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ONE-PARTICLE EXCHANGE FORCES AND
MESON SUPERMULTIPLETS IN AN SU_3 INVARIANT MODEL

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The following three dynamical hypotheses have frequently been made in current work on strong interactions:

- A) at sufficiently low energies it is a fair approximation to consider only those reactions with the lowest physical thresholds;
- B) in processes involving more than two particles between which strong resonances exist, the interaction is dominated by the mechanism in which the particles emerge in two resonating groups (quasi-two-particle processes);
- C) long-range forces are responsible for binding 'particles' or resonances and are qualitatively well represented by forces due to 'particle' or 'resonance' exchange.

The purpose of this note is to report briefly some preliminary results of an attempt, within the framework of these assumptions, to understand the spectrum of mesonic resonances up to energies below the production threshold of baryon pairs. Our model is constructed to be SU_3 invariant so that all our supermultiplets are degenerate in mass. Although this approximation may not be necessary in such a dynamical model, it simplifies the problem sufficiently for us to be able to take into account all possible coupled channels.

We start from the assumption that the pseudoscalar mesons (π, K, η) and the vector resonances (ρ, K^*, ϕ) exist and have the usual assignment of quantum numbers. The problem of mutual scattering of the pseudoscalar meson octet has been investigated in a recent work¹⁾ where it was predicted that at low energies no $10, \overline{10}$ vector (1^-) resonances or 27 scalar (0^+) resonances exist, whereas scalar resonances in the 1 and 8 channels are expected. In the calculation these scalar resonances were found to be weakly coupled to pseudoscalar mesons and have narrow widths, and therefore they may be difficult to produce or observe experimentally. Their Regge recurrences however, a 2^+ singlet and a 2^+ octet at energies $\gtrsim 1$ GeV, may be observable, and the recent confirmation of the quantum numbers $J^{PG} = 2^{++}$ ($I=0$) for the f_0 ²⁾ and the discovery of the 2^{+-} ($I=1$) resonance at 1320 MeV³⁾ seem to confirm this prediction.

We wish here to extend the discussion to higher energies where the production of mesons become appreciable. This is of particular interest because of the recent discovery of more resonances above 1 GeV⁴⁻⁷). Also an extension in this direction will allow us to study states of 'wrong' symmetry (e.g. the singlet vector ω) or 'wrong' parity, $P = (-1)^{J+1}$, which did not occur in the model of Ref. 1.

Assumption (B) allows us to replace multimeson processes approximately by the quasi-two-particle production of resonances, in particular (ps,V). We shall neglect the production of vector meson pairs or other resonances (e.g. f_0, A_2) which will be appreciable only at energies comparable to that of baryon pair production. The effects of the production of the scalar particles predicted in Ref. 1 have been studied in a recent work by one of us⁸), where it was suggested that at least in the parity $P = (-1)^J$ states such effects are negligible within the present framework. The pseudovector mesons also predicted in Ref. 8 have been tentatively assigned to the B and C resonances^{4,5}), and if so, their higher masses (~ 1250 MeV) will make their production and their exchange possibly negligible in the spirit of the present model. In any case since these predicted resonances have not been definitely identified, we shall ignore them in the present semi-phenomenological treatment. We are thus led (A), in the energy range under consideration, to an S-matrix of the form shown in Fig. 1⁹).

Fig. 1

We shall approximate the forces by one-particle exchange graphs [assumption (C)]. The exchange of the 'scalar particles' we neglect in view of their weaker coupling with the pseudoscalar mesons found in the calculation of Ref. 1. We shall also neglect the exchange of higher resonances (e.g. f_0) because their exchange contribution will be of shorter range and may thus conceivably be less important for the binding of resonances. We are then left with only pseudoscalar and vector exchange. All such possible graphs are listed in Fig. 2.

Three types of couplings are involved: (ps,ps,V), (V,V,ps), and (V,V,V). The couplings are restricted by Lorentz invariance and SU_3 symmetry to be of the following forms⁽¹⁰⁾:

$$(ps,ps,V): \sum_{ab\alpha} iG_1 F_{ab\alpha} (p_a - p_b)_\mu \phi_a \phi_b V_\alpha^\mu \quad (1)$$

$$(V,V,ps): \sum_{\alpha\beta a} iG_2 D_{\alpha\beta a} \epsilon_{\mu\nu\rho\sigma} (p_a^\mu V_\alpha^\nu p_\beta^\rho V_\beta^\sigma) \phi_a \quad (\text{cont.})$$

$$\begin{aligned}
(V,V,V): \sum_{\alpha\beta\gamma} iG_3 F_{\alpha\beta\gamma} (p_\beta^\nu V_{\alpha\nu})(p_\gamma^\lambda V_{\beta\lambda})(p_\alpha^\mu V_{\gamma\mu}) \\
+ \sum_{\alpha\beta\gamma} (-i)G_4 F_{\alpha\beta\gamma} (p_\alpha - p_\beta)^\mu V_\alpha^\nu V_{\beta\nu} V_{\gamma\mu}
\end{aligned}
\tag{1}$$

Here F_{abc} and D_{abc} ¹¹⁾ are, respectively, the totally antisymmetric and totally symmetric C-G coefficients in $SU_3 : 8 \times 8 \rightarrow 8$. We have, without loss of generality, employed the real fields. There are four coupling parameters, G_1, G_2, G_3, G_4 .

The evaluation of the Born terms (Fig. 2 a-e) and their decomposition into the various spin-parity states and SU_3 channels are straightforward though rather lengthy. The results of our calculation relevant for the present discussion are summarized in Figs. 3-5 and Table 1. Here we note only two points:

- i) in the scattering of pseudoscalar mesons, only intermediate stages of parity $P = (-1)^J$ exist, and, because of the Pauli principle even (odd) J states can only be coupled symmetrically (antisymmetrically) to the external pseudoscalar mesons;
- ii) self-conjugate representation $(1, 8, 27)$ of SU_3 for mesons have definite charge conjugation parities. Meson octets of opposite charge conjugation parities may both be coupled to the external ps (π, K, η) and V (ρ, K^*, ϕ) octets but their couplings are different (F or D). We shall denote our various channels by $J^P(X^C)$ ¹²⁾

To obtain the contribution of the graphs (Fig. 2 a-e) in any direct channel $J^P(X^C)$, one multiplies the crossing factors listed in Table 1 by the appropriate coupling constants and the values given by the curves (Figs. 3-5) for the required spin-parity states (note caption on figures). For example, the contribution of the pseudoscalar exchange graph (Fig. 2 d) to the supermultiplet $0^-(8^+)$ [i.e. the original pseudoscalar octet (π, K, η)], is $+1/2 G_1^2 \times$ curve (d) on Fig. 3. An over-all +ve (-ve) sign

means attraction (repulsion), i.e. in the determinantal approach, the denominator function may (may not) vanish giving rise to pole(s) in the amplitude.

Only the results in the partial waves $0^+, 0^-, 1^-, 2^+$ will be given in this preliminary report. The partial waves $1^+, 2^-$, though of equal interest and also investigated in our programme, are more difficult to analyse because of more complicated kinematics. Conclusion in these states can only be reached by more detailed calculations.

In principle, the coupling parameters, $G_1 - G_4$, may all be calculated self-consistently by a bootstrap mechanism. Such a calculation is, in fact, underway. However, since all existing bootstrap mechanisms are rather crude, it is worthwhile first to investigate the system by supplying a limited amount of experimental information.

The approximate value of the coupling constant G_1 is well known from ρ decay. The value of the coupling G_2 may be estimated using the model of Gell-Mann et al.¹³⁾ for $\omega \rightarrow (3\pi)$ decay. This yields a value for G_2 of the same order of magnitude as G_1 ($G_2^2/G_1^2 \sim 2.5$). Because of the smallness of the Born term of graph (c) compared with graph (d) (see curves), the ratio $G_2^2/G_1^2 \sim 1$ implies that the effect of graph (c) is small compared with (d) in essentially all channels. Conclusions concerning the binding of resonances deduced from graph (c) alone^{14,15)} seems thus not very likely to be accurate, even within the framework of the one-particle exchange model.

At present, almost no information can be obtained directly from experiments concerning the coupling constants G_3 and G_4 in the (V,V,V) coupling. It turns out, however, that within the framework of the present model and using some experimental facts, one can make a crude estimate of the values of G_3 and G_4 , and that with these crude estimates one is already able to make quite definite predictions concerning the 'binding' or otherwise of meson supermultiplets in the $0^+, 0^-, 1^-, 2^+$ partial wave states

One uses the following two facts:

- i) the ps octet (π, K, η), or $0^-(8^+)$ in the present notation, is very tightly bound;
- ii) no pseudoscalar meson state with wrong charge conjugation parity (-) is known to exist below 1 GeV.

The argument runs as follows. The specially tight binding of the $0^-(8^+)$ supermultiplet suggests that all forces in this channel add up constructively to give a strong attraction. From Table 1 and Fig. 3, one sees that the forces (c) and (d) are attractive, but (e1) and (e2) depend on the signs of G_3 and G_4 . For attraction one wants $G_1 G_3, G_1 G_4 > 0$. These last constants, however, cannot be much larger than G_1^2 , (say $G_1 G_3, G_1 G_4 \lesssim 2G_1^2$) otherwise one would get by Table 1 an attraction in the $0^-(1^-)$ channel comparable to that in $0^-(8^+)$, giving rise to a low-lying $0^-(1^-)$ state, contradicting (ii). They cannot be too small either, otherwise one gets binding in $0^-(27^-)$. A choice of $G_1 G_3, G_1 G_4 \sim G_1^2$ will guarantee a strong binding in the $0^-(8^+)$ state, and no unwanted state with wrong charge conjugation parity in 0^- .

With this information one can proceed to analyse the $0^+, 0^-, 1^-$ and 2^+ states. We note first that the (ps,V) system has no 0^+ intermediate state so that the forces in the 0^+ state in the present model are the same as in the restricted model of Ref. 1, the conclusion of which thus remains valid and independent of our estimate of G_3, G_4 . In 0^- , no other channel beside the input 8^+ shows attraction. For 1^- and 2^+ , one sees from Figs. 4 and 5 that the Born terms for graphs (c) and (e1) are small, so that for G_2^2 and $G_1 G_3 \sim G_1^2$, these graphs are negligible compared with (a), (d) and (e2), while (d) and (e2) themselves are comparable in magnitude. It is clear then that for channels with no (a) contribution strong attraction (binding) can occur only when both (d) and (e2) are +ve. This leads to the conclusion listed in Table 1. It is interesting that in the many possible channels listed, only the $1^-(1^-)$ channel shows a strong attraction besides the input $0^-(8^+)$ and those already discussed in Ref. 1. This state $1^-(1^-)$ is usually assigned to the meson ω , and because of the Pauli principle was not present in the model of Ref. 1.

Two points remain to be clarified:

- i) The states $1^-(8^-)$ and $2^+(8^+)$ occupy rather special positions. They are each coupled both to the (ps,V) and to the (ps,ps) channels, the forces in each being attractive in both channels. It is not excluded that the attraction may then be strong enough to bind two sets of poles of the same quantum numbers.
- ii) Because of the interference of two different helicity states of the external vector mesons, and the possible further complication due to the production of 'scalar mesons'⁸⁾, no definite conclusion can be drawn concerning the 1^+ and 2^- partial wave states from such a crude estimate of the coupling constants, though one is tempted to see pronounced attraction in $1^+(8^-)$ and $2^-(8^+)$. Both these points have to await clarification by detailed calculations.

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Table 1 - Crossing factors, conclusion on binding, and tentative assignments of the various supermultiplet channels. Our crossing factors may differ by a sign from usual definition of crossing matrix. A +ve crossing factor means attraction.

Supermultiplets	Graphs, Fig. 2 a-e.						Conclusion	Assignment
$J^P(X^C)$	a G_1^2	b $G_1 G_2$	c G_2^2	d G_1^2	e1 $G_1 G_3$	e2 $G_1 G_4$		
$0^+(1^+)$	+2	-	-	-	-	-	Binding	Pomeranchuk
(8^+)	+1	-	-	-	-	-	Binding	$K^*(725)$?
(27^+)	$-2/3$	-	-	-	-	-	No binding	-
$0^-(1^-)$	-	-	-1	-1	+1	+1	No binding	-
(8^-)	-	-	$+3/10$	$-1/2$	$+1/2$	$+1/2$	No binding	-
(8^+)	-	-	$+1/2$	$+1/2$	$+1/2$	$+1/2$	Binding	π, K, η
(10)	-	-	$-2/5$	0	0	0	No binding	-
$(\bar{10})$	-	-	$-2/5$	0	0	0	No binding	-
(27^-)	-	-	$-1/5$	$+1/3$	$-1/3$	$-1/3$	No binding	-
$1^-(1^-)$	-	-	+1	+1	-1	+1	Binding	ω
(8^+)	-	-	$-1/2$	$-1/2$	$-1/2$	$+1/2$	No binding	-
(8^-)	+1	+1	$-3/10$	$+1/2$	$-1/2$	$+1/2$	Binding	ρ, K^*, ϕ
(10)	0	$-2/5$	$+2/5$	0	0	0	No binding	-
$(\bar{10})$	0	$-2/5$	$+2/5$	0	0	0	No binding	-
(27^-)	-	-	$+1/5$	$-1/3$	$+1/3$	$-1/3$	No binding	-
$2^+(1^+)$	+2	-	-	-	-	-	Binding	f_0
(8^+)	+1	-1	$+1/2$	$+1/2$	$-1/2$	$+1/2$	Binding	$A_2(1320)$?
(27^+)	$-2/3$	-	-	-	-	-	No binding	-
(1^-)	-	-	-1	-1	-1	+1	No binding	-
(8^-)	-	-	$+3/10$	$-1/2$	$-1/2$	$+1/2$	No binding	-
(10)	-	-	$-2/5$	0	0	0	No binding	-
$(\bar{10})$	-	-	$-2/5$	0	0	0	No binding	-
(27^-)	-	-	$-1/5$	$+1/3$	$+1/3$	$-1/3$	No binding	-

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- 9) In Fig. 1, as in all subsequent figures, a dotted line represents a pseudoscalar meson and a wavy line a vector meson.
- 10) We use the metric $p_\mu p^\mu = p_0^2 - \underline{p}^2$.
- 11) In our convention the C-G coefficients F and D are normalized as follows:

$$\sum_{bc} F_{abc} F_{a'bc} = \delta_{aa'}, \quad \sum_{bc} D_{abc} D_{a'bc} = \delta_{aa'},$$

they differ therefore from Gell-Mann's f and d by factors $1/\sqrt{3}$ and $\sqrt{(3/5)}$, respectively.
- 12) J = spin, P = space parity, X = dimension of SU_3 representation, c = charge conjugation parity. The charge conjugation properties of the supermultiplets can readily be seen from the symmetry properties of the C-G coefficients tabulated, for example, by J.J. de Swart, Rev.Mod.Phys. 35, 916 (1963). In de Swart's notation we define charge conjugation parity $c = \pm 1$ by

$$C\Psi_\nu^\mu = c(-1)^{I_z + Y/2} \Psi_{-\nu}^\mu$$

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Figure caption (Figs. 3,4,5)

Our model is constructed to be completely SU_3 invariant so that the supermultiplets are degenerate. We have taken the degenerate masses of the octets to be the experimental root-mean-square masses, thus $m_{ps} = 410$ MeV, $m_V = 840$ MeV. The graphs are plotted in units of $m_V = 1$. We have plotted against s (= the C.M. energy squared) the invariant Born amplitude $\times q^2/s^{3/2}(s-1)$. Thus the area under a curve is the contribution to the dispersion integral for the D-function at $s = 1$, subtracted once at $s = 0$, which in models of the type proposed by Zachariasen and Zemach¹⁶⁾ is a measure of the strength of attractive or repulsive forces in the partial wave state.

FIG. 3

FIG. 4

FIG. 5