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EXTERNAL SOURCES IN GAUGE THEORIESDavid G. Boulware ^{*)} and S. Deser ⁺⁾

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A B S T R A C T

The notion of prescribed external sources is discussed for gauge fields. Such sources necessarily obey conservation laws which generally involve the fields themselves. The implications of these conservation requirements are analyzed for classical and quantized vector multiplets (such as the Yang-Mills field) and for (classical) general relativity. A number of the vector source components may be prescribed independent of the field. The remaining components are those coupled to the gauge parts of the field, and may be eliminated by a suitable choice of gauge. On the other hand, although the gravitational conservation laws also leave a number of source components unconstrained, the rest depend non-locally in time on the gravitational field variables. It is shown, however, that there exists a choice of co-ordinate frame in which these non-local components do not appear explicitly in the theory.

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I. EXTERNAL CURRENTS IN ELECTRODYNAMICS

The notion of a classical, arbitrarily prescribed external current $J^\mu(x)$ acting on, but not affected by the electromagnetic fields it generates is an extremely valuable one in both classical and quantum electrodynamics. Viewed as the macroscopic limit of a system of many heavy charges, such a current is affected arbitrarily little by the Maxwell field, the only restriction on it being that the microscopic conservation law, $\partial_\mu J^\mu = 0$ remains valid for the macroscopic sum of charges as well. Aside from this requirement, the function $J^\mu(x)$ may be assigned arbitrarily throughout space-time. Clearly, charge conservation is necessary in order to satisfy the identity

$$\partial_\mu J^\mu = \partial_\mu F^{\mu\nu},_{\nu} \equiv 0. \quad (1)$$

Equation (1) is a single condition to be fulfilled by the four J^μ components. It is not surprising that a source of A_μ has only three independent components, since, as is well known, A_μ itself is only defined physically to within its longitudinal parts. The corresponding redundancy in J^μ , then, is expressed by the fact that its longitudinal part, in a given Lorentz frame, is fixed by the conservation law as a function of the other components ¹⁾

$$\partial_\mu J^\mu = \partial_0 J^0 + \nabla^2 J^L = 0, \quad J^L = \mathcal{D} \partial_0 J^0. \quad (2)$$

Here we have introduced the breakup $J^k = J^{kT} + \nabla_k J^L$ where $\nabla_i J^{iT} = 0$ so that $J^L = -\mathcal{D} \nabla_i J^i$. Actually, then, the three components

2.

of J^μ , (J^0, J^{kT}) , are arbitrarily prescribable and J^L is then fixed by $\partial_0 J^0$.

The restriction (2) is independent of the state of the Maxwell field, of course, and one may alternately consider J^μ as the projection of four unrestricted components $\mathcal{J}^\mu$ where

$$J^\mu = (\delta^\mu_\nu + \nabla^\mu \partial_\nu) \mathcal{J}^\nu \equiv P^\mu_\nu \mathcal{J}^\nu \quad (3)$$

and P is a purely numerical operator. In terms of $\mathcal{J}$, the coupling $\int A J dx$ may be written as $\int dx A_\mu P^\mu_\nu \mathcal{J}^\nu$. Thus, when the transition to a quantized Maxwell field is made, the notion of a c-number external J^μ is completely undisturbed and is very useful both in discussing the theory of measurement for the field and in examining the response of the system to external sources. Since J^μ commutes with all operators, it may be varied and manipulated freely.

II. EXTERNAL CURRENTS IN NON-ABELIAN GAUGE THEORIES ²⁾

The familiar situation described in Section I. is profoundly altered when the gauge invariance becomes more complicated. We discuss first the general case of vector boson families. The formalism is identical in all cases ³⁾ (one may think of the Yang-Mills ⁴⁾ field for concreteness). The invariance group here is non-Abelian and as a result the external current is subject to a quite different sort of conservation requirement.

a) Classical Theory

The equation satisfied by the source ⁵⁾ J^μ is obtained from the field equation

$$(\partial_\nu - i't\phi'_\nu) G^{\mu\nu} = J^\mu \quad (4)$$

by applying the "covariant differentiation" operator $(\partial_\mu - i't\phi'_\mu)$ to it

$$\begin{aligned} (\partial_\mu - i't\phi'_\mu)(\partial_\nu - i't\phi'_\nu) G^{\mu\nu} &= \frac{i}{2} f^2 G^{\mu\nu} t G_{\mu\nu} = 0 \\ &= (\partial_\mu - i't\phi'_\mu) J^\mu \end{aligned} \quad (5)$$

Thus, the "covariant" conservation law for J^μ now contains the Yang-Mills field ϕ_μ explicitly ⁶⁾, and the external current must depend on the state of the field. It would therefore seem to be no longer specifiable a priori. To see in detail how much freedom is left in prescribing J^μ , we employ once more the transverse-longitudinal breakup for J^k . The equation satisfied by J^L is then

$$(\vec{\nabla} - i't\vec{\phi}') \cdot \vec{\nabla} J^L = -[(\partial_0 - i't\phi'_0) J^0 + (\vec{\nabla} - i't\vec{\phi}') \cdot \vec{J}^T] \quad (6)$$

In terms of the Green's function $D_\phi(r, r')$ defined by

$$(\vec{\nabla} - i't\vec{\phi}') \cdot \vec{\nabla} D_\phi(\vec{r}, \vec{r}') = -\delta(\vec{r} - \vec{r}') \quad (7)$$

4.

we may write

$$J^L = \mathcal{D}_\varphi [(\partial_0 - i' t \varphi_0') J^0 + (\vec{\nabla} - i' t \vec{\varphi}') \vec{J}^T] . \quad (8)$$

Alternately J^μ will be manifestly conserved if it has the form,

$$J^\mu = P_\varphi^\mu{}_\nu J^\nu$$

$$P_\varphi^\mu{}_\nu = \delta^\mu_\nu + \nabla^\mu \mathcal{D}_\varphi (\partial_\nu - i' t \varphi_\nu') . \quad (9)$$

The projection operator P_φ depends explicitly on φ , and J^ν is entirely arbitrary. Equation (9) expresses the fact that J^T and J^0 may still be given arbitrarily, but that J^L now depends on both these quantities and on φ_μ . The number of independent components of J^μ (3 isovectors in the Yang-Mills case) corresponds, as in electrodynamics, to that of the physical components of the φ -field. However, the redundant component J^L now depends (through φ) on the state of the system.

This dependence enters both in the replacement of ∂_μ by $\partial_\mu - i' t \varphi_\mu'$ and in the change from the Coulomb Green's function $\mathcal{D}(r - r')$ to that in the field $\vec{\varphi}$, $\mathcal{D}_\varphi(r, r')$. The latter quantity is well defined in the classical case for well-behaved $\vec{\varphi}$, despite the lack of commutativity of the t matrices.

b) Quantized Theory

The quantization of a non-Abelian field involves a number of delicate questions concerning operator orderings ⁷⁾. We shall only discuss them here as they affect our specific problem. In this connection, since quantization performed in gauges differing by a q-number may not be equivalent, we restrict ourselves to the radiation gauge, $\nabla_\mu \varphi^\mu = 0$. Clearly, we must be resigned to having at least J^L a q-number since it depends on φ . Further, since φ_k and φ_0 fail to commute at equal times, difficulties could arise in solving for J^L itself.

The quantum form of the classical field Eq. (4), in which the $\varphi_\nu' G^{\mu\nu}$ product is taken to be symmetrized, is actually inconsistent ⁷⁾. In addition $(\partial_\mu - i\varphi_\mu')((\partial_\nu - i\varphi_\nu')G^{\mu\nu})$ is no longer identically zero, (where $\partial_\mu - i\varphi_\mu'$ must be symmetrized with $(\partial_\nu - i\varphi_\nu') \cdot G^{\mu\nu}$) due to the appearance of terms of the form

$$[\varphi_\mu', [\varphi_\nu', G^{\mu\nu}]]$$

which are of order $\hbar^2$ (note that the commutators here refer to the operators, not the φ matrices). A consistent field equation is obtained, however ⁷⁾, upon addition of a certain term $\hbar \frac{\delta}{\delta \varphi}$ to the left side of Eq. (4). This term, also of order $\hbar^2$, is dictated by the requirement of Lorentz covariance and is just such as to reinstate the operator equivalent of the identity (5). Thus, the conservation law for J^μ is unchanged, so that we still have Eq. (6), but with an operator φ .

It should be noted that we do not know a priori that J^{kT} and J^0 can still be c-numbers; however, we shall assume this and check the consistency of the assumption afterwards. While the existence of an operator $\mathcal{D}\varphi$ satisfying Eq. (7) has not been established, we make

the reasonable assumption that there is one. In any case, this hypothesis is necessary for the possibility of radiation gauge quantization. The remaining problem then is that of the lack of commutativity of $\vec{\varphi}$ and φ_0 which might invalidate Eq. (8) as a solution of the J conservation law. Fortunately, one can show using the constraint equation for φ^0 , that :

$$[\vec{\varphi}(\vec{r}), [\vec{\varphi}(\vec{r}'''), \varphi^0(\vec{r}''')]] = 0. \quad (10)$$

Equation (10) may then be used to establish that Eq. (8) remains a solution, where now $\mathcal{D}_\varphi$ is symmetrized as a unit with φ_0 .

We may then write the projection operator form of J^μ

$$J^\mu = P_\varphi^{\mu\nu} \mathcal{J}^\nu \quad (11)$$

where $P_\varphi^{\mu\nu}$ is identical, apart from appropriate symmetrization, with its classical form Eq. (9) and $\mathcal{J}^\nu$ are four unrestricted c-numbers. Since $(\partial_\mu - i't\varphi'_\mu) J^\mu$ now correctly vanishes as an operator equation, our assumption that J^{kT} and J^0 were c-numbers is verified.

The above results are quite satisfactory, since only J^{kT} and J^0 are coupled to the physically significant amplitudes φ^{kT} and φ^0 . Further, while the equation of motion

$$(\partial_\nu - i't\varphi'_\nu) G^{\mu\nu} + t_\varphi^\mu = P_\varphi^{\mu\nu} \mathcal{J}^\nu \quad (12)$$

now has an operator source in any gauge, the coupling term in the action $\int (dx) \varphi_{\mu} P_{\varphi}^{\mu} \mathcal{J}^{\nu}$ just reduces to $\int (dx) \varphi_{\mu} \mathcal{J}^{\mu}$ in the radiation gauge where $\nabla_{\mu} \varphi^{\mu} = 0$. This does not mean, however, that all operator aspects of the current vanish in this gauge, since the reduced Lagrangian only yields the transverse and time components of Eq. (12). The operator dependence of J^{μ} reappears upon computing the longitudinal part of Eq. (12). This latter part is found by solving the constraint equation $(\nabla_k - i' t \varphi'_k) G^{ok} = J^0$ (obtained by varying φ_0) for G^{okL} and taking its time derivative. One then finds

$$\partial_0 G^{okL} - (i' t \varphi'_0 G^{ok})^L - \left[(\partial_0 - i' t \varphi'_0) G^{ek} \right]^L + \mathcal{L}_{\varphi}^{kL} = J^{kL}.$$

To summarize, we have found that for quantized non-Abelian vector fields, the notion of external source is still usefully defined, despite the operator nature of the conservation law involved. More precisely, three components of the current may be arbitrarily prescribed and these are coupled to the three physical components of φ . The properties of J^{μ} are conveniently summarized by the equation $J^{\mu} = P_{\varphi}^{\mu} \mathcal{J}^{\nu}$, $\mathcal{J}^{\nu}$ arbitrary. Further, in radiation gauge, we may consider the non-Abelian (and also, of course, the Maxwell) field to be coupled to four independent c-number functions, which are denoted by $\mathcal{J}^{\nu}$, in such a way that the longitudinal part and its variations never appear in the theory. The functions J^T and $J^0 = \mathcal{J}^0$ constitute useful local probes of the system, in that the coefficients of their variations are local observables describing its response. The longitudinal part of J^k is the "source" of the longitudinal part of the field equations which are derived from the constraint equation for G_{oi}^L . There, it automatically appears as the appropriate function of J^T , J^0 and the field variables.

$$\tau^{0i} + \tau^{00} = \nabla^i \tau^{0L} + \tau^{00} = 0 \quad (18b)$$

just express the four components τ^{0L} and τ^i in terms of the remaining six, τ^{ijT} , τ^{oiT} and τ^{oo} (actually, τ^{ijT} is not even involved). Note that in the τ^i equation, there appears $\tau_{,0}^{0L}$, which in the linearized case is just τ^{00} , oo -dependent. Accordingly, one could vary the six unconstrained quantities independently in the coupling term, $\int (dx) \tau^{\mu\nu} g_{\mu\nu}$. Further, the transverse parts, which couple to the dynamical parts of the spin 2 field, appear only through $\int dx \tau^{ijT} g_{ijT}$ owing to the orthogonality of transverse and "LT" tensors when integrated. However, the linearized theory is not co-ordinate invariant, and the $\int \tau$ terms in the full theory alter the whole analysis. For, not only do the equations become non-linear through the appearance of the constraint variables themselves in $\int \tau$, but, what is worse, non-local in time. This means that the four constraint variables (which may still be taken to be τ^{0L} and τ^i as in the linearized case) are determined as functions of the metric and the other source components at all times. This time non-locality is an essential part of the theory. As an example, consider the motion (i.e., time-development) of an arbitrary point source; it is completely determined by $\tau^{\mu\nu}_{;\nu} = 0$. Thus, if one has a source of the form $\tau^{\mu\nu} = \int t^{\mu\nu}(s) \delta(x-x(s)) ds$, an a priori arbitrary function, the conservation law implies 8) first that $t^{\mu\nu} = v^\mu v^\nu$ and secondly the geodesic equation

$$\frac{dv^\mu}{ds} + \Gamma_{\alpha\beta}^\mu v^\alpha v^\beta = 0. \quad (19)$$

The solution of Eq. (19) clearly depends on the field along the whole trajectory of the particle. To exhibit most simply the implications of the conservation laws, we may use Gaussian normal co-ordinates, $g_{oi} = 0$, $g_{oo} = -1$. Then the only surviving components of $\Gamma_{\mu\nu}^\alpha$ are $\Gamma_{ij}^0 = \frac{1}{2} g_{ij,0} = \Gamma_{oj}^k g_{ki}$ and Γ_{jk}^i so that Eqs. (16) now read

$$\tau^{0i}_{,i} + \tau^{00}_{,0} + \frac{1}{2} g_{ij,0} \tau^{ij} = 0 \quad (20a)$$

$$\tau^{ij}_{,j} + \tau^{0i}_{,0} + \Gamma^i_{jk} \tau^{jk} + g^{ik} g_{ik,0} \tau^{0j} = 0, \quad (20b)$$

This coupled system of equation for τ^{0L} and τ^i is parabolic in time due to the $\tau^{0i}_{,0}$ term in Eq. (20b). When Eq. (20a) is solved for τ^{0L} ,

$$\tau^{0L} = \mathcal{D} \left[\frac{1}{2} g_{ij,0} \tau^{ij} + \tau^{00}_{,0} \right] \quad (21)$$

and this result inserted in Eq. (20b), time derivatives of τ^{ij} and of $g_{ij,0}$ appear. The former are responsible for the parabolic character of the equation; the latter yield non-linear terms in τ^i when the field equations (13) are used to eliminate $g_{ij,00}$. Restricting oneself to weak external sources does not resolve the problem, although it does of course remove the explicit non-linearity. However, the time non-locality persists in any case and cannot be removed by taking a static $\tau^{\mu\nu}$, unless the metric is also static. In fact, as can be seen from Eqs. (20), a static $\tau^{\mu\nu}$ is not even consistent with a general non-static metric.

Although the above discussion involved a particular frame, the time non-locality and non-linearity would persist in other frames. In particular, the term $\Gamma^0_{ij} \tau^{ij}$ in Eq. (16b) cannot be removed since the Γ^0_{ij} are part of the Cauchy data for the gravitational field⁹⁾.

In the case of the vector fields, it was found that the constraint variables could be completely eliminated from the action by appropriate choice of gauge. In order to see whether, analogously, a co-ordinate frame may be found such that the constrained variables do not appear in the Einstein action, we make the orthogonal decomposition of the coupling term

$$\int (dx) g_{\mu\nu} \tau^{\mu\nu} = \int (dx) \left\{ g_{00}^T \tau^{00} + 2 g_{0i}^T \tau^{0i} + g_{ij} \tau^{ij} - 2 g_{0i,j} \tau^{ij} - 2 g_{0i,j} \tau^{0j} \right\}. \quad (22)$$

A choice of spatial co-ordinates which is quite analogous to radiation gauge, namely ¹⁰⁾ $g_{ij,j} = 0$ indeed removes the τ^{ij} term. Similarly, one could choose as the time co-ordinate condition $g_{0i,i} = 0$ to eliminate the τ^{0i} term as well. The latter condition is formally compatible with Minkowskian co-ordinates asymptotically. Thus, in this frame, the dependent variables do not appear explicitly in the action.

To understand the qualitative difference between the constraints on the vector sources and those on an external stress energy tensor for general relativity, it is instructive to consider further the point source. In this case, as we have seen, there are only four (rather than ten) a priori independent functions determining $\tau^{\mu\nu}$, which are then all constraint variables, determined by the geodesic Eq. (19). Viewed as a coupled system of dynamics equations for gravitational and matter variables, Eqs. (19) and (13) are time local. One can then specify Cauchy data for the gravitational field and the matter variables on any given space-like surface; their evolution in time is determined by the equations of motion (13) and (19). From the point of view of external sources, however, the four equations (19) specify the constraint variables $\tau^{\mu\nu}$ as functionals of the metric which are non-local in time.

When the corresponding $T^{\mu\nu} = \int v^\mu v^\nu \delta(x-x(s))ds$ is inserted into the field equations (13), the latter become a highly non-local system of equations for the metric ¹¹⁾.

Having seen that the introduction of an external stress-tensor in general involves loss of time locality for the system when the covariant conservation law is taken into account, it may be concluded that the general usefulness of the prescribed source concept is lost for the gravitational field. It should perhaps be restated that in general there are still six unconstrained components allowed in the $T^{\mu\nu}$. Varying these arbitrarily will still yield the response of the gravitational system, but in terms of its whole history rather than of its local properties, so that one would be led to problems beyond the framework of local field theory. Our approach here has, of course, been entirely within classical theory; the corresponding problem in a quantum treatment would be, needless to say, still more formidable ¹²⁾.

There exists a formulation of general relativity whose structure is particularly close to the non-Abelian vector situation. The variables here are "vierbeins", a set of four vector fields defined throughout space-time, and the metric is a quadratic function of these. In addition to co-ordinate invariance there is a further invariance in this description with respect to local Lorentz relations of the vierbeins. The Appendix discusses the external source problem in vierbein language. The external source must satisfy "conservation laws" due to both co-ordinate and local Lorentz invariance. Either of those requirements turns out to be separately soluble, but taken simultaneously they lead once more to the difficulties encountered in the purely metric formulation of this section.

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A P P E N D I X

The gravitational field equations expressed in terms of vierbein variables $e_{a\mu}$ and $\omega_{\mu ab}$ ($e_{a\mu} e^a_\nu = g_{\mu\nu}$) (a ranges from 0 to 3) are

$$G_a{}^\mu = R_a{}^\mu - \frac{1}{2} e_a{}^\mu R = t_a{}^\mu \quad (\text{A.1})$$

where

$$R_{a\mu} = (\det e_\nu) e^{c\lambda} (\omega_{\lambda ac, \mu} - \omega_{\mu ac, \lambda} - \omega_{\mu ad} \omega_{\lambda}{}^d{}_c + \omega_{\mu cd} \omega_{\lambda}{}^d{}_a) \quad (\text{A.2})$$

and

$$\begin{aligned} \omega_{\mu ab} = \frac{1}{2} \{ & e_{b\lambda} e_a{}^\lambda{}_{, \mu} - e_{a\lambda} e_b{}^\lambda{}_{, \mu} \\ & + (e_a{}^\lambda e_b{}^\sigma - e_b{}^\sigma e_a{}^\lambda) g_{\mu\sigma, \lambda} \} \end{aligned} \quad (\text{A.3})$$

Now, $G_a{}^\mu$ satisfies the algebraic identities

$$G^{\mu\nu} \equiv e^{\nu a} G_a{}^\mu \equiv G^{\nu\mu} \quad (\text{A.4})$$

and

$$g^{\mu}_{a, \mu} - \omega_{\mu ab} g^{b\mu} = 0 \quad (\text{A.5})$$

The symmetry requirement (A.4) is related to the invariance of the theory under arbitrary local Lorentz transformations of the vierbeins, and the conservation equation to invariance under co-ordinate transformations.

Correspondingly, t_a^{μ} must satisfy

$$e^{\nu 0} t_0^{\mu} = \sigma^{\mu\nu} = \sigma^{\nu\mu} \quad (\text{A.6})$$

and

$$t_a^{\mu}_{, \mu} - \omega_{\mu ab} t^{b\mu} = 0 \quad (\text{A.7})$$

Equation (A.6) can be satisfied at the price of returning to the metric formulation, where we have already noted the attendant difficulties. On the other hand, we can solve Eq. (A.7) for the longitudinal part of t_a^k in terms of its other components and an appropriately defined $\mathcal{D}_\omega$ function. However, we still must impose the symmetry requirement on the solution of (A.7). If we write it as

$$t_a^{\mu} = \sigma_0^{\mu} + \nabla_{a\omega}^{\mu} \mathcal{D}_\omega^b (\delta_b^c \sigma_c - \omega_{b c}^c) \gamma_c^{\nu} \quad (\text{A.8})$$

where γ_a^{μ} are entirely arbitrary, the conditions which (A.6) imposes on γ_a^{μ} reintroduce the time non-locality.

REFERENCES AND FOOTNOTES

- 1) We use the notation $\nabla_\mu = \partial_\mu - g_\mu^\alpha \partial_\alpha$. Greek indices range from 0 to 3, Latin indices from 1 to 3. The Coulomb Green's function is denoted by $\mathcal{L}$.
- 2) Many of the results of this section have been obtained previously by J. Schwinger, Seminar on theoretical physics, Trieste (1962), and are included here for completeness.
- 3) For a general discussion of non-Abelian gauge fields see, for example, M. Gell-Mann and S.L. Glashow, Ann. Phys. 15, 437 (1961). Different multiplet theories will be characterized by different structure constants, but our discussion never involves more than general properties common to all such fields.
- 4) C.N. Yang and R.L. Mills, Phys. Rev. 96, 191 (1954).
- 5) We use the notation of J. Schwinger, Phys. Rev. 125, 1043 (1962) and Ref. ²⁾. The gauge fields and their sources are n -component vectors in an internal space. The structure constants of the associated Lie algebra t_a , $a = \dots, n$ are $n \times n$ matrices. We write $(t_b)_{ac} \equiv t_{abc} = -t_{acb} = -t_{bac}$. All products are matrix products and we use the notation $t \cdot \varphi \equiv \sum_{a=1}^n t_a \varphi_a$. The field amplitude and associated field strength are denoted by φ_μ and $G^{\mu\nu}$ respectively, so that

$$-e f^{\mu\nu} \mathcal{L} G_{\mu\nu} = [\partial_\mu - i t \cdot \varphi_\mu, \partial_\nu - i t \cdot \varphi_\nu]$$

The commutator refers only to the t -matrices and to the differentiations, not to possible operator properties of φ which are discussed later.

- 6) The physical origin of the φ -dependence of the J "conservation" law reflects, (say for the Yang-Mills case) the fact that the φ -field itself carries isotopic spin. It is correspondingly absent in the simpler Maxwell case, whose quanta are electrically neutral. One may, of course, also define a normally conserved "current" k_μ , as the source of $\partial_\mu G^{\mu\nu}$. This "current" is the total isotopic current and hence carries the charge associated with φ as well. Its normal conservation law, $\partial_\mu k^\mu = 0$, is equivalent to the covariant conservation law of J^μ .
- 7) J.S. Schwinger, Phys. Rev. 127, 324 (1962) ;
R.L. Arnowitt and S.I. Fickler, Phys. Rev. 127, 1821 (1962).
- 8) W. Tulczyjew, Bull. Acad. Polon. Sci. Cl. III, 5, 279 (1957) ;
See also L. Infeld and S. Plebanski, Motion and Relativity (Warsaw 1960).
- 9) For a discussion of the Cauchy problem and dynamics of the gravitational field, see, for example, articles by Y. Bruhat and by R.L. Arnowitt, S. Deser and C.W. Misner in "Gravitation", L. Witten ed., (Wiley, New York 1962).
- 10) The co-ordinate conditions $g_{ij,j} = 0$ are discussed in detail in the second article of Ref.⁹⁾. The condition $g_{0i,i} = 0$ differs somewhat from the time co-ordinate specifications considered there.
- 11) Alternatively, one can specify v^μ for all time ; Eq. (19) then determines certain components of $\Gamma^\mu_{\alpha\beta}$ along the particle's trajectory and thereby selects the set of metrics compatible with the chosen trajectory.
- 12) In contrast, the inverse notion, that of using a prescribed external metric field as a probe of energy-density properties of matter can be quite useful. See for example, J.S. Schwinger, Phys. Rev. 130, 406 (1963).