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THE PREDICTION OF 1^+ , 2^- , 2^+ MESON SUPERMULTIPLETS
IN THE GEV REGION BY A SELF-CONSISTENT BOOTSTRAP MODEL

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In a recent paper by De Celles, Paton and the present author¹⁾, the consequences of the bootstrap self-consistency hypothesis were investigated in the scattering of pseudoscalar mesons under conditions equivalent to SU_3 invariance. Starting from the pseudoscalar (0^-) mesons (π, K, η) and the vector (1^-) resonances (ρ, K^*, Φ) whose existence we assumed, together with the usual assignment of quantum numbers, it was found that the bootstrap hypothesis requires, for consistency, the existence of an octet and a singlet of scalar (0^+) particles in addition to the 0^- and 1^- octets we started with. Within the framework of the model, it was found that no further supermultiplets were then required besides those mentioned and their Regge recurrences, so that the system was closed as far as the scattering of pseudoscalar mesons was concerned. The calculation of the dynamical parameters using Zachariasen and Zemach's mechanism²⁾ yields the mass values $m_V^8 = 2.33\mu$, and $m_S^8 = 1.2\mu$ ($\mu =$ degenerate pseudoscalar meson mass), and the ratio for the coupling strengths $(g_S^8)^2 / (g_V^8)^2 \sim 0.04$.

Although no scalar resonance has so far been definitely identified experimentally, the fact is not in contradiction with the predictions of the present model, for if the (ps,ps,s) coupling is indeed weak then the 0^+ resonances will have narrow widths and their production from, e.g., π -N reactions may only be a few percent of the production of vector resonances. Besides such possible candidates as the $K^*(725)$ for the 0^+ octet, one might verify the existence of such a $J = \text{even} + (\text{octet})$ Regge trajectory by looking for its higher recurrences. Our model¹⁾ predicts a 2^+ octet at energies ~ 1.5 GeV. It is interesting that the recent assignment of $J^{PG} = 2^{+-}$ ($I = 1$) to the A_2 resonance at 1320 MeV³⁾ coincides with that of the $I = 1$ members of the 2^+ octet predicted by our model¹⁾. In any case it seems difficult to avoid the conclusion on the existence of the scalar mesons if we accept the present bootstrap hypothesis and approximate SU_3 symmetry. It is the purpose of this note therefore, to investigate further the consequences if we accept the predicted 'scalar mesons' as genuine particles. We shall consider here only the 0^+ octet. The embarrassingly tight-bound 0^+ singlet has been identified by some authors³⁾ as the Pomeranchon, i.e., the ghost state with zero residue.

If so, it is not a particle and has no physical effects. The exchange of its higher Regge recurrences (e.g., the f_0) will give rise to short range forces possibly dominating the high-energy behaviour, but may be negligible for the binding of other particles. In any case we shall neglect its existence in the following discussion.

If the 0^+ octet is accepted as genuine particles, one should consider, for consistency, also their interaction with one another and with the pseudoscalar mesons. Neither of these processes had been studied in the model of Ref. 1. The mutual scattering of the scalar mesons is coupled by unitarity through production processes to the scattering of pseudoscalar mesons, while the scattering of the scalar with the pseudoscalar mesons, involving states of parity $P = (-1)^{J+1}$, is not coupled to other low-energy processes. (The processes involving vector mesons occur at higher average energies and are neglected in the present discussion). We have thus, respectively, for the partial wave states $P = (-1)^J$ and $P = (-1)^{J+1}$, S matrices of the form shown in Figs. 1a and 1b⁴⁾.

We seek a bootstrap solution completely self-consistent with respect to all the related processes in Fig. 1. Such an investigation will allow us (i) to check the validity of the results of Ref. 1 under the present more general self-consistency requirement; (ii) to determine the 'binding' or otherwise of resonances in the $P = (-1)^{J+1}$ partial wave states which did not occur for kinematical reasons in Ref. 1, and in particular (iii) to bootstrap the original 0^- octet.

A very simple solution to the problem has been found which (i) maintains the results of Ref. 1 to a good approximation within the framework of the model; (ii) bootstraps the original 0^- octet yielding a mass value consistent with the scheme; (iii) predicts the existence of a pseudovector 1^+ octet in the GeV region and (iv) excludes the occurrence of 10 , $\overline{10}$ and 27 meson supermultiplets at low energies.

We consider first the problem of Fig. 1a for partial wave states with $P = (-1)^J$. Following the usual one-particle exchange (OPE) bootstrap hypothesis, we insert as input forces all those OPE diagrams which are kinematically allowed, with any member of the three known octets 0^+ , 0^- , 1^- as the particles exchanged. Now, only pseudoscalar particles (0^-) can be exchanged in the off-diagonal elements of Fig. 1a; the exchange of either the 0^+ or the 1^- is forbidden by the conservation of angular momentum and parity. This 0^- exchange graph (Fig. 2) however, involves the coupling $(\mathcal{G}_S^8)^2$, which according to Ref. 1, is much smaller than the vector coupling $(\mathcal{G}_V^8)^2$, ($\rho = 0.04$), and may, to a good approximation be neglected. Our system then becomes decoupled.

Fig. 2

This approximate decoupling of the two channels means of course that our calculation in Ref. 1 remains valid, and that the calculated parameters for the vector and pseudoscalar octets remain good to $\sim 5\%$ within the framework of the present model.

Moreover, one is allowed to consider the scattering of scalar mesons in the same way separately, neglecting the production of pseudoscalar meson pairs. This bootstrap problem is kinematically identical

to that of Ref. 1, so that we have immediately $m_V^8 \sim 2.33\mu'$, $m_S^8 \sim 1.2\mu'$, $(\mathcal{G}_S^{8'})^2/(\mathcal{G}_V^{8'})^2 \sim 0.04$, where μ' is now the mass (degenerate) of the external scalar octet, and $\mathcal{G}_V^{8'}$, $\mathcal{G}_S^{8'}$ are respectively the vector and scalar coupling strengths with the scalar mesons. Again the system is closed in itself and requires no further meson supermultiplets (excepting the Pomeranchon) for bootstrap self-consistency.

Identifying the particles in the two bootstrap problems we should then have $\mu' = \mu = m_S^8$ within the error of the decoupling approximation, and it is amusing that the calculated value of $m_S^8 = 1.2\mu$ from the Zachariasen-Zemach²⁾ mechanism is indeed so close.

Next we consider the problem of Fig. 1b for $P = (-1)^{J+1}$ partial wave states. Starting from the known particles, we wish, as in Ref. 1, to find a 'closed' system such that all particles bound as poles in the direct channel are also exchanged, contributing to the interaction forces. The OPE diagrams exchanging the previously known particles are shown in Fig. 3. The diagrams (a) and (b) of Fig. 3 are proportional respectively to $(\mathcal{G}_S^8)^2$ and $(\mathcal{G}_S^8 \mathcal{G}_S^{8'})$ and are, by previous result, small. The graph (c) is kinematically identical to the vector exchange graph in pseudo-scalar meson scattering¹⁾ with the same crossing matrix elements^{1,5)} namely,

$$\lambda_1 = 1, \lambda_8 = 1/2, \lambda_{8'} = 1/2, \lambda_{10} = \lambda_{\overline{10}} = 0, \lambda_{27} = -1/3. \quad (1)$$

The net forces of Fig. 3 are thus, as before, attractive in 1, 8, 8', repulsive in 27 and zero in 10, $\overline{10}$, SU_3 channels, and may bind meson supermultiplets with the following quantum numbers⁶⁾:

$$J^P(X^c) = 0^-(1^+), 0^-(8^\pm), 1^+(1^+), 1^+(8^\pm). \quad (2)$$

Any of these may give rise to further interaction forces in the (s,ps) system.

Fig. 3

Consider in particular the exchange diagrams of the pseudo-vector $1^+(8^-)$ supermultiplet (Fig. 4a). It is kinematically similar to the crossed vector exchange graph in the (ps,ps) bootstrap (Fig. 4b). Moreover, the charge conjugation parity (-) for $1^+(8^-)$ requires that the coupling (ps,s,pv) be the antisymmetric F coupling⁶⁾, so that the crossing matrix elements are also identical to those of Fig. 4b, namely for $\ell = 0, 2, \dots$

$$\lambda'_1 = 1, \lambda'_8 = 1/2, \lambda'_8 = -1/2, \lambda'_{10} = \lambda'_{\overline{10}} = 0, \lambda'_{27} = -1/3, \quad (3)$$

and for $\ell = 1, 3, \dots$

$$\lambda'_1 = -1, \lambda'_8 = -1/2, \lambda'_8 = 1/2, \lambda'_{10} = \lambda'_{\overline{10}} = 0, \lambda'_{27} = 1/3. \quad (4)$$

The pseudovector $1^+(8^-)$ exchange forces are thus attractive in the $0^-(1^+)$, $0^-(8^+)$, $1^+(8^-)$, $1^+(27^+)$ and repulsive or zero in all other supermultiplets. If we take the mass of the $1^+(8^-)$ octet to be the same as the $1^-(8^-)$, the repulsive [attractive] forces in the channels $0^-(8^-)$, $1^+(1^+)$, $1^+(8^+)$, [$1^+(27^+)$] due to graph 4a exactly cancel the attractive [repulsive] forces in these channels (1) due to vector exchange (Fig. 3c). It is easy to see that the system again becomes identical to the scattering of pseudoscalar mesons¹⁾. The inclusion of the weaker forces, Fig. 3a and 3b, does not alter this similarity. It is then trivial to note that we have a self-consistent solution to the present bootstrap

problem with $m[0^-(8^+)] \sim 1.2\mu$, $m[1^+(8^-)] \sim 2.33\mu$, $(g_{pV}^8)^2 = (g_V^8)^2$, while all other parameters remain the same as before. Besides the original octets $0^-(8^+)$, $0^+(8^+)$, $1^-(8^-)$ and the singlet $0^+(1^+)$, we have added two other supermultiplets, $0^-(1^+)$ and $1^+(8^-)$, but no further supermultiplets are then required for self-consistency. We have then a 'closed' system as desired.

Fig. 4

This solution of the (ps,s) bootstrap problem is the simplest possible, though not necessarily the only solution. Of the supermultiplets (2) 'bound' by the vector exchange force (Fig. 3c) the exchange of any, gives rise to an attraction in the $1^+(8^-)$ so that no self-consistent solution could exist without also admitting the $1^+(8^-)$ supermultiplet. Whereas, the above solution exists with only the $1^+(8^-)$ and no other (excepting the $0^-(1^+)$), since the exchange of $1^+(8^-)$ gives rise to repulsion in other relevant channels.

Also the solution is consistent with the related problem of Fig. 1a within the decoupling approximation, with a mass for the $0^-(8^+)$, i.e., (π, K, η) , $\sim 1.2\mu$, close to the required value μ . Thus, we have obtained a solution apparently self-consistent in all directions in all related bootstrap problems at low energy. Though such a degenerate solution is conceivable from the beginning because of the similarity in kinematics between particles and pseudoparticles (i.e., opposite space parity), it is interesting and non-trivial that starting from pseudo-scalar and vector mesons alone, we were forced essentially at every step by bootstrap requirements to arrive at such a conclusion. It is also

non-trivial that no further supermultiplets are required for consistency within the low-energy approximation.

Of the supermultiplets found necessary [$0^+(1^+)$, $0^-(1^+)$, $0^+(8^+)$, $0^-(8^+)$, $1^-(8^-)$, $1^+(8^-)$], the $0^-(8^+)$ octet (π, K, η) and the $1^-(8^-)$ octet (ρ, K^*, Φ) are well known⁷⁾. The $0^+(1^+)$, identified as the Pomeranchon³⁾, is not observable, while its Regge recurrence, $2^+(1^+)$, predicted in our model¹⁾ at ~ 1 GeV, may be identified as the $f_0(1250)$ ⁸⁾. No scalar resonances have so far been found experimentally for definite assignment to $0^+(8^+)$, but this fact may be explained within the framework of the model, (see previous discussion). The recently identified A_2 resonance $J^{PG} = 2^{+-}$, $I = 1^0$, fits well as the $I = 1$ member of the Regge recurrence $2^+(8^+)$ predicted at ~ 1.5 GeV¹⁾. No 1^+ resonance has so far been definitely identified, but the $B(\pi-\omega)$, $I = 1$, G-parity $+^{10)$, and $C(K\pi\pi)$, $I = \frac{1}{2}$ (?)¹¹⁾ resonances at ~ 1250 MeV are possible candidates. Also the Regge recurrence $2^-(8^+)$ of the (π, K, η) octet is expected to occur around this region of energy. There remains the embarrassing $0^-(1^+)$ singlet, the occurrence of which, together with the $0^+(1^+)$, is the main weakness of our model. Both of these singlets are tightly bound, with masses lower even than the $0^-(8^+)$ octet! If we are willing to accept the $0^+(1^+)$ as the ghost-state Pomeranchon, then the $0^-(1^+)$ is to be interpreted as a pseudo-Pomeranchon, and thus unobservable. Its Regge recurrence, $2^-(1^+)$, may then show up as a resonance at energies comparable to f_0 , [the $H(\eta\pi\pi)$ at 960 MeV¹²⁾?], while its trajectory will dominate backward (ps,s) scattering!

Apart from its possible relation to the experimental spectrum of mesonic resonances, the present prediction is of interest in another direction. The supermultiplets found necessary for self-consistency are exactly those recently suggested by Gell-Mann in his W_3 invariant model¹³⁾ and by Salam and Ward¹⁴⁾.

One last point which requires clarifying is the approximate decoupling of the problem in Fig. 1, where we have claimed that the off-diagonal elements are small because of the smallness of the graph (Fig. 2).

If the pseudovector $1^+(8^-)$ mesons exist, such a claim is no longer valid since the graph (Fig. 5) is not negligible. We have then a genuine two-channel bootstrap problem the solution of which will split the particle-pseudoparticle degeneracy, hopefully in the right direction. The conclusion has to await more detailed analysis.

Fig. 5

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