta(x)) \left(\frac{a + ib}{x} \right)^{n-1} dx$$

λ is a very small but finite number and $a+ib$ is an arbitrary complex number. If one is able to give a meaning to the sum $M^{(2)} + \dots + M^{(n)}$ one is tempted to say that the result should be a complex number. We can see very easily that it is not the case. The series $M = M^{(1)} + \dots + M^{(n)}$ is a geometrical series which can be readily summed. The sum is a finite number which is given to the order λ^2 by :

$$M = \sum_{n=1}^{\infty} M^{(n)} = \int_{-1/2}^{1/2} \frac{\lambda (1 + \delta(x))}{1 - \lambda \frac{a+ib}{x}} dx = \lambda + O(\lambda^2)$$

which is to be compared with the first order result :

$$M^{(1)} = \int_{-1/2}^{1/2} \lambda (1 + \delta(x)) dx = 2\lambda$$

Although each term of the formal series expansion of $M - M^{(1)}$ is a complex number, the sum is a real number when terms of the order λ^2 are neglected.

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