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ON THE AZIMOV CONDITIONS FOR NUCLEON-NUCLEON SCATTERING

John M. Charap *)

CERN - Geneva

A B S T R A C T

The analytic continuation of the partial wave scattering amplitudes for nucleon-nucleon elastic scattering are stated. The Gribov - Pomeranchuk - Azimov essential singularity at $J = 0$ is investigated, and by considering the two-meson exchange contributions it is shown that Azimov's conditions for the absence of the essential singularity are not satisfied.

*) Present address : Imperial College, London.

1. INTRODUCTION

It was shown by Gribov and Pomeranchuk ¹⁾ that if it is possible, in a relativistic theory with crossing and unitarity, to continue analytically the partial wave scattering amplitude to the vicinity of $\ell = -n$, for $n = 1, 2, 3, \dots$, an accumulation of poles should in general be found there. They further showed that for identical scalar particles the integral condition on the double spectral function needed to eliminate that essential singularity at $\ell = -1$ was certainly not satisfied. As was discussed by Azimov ²⁾ when the particles have spin, these essential singularities still occur when ℓ has a negative integer value, but because $J = \ell + s$, they may occur at the points $J = s_{\max}^{-1}, s_{\max}^{-2}, \dots$, where $s_{\max}$ is the maximum total spin which may occur. Because of the unitarity condition, which couples amplitudes together, these essential singularities would then appear at physical values of J in some amplitudes. The only way to prevent this situation would be for certain integral conditions to be satisfied by the double spectral functions, which would eliminate the essential singularities at $J = s_{\max}^{-1}, s_{\max}^{-2}, \dots, 0$ (or $\frac{1}{2}$ in the case when $2s_{\max}$ is odd). The number of such conditions becomes very large as $s_{\max}$ increases, and Azimov thought it unlikely that so many conditions are in fact satisfied.

Calogero and Charap ³⁾ in an amplification of Azimov's considerations, suggested that such conditions were after all not too disturbing since one hopes that the conditions of unitarity and maximal analyticity determine the S matrix completely: constraints are therefore to be welcomed.

In the present paper these conditions are investigated for the case of nucleon-nucleon scattering. It is found that they are not satisfied. A similar investigation by Mandelstam ⁴⁾ for the $\pi\pi \rightarrow \bar{N}N$ amplitude leads to similar conclusions: in the absence of cuts in the complex angular momentum plane the analytically continued partial wave amplitude will exhibit an essential singularity at $J = 0$.

2. THE PARTIAL WAVE AMPLITUDES

In the nucleon-nucleon scattering problem, for each value of the total isotopic spin $I = 0, 1$, there are five linearly independent amplitudes used to describe the elastic scattering process. It is convenient to start with the helicity amplitudes (see, e.g., Goldberger et al. ⁵⁾)

$$\phi_i^I(p, \theta) \quad i=1, 2, \dots, 5 \quad (2.1)$$

which are so normalized that they square to give the differential cross-section. The quantities p and θ are respectively the barycentric momentum and scattering angle. These amplitudes do not satisfy the Mandelstam representation, since they contain trivial kinematic singularities. However, it is plausible that a related set of amplitudes

$$\begin{aligned} \chi_1 &= E p^2 \phi_1, \\ \chi_2 &= E p^2 \phi_2, \\ \chi_3 &= E m^2 (1 + \cos \theta)^{-1} \phi_3, \\ \chi_4 &= E m^2 (1 - \cos \theta)^{-1} \phi_4, \\ \chi_5 &= m^3 (\sin \theta)^{-1} \phi_5, \end{aligned} \quad (2.2)$$

do (see Charap, Lubkin and Scotti ⁶⁾). the amplitudes χ_i are examples of the amplitudes $F_{\lambda'\lambda}$ considered in Ref. ³⁾). We have dropped reference to the isotopic spin index I , and will frequently do so in subsequent equations.

The partial wave amplitudes $A_i^J(p)$ are defined by the expansions

$$\phi_i(p, \theta) = \sum_J (2J+1) d_i^J(\theta) A_i^J(p), \quad (2.3)$$

where

$$\begin{aligned}
 d_1^J(\theta) &= d_2^J(\theta) = d_{0,0}^J(\theta), \\
 d_3^J(\theta) &= d_{1,1}^J(\theta), \\
 d_4^J(\theta) &= d_{-1,1}^J(\theta), \\
 d_5^J(\theta) &= d_{1,0}^J(\theta);
 \end{aligned}
 \tag{2.4}$$

we shall suppose that they may be analytically continued in J , to give functions $A_{iI}^{J\pm}$ of J . The index $\pm$ indicates the usual separation into two continuations, the $+$ amplitude agreeing with the physical values for J even and the $-$ amplitude for J odd. For the nucleon-nucleon problem the Pauli principle reduces the number of independent functions $A_{iI}^{J\pm}$ of J from 20 to 10; e.g., $A_{10}^{J-} = A_{20}^{J-}$.

If we define a set of modified quantities

$$\begin{aligned}
 B_1^{J\pm} &= (4p^2)^{-J} E p^2 A_1^{J\pm}, \\
 B_2^{J\pm} &= (4p^2)^{-J} E p^2 A_2^{J\pm}, \\
 B_3^{J\pm} &= (4p^2)^{-J+1} E m^2 A_3^{J\pm}, \\
 B_4^{J\pm} &= (4p^2)^{-J+1} E m^2 A_4^{J\pm}, \\
 B_5^{J\pm} &= (4p^2)^{-J+1} m^3 A_5^{J\pm},
 \end{aligned}
 \tag{2.5}$$

then ³⁾ the analogous definition of the analytically continued partial wave amplitude to that given by Gribov ⁷⁾ and Froissart ⁸⁾ for the no-spin case, enables us to write [cf. Muzinich ⁹⁾]

4.

$$B_1^{J\pm} = \frac{2}{\pi} \int_{4\mu^2}^{\infty} dt (4p^2)^{-J-1} [D_1^{(1)}(s, t) \pm D_1^{(2)}(s, t)] Q_J^{(0,0)} \left(1 + \frac{t}{2p^2}\right),$$

$$B_2^{J\pm} = \frac{2}{\pi} \int_{4\mu^2}^{\infty} dt (4p^2)^{-J-1} [D_2^{(1)}(s, t) \pm D_2^{(2)}(s, t)] Q_J^{(0,0)} \left(1 + \frac{t}{2p^2}\right),$$

$$B_3^{J\pm} = \frac{2}{\pi} \int_{4\mu^2}^{\infty} dt (4p^2)^{-J} \left[\left(\frac{4p^2+t}{4p^2}\right)^2 D_3^{(1)}(s, t) Q_{J-1}^{(0,2)} \left(1 + \frac{t}{2p^2}\right) \right. \\ \left. \mp \left(\frac{t}{4p^2}\right)^2 D_3^{(2)}(s, t) Q_{J-1}^{(2,0)} \left(1 + \frac{t}{2p^2}\right) \right],$$

$$B_4^{J\pm} = \frac{2}{\pi} \int_{4\mu^2}^{\infty} dt (4p^2)^{-J} \left[\left(\frac{t}{4p^2}\right)^2 D_4^{(1)}(s, t) Q_{J-1}^{(2,0)} \left(1 + \frac{t}{2p^2}\right) \right. \\ \left. \mp \left(\frac{4p^2+t}{4p^2}\right)^2 D_4^{(2)}(s, t) Q_{J-1}^{(0,2)} \left(1 + \frac{t}{2p^2}\right) \right] \quad (2.6)$$

$$B_5^{J\pm} = -\left(\frac{J+1}{J}\right)^{1/2} \frac{2}{\pi} \int_{4\mu^2}^{\infty} dt (4p^2)^{-J} [D_5^{(1)}(s, t) \mp D_5^{(2)}(s, t)] \\ \left(\frac{t}{4p^2}\right) \left(\frac{4p^2+t}{4p^2}\right) Q_{J-1}^{(1,1)} \left(1 + \frac{t}{2p^2}\right).$$

We have introduced the familiar notation

$$\begin{aligned} s &= 4(m^2 + p^2) = 4E^2, \\ t &= -2p^2(1 - \cos \theta), \\ u &= -2p^2(1 + \cos \theta). \end{aligned} \quad (2.7)$$

The quantities $D_i^{(1)}(s, t)$ and $D_i^{(2)}(s, u)$ are the imaginary parts of $\chi_i(s, t, u)$ taken for $t > 0$ and $u > 0$ respectively. The functions $Q_{\nu}^{(a, b)}(z)$ are Jacobi functions of the second kind: they generalize Legendre functions of the second kind, and indeed

$$Q_{\nu}^{(0, 0)}(z) \equiv Q_{\nu}(z). \quad (2.8)$$

The absorptive parts D should include s independent δ functions at $t = \mu^2$ (or $u = \mu^2$) corresponding to the one-pion poles in the amplitudes χ : the contribution of these terms to the partial wave amplitudes is trivial, well-known, and has been omitted. Accordingly the integrals start from $4\mu^2$, the threshold for the two-pion contributions.

The partial wave dispersion relations read

$$\begin{aligned} B_i^{J\pm}(s) &= \frac{1}{\pi} \int_{4m^2}^{\infty} \frac{ds'}{s' - s} \operatorname{Im} B_i^{J\pm}(s') \\ &+ \frac{1}{\pi} \int_{-\infty}^{4(m^2 - \mu^2)} \frac{ds'}{s' - s} \Delta B_i^{J\pm}(s'), \end{aligned} \quad (2.9)$$

where again the left-hand cut contribution coming from the one-pion term (which extends up to $s' = 4m^2 - \mu^2$) has been dropped. The subtractions which may be necessary do not affect the subsequent discussion.

It is easy to write down explicitly the quantities $\Delta B_i^{J\pm}$. We have

6.

$$\Delta B_i^{\pm} = - \int_{4\mu^2}^{-4p^2} dt (-4p^2)^{-J-1} \left[D_i^{(1)}(s-i\epsilon, t) \pm D_i^{(2)}(s-i\epsilon, t) \right] P_J \left(-1 - \frac{t}{2p^2} \right) \\ + \frac{2}{\pi} \int_{t_1}^{t_2} dt (-4p^2)^{-J-1} \left[\rho_i^{(3)}(t, -4p^2-t) \pm \rho_i^{(3)}(-4p^2-t, t) \right] Q_J \left(-1 - \frac{t}{2p^2} + i\epsilon \right), \quad (2.10a)$$

with an equivalent expression for $i = 2$,

$$\Delta B_3^{\pm} = - \int_{4\mu^2}^{-4p^2} dt (-4p^2)^{-J} \left[\left(\frac{4p^2+t}{4p^2} \right)^2 D_3^{(1)}(s-i\epsilon, t) P_{J-1}^{(2,0)} \left(-1 - \frac{t}{2p^2} \right) \right. \\ \left. \mp \left(\frac{t}{-4p^2} \right)^2 D_3^{(2)}(s-i\epsilon, t) P_{J-1}^{(0,2)} \left(-1 - \frac{t}{2p^2} \right) \right] \\ + \frac{2}{\pi} \int_{t_1}^{t_2} dt (-4p^2)^{-J} \left[\left(\frac{4p^2+t}{4p^2} \right)^2 \rho_3^{(3)}(t, -4p^2-t) Q_{J-1}^{(2,0)} \left(-1 - \frac{t}{2p^2} + i\epsilon \right) \right. \\ \left. \mp \left(\frac{t}{-4p^2} \right)^2 \rho_3^{(3)}(-4p^2-t, t) Q_{J-1}^{(0,2)} \left(-1 - \frac{t}{2p^2} + i\epsilon \right) \right], \quad (2.10b)$$

with a similar expression for $i = 4$: and

$$\Delta B_5^{\pm} = - \left(\frac{J+1}{J} \right)^{\frac{1}{2}} \left\{ - \int_{4\mu^2}^{-4p^2} dt (-4p^2)^{-J} \left(\frac{t}{-4p^2} \right) \left(\frac{4p^2+t}{4p^2} \right) \right. \\ \left[D_5^{(1)}(s-i\epsilon, t) \mp D_5^{(2)}(s-i\epsilon, t) \right] P_{J-1}^{(1,1)} \left(-1 - \frac{t}{2p^2} \right) \\ - \frac{2}{\pi} \int_{t_1}^{t_2} dt (-4p^2)^{-J} \left(\frac{t}{-4p^2} \right) \left(\frac{4p^2+t}{4p^2} \right) \\ \left. \left[\rho_5^{(3)}(t, -4p^2-t) \mp \rho_5^{(3)}(-4p^2-t, t) \right] Q_{J-1}^{(1,1)} \left(-1 - \frac{t}{2p^2} + i\epsilon \right) \right\}. \quad (2.10c)$$

In these expressions $P_{\nu}^{(a,b)}(z)$ are Jacobi functions of the first kind. The functions $\rho_i^{(3)}(t,u)$ are the (t,u) double spectral functions of the amplitudes $\chi_i(s,t,u)$. We also have

$$4\mu^2 < t_1 \leq t_2 < -4p^2 \quad (2.11)$$

the quantities t_1 and t_2 being defined by the support of the double spectral functions $\rho_i^{(3)}$. It is important to observe that although the definitions (2.6) of B_i^{J+} break down when $\text{Re } J$ is too small, since then the integrals may diverge, the definitions (2.10) of ΔB_i^{J+} may be continued to the whole J plane, since the integrals run over finite intervals and the integrands are, except for isolated singularities, analytic functions of J .

3. THE UNITARITY CONDITION

For energies such that

$$4m^2 < s < (2m + \mu)^2 \quad (3.1)$$

and for physical values of J , we have

$$\frac{1}{2ip} \left[A_{iI}^J - (A_{iI}^{J*})^* \right] = \sum_{jk} M_{ijk} (A_{jI}^{J*})^* A_{kI}^J, \quad (3.2)$$

where the non-zero elements of the $5 \times 5 \times 5$ matrix M_{ijk} are given by

$$\begin{aligned} M_{111} = M_{122} = M_{212} = M_{221} = M_{333} = M_{344} = M_{434} = M_{443} = M_{515} = \\ = M_{525} = M_{544} = -M_{553} = 1, \end{aligned} \quad (3.3)$$

$$M_{155} = M_{255} = M_{355} = -M_{455} = 2.$$

Using the familiar arguments based on Carlson's theorem, we may continue Eq. (3.2) for each signature to all values of J for which the quantities involved have an analytic continuation. Care must be taken about the continuation if there are branch cuts in the J plane. On the basis of potential theory it is reasonable to expect ¹⁰⁾ that $A_5^{J\pm}$ has kinematical square-root type branch points at $J = 0$, $J = -1$. If we assume that there are no other branch points at real values of J , we conclude that for all real J and for s satisfying the condition (3.1) we have

$$\frac{1}{p} \operatorname{Im} A_1^{J\pm} = |A_1^{J\pm}|^2 + |A_2^{J\pm}|^2 + 2 |A_5^{J\pm}|^2, \quad (3.4)$$

$$\frac{1}{p} \operatorname{Im} A_3^{J\pm} = |A_3^{J\pm}|^2 + |A_4^{J\pm}|^2 + 2 |A_5^{J\pm}|^2,$$

where we must change the sign of the last term in each of these equations for $-1 < J < 0$. In any case, so long as we assume that there are no branch points in the amplitude for J real and positive, and that we may continue down to $J = 0$, we deduce that

$$P^2 |A_i^{J\pm}|^2 \leq 1 \quad \text{as } J \rightarrow 0+. \quad (3.5)$$

This condition, with $i = 5$, implies that the branch point in $A_5^{J\pm}$ is of the form $J^{\frac{1}{2}}$ rather than $J^{-\frac{1}{2}}$, so that

$$A_{5I}^{0\pm} = 0. \quad (3.6)$$

The vanishing of A_{5I}^{0+} is also required by the fact that we wish the first of Eqs. (3.4) to agree with

$$\frac{1}{P} \text{Im } A_i^{0+} = |A_1^{0+}|^2 + |A_2^{0+}|^2, \quad (3.7)$$

which is the $J = 0$ unitarity equation one would obtain without analytic continuation. This is an example of Gell-Mann's requirement of "decoupling of sense from nonsense" ¹¹⁾. Similar conclusions may be drawn for the quantities $B_{iI}^{J\pm}$, namely that subject to the condition (3.1) they are bounded uniformly in J as $J \rightarrow 0+$, and that for $i = 5$ they vanish at $J = 0$.

4. THE ESSENTIAL SINGULARITIES

The Gribov-Pomeranchuk-Azimov essential singularities have their origin in infinities in ΔB , fixed in J . Thus, for example for $i = 1, 2$, when $J = -1, -2, \dots$, where Q_J has simple poles, ΔB becomes unbounded. We are in this paper concerned with the situation at $J = 0$, where no trouble arises for $i = 1, 2$. However, for $i = 3$ or 4 we must consider the functions $Q_{J-1}^{(2,0)}$ and $Q_{J-1}^{(0,2)}$ which have simple poles at $J = 0$; so a priori we would say that $\Delta B_{iI}^{0\pm}$ is unbounded for $i = 3, 4$. In the case $i = 5$, where the relevant Jacobi function is $Q_{J-1}^{(1,1)}$ which has no pole at $J = 0$, the problem arises from the over-all factor $J^{-\frac{1}{2}}$.

Let us recapitulate the argument of Gribov and Pomeranchuk as applied to this situation. We start with the partial wave dispersion relation (2.9), and as we have remarked, the inclusion of any necessary subtractions will not affect our argument. As we continue this equation analytically in J for fixed s to the vicinity of $J = 0$, it is possible that Regge poles move up through the right-hand cut onto the physical sheet. We will not consider the possibility of encountering less well-behaved singularities in the J plane, although Gribov and Pomeranchuk's discussion of fixed branch points may be adopted mutatis mutandis.

We obtain then

$$\begin{aligned}
 B_i^{J\pm}(s) &= \frac{1}{\pi} \int_{4m^2}^{\infty} \frac{ds'}{s'-s} \operatorname{Im} B_i^{J\pm}(s') \\
 &+ \frac{1}{\pi} \int_{-\infty}^{4(m^2-\mu^2)} \frac{ds'}{s'-s} \Delta B_i^{J\pm}(s') \\
 &+ \sum_n \frac{b_{ni}^{\pm}(J)}{s_n^{\pm}(J) - s},
 \end{aligned} \tag{4.1}$$

where the last term is the sum over the Regge poles just mentioned. This equation then is valid so long as nothing more disturbing than moving Regge poles is encountered as the J continuation is made, and we have assumed this to be true as $J = 0$ is approached from the right. Suppose that ΔB_i^{J+} is unbounded as $J \rightarrow 0+$. Gribov and Pomeranchuk argue that B_i^{J+} is bounded, and so also is the first integral. It is not obvious that the inelastic contributions to the right-hand cut integral have to be bounded, but in any case they cannot cancel the unboundedness of the left-hand cut integral. Hence there must be an infinite number of terms in the sum, in order that it may be unbounded for all s as $J \rightarrow 0+$: i.e., there is an accumulation of (moving) poles at $J = 0$, or an essential singularity of the second kind there.

Thus, if any of the ΔB_{iI}^{J+} are unbounded at $J = 0$, there will be an essential singularity in the corresponding amplitude there: alternatively some singularity in the J plane more severe than a moving pole always exists to the right of $J = 0$. In order to eliminate these equally distressing alternatives we must discuss the conditions under which the ΔB_{iI}^{J+} remain bounded at $J = 0$. As already remarked this is already true a priori for $i = 1, 2$. For $i = 3, 4, 5$, the conditions are easily obtained ³⁾: they are

$$\int_{t_1}^{t_2} dt \left[f_{iI}^{(3)}(t, -4p^2 - t) \mp f_{iI}^{(3)}(-4p^2 - t, t) \right] = 0 \quad (4.2)$$

The stronger condition for the vanishing of B_{5I}^{0+} is

$$\int_{4p^2}^{\infty} dt \left[D_{5I}^{(1)}(s, t) \mp D_{5I}^{(2)}(s, t) \right] = 0 ; \quad (4.3)$$

the imaginary part of this equation taken for $s < 0$ reproduces one of the previous set. Note that the one-pion contribution to D_{5I} , omitted above, vanishes identically. The Pauli symmetry of the nucleon-nucleon amplitudes reduces the number of linearly independent conditions.

It would be extremely difficult to examine the above conditions in general. However, it is not hard to write down the contributions to the double spectral functions coming from the two-pion exchange dispersion theory diagrams. For large enough values of $4p^2$ this is the only contribution, so that a necessary condition is that the equations (4.2) be satisfied with the $\rho_i^{(3)}$ replaced by their two-pion parts. We investigate this in the next Section.

5. TWO-PION CONTRIBUTIONS

In this Section we investigate whether the conditions (4.2), with only the two-pion contributions to the double spectral functions $\rho^{(3)}$ retained, are satisfied. These two-pion contributions may be derived from formulas given in Ref. ⁶⁾ for the two-pion contributions to the absorptive parts of the G amplitudes. We have

$$\begin{aligned} \chi_3 &= \frac{m^2}{2} [m^2 G_2 - p^2 G_3 + E^2 G_4], \\ \chi_4 &= \frac{m^2}{2} [m^2 G_2 + p^2 G_3 + E^2 G_4], \\ \chi_5 &= -\frac{m^2}{2} [m^2 G_2 + m^2 G_4], \end{aligned} \quad (5.1)$$

so that a set of conditions equivalent to those of (4.2) is

$$\int_{t_1}^{t_2} dt \sigma_{jI}^{(3)}(t, -4p^2 - t) = 0; \quad j = 2, 3, 4; \quad (5.2)$$

here $\sigma_{jI}^{(3)}(t, u)$ is the (t, u) double spectral function of the amplitude $G_j^I(s, t, u)$, and we have used the Pauli symmetry to reduce the number of conditions by one-half.

Now the double spectral functions σ are given by

$$\begin{aligned} \sigma_{iI}^{(3)}(t, u) &= \text{Im}_{t>0} \text{Im}_{u>0} G_i^I(s, t, u) \\ &= \sum_{I'j} B_{II'} \Delta_{ij} \text{Im}_{t>0} \text{Im}_{u>0} \bar{G}_j^{I'}(s, t, u), \end{aligned} \quad (5.3)$$

where $\underline{B} \Delta$ is the crossing matrix and the amplitudes $\bar{G}$ are defined in the nucleon-antinucleon channel in a manner analogous to the G in the nucleon-nucleon channel. The contributions to the absorptive parts

$$I_m \quad \bar{G}(s, t, u) \quad (5.4)$$

coming from two-pion intermediate states are given in Ref. 6), and from the expressions given there one may in a straightforward way derive

$$\sigma_{2I}^{(3)} \Big|_{2\pi} = \frac{-g^4}{128\pi^2} \left[4 - (-1)^I \right] \frac{1}{st} K \times \left\{ t(u-4\mu^2)^2 + u[(u-4\mu^2)(t-4m^2) - 4\mu^4] \right\}, \quad (5.5)$$

$$\sigma_{3I}^{(3)} \Big|_{2\pi} = \sigma_{4I}^{(3)} \Big|_{2\pi} = \frac{-g^4}{128\pi^2} \left[4 - (-1)^I \right] \frac{1}{st} K \times \left\{ 4\mu^4(4m^2-s) + 4m^2u(u-4\mu^2) \right\}.$$

Here K is given by

$$K = \left\{ t u [(u-4\mu^2)(t-4m^2) - 4\mu^4] \right\}^{-1/2} \quad (5.6)$$

if the radicand is positive and vanishes otherwise. It is clear then that the expressions (5.5) for $\sigma|_{2\pi}$ are positive-definite, so that the conditions (5.2) cannot be satisfied. Similar conclusions may be reached for the corresponding conditions for the nucleon-antinucleon channel: none of them are satisfied.

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R E F E R E N C E S

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