DIFFERENCE IN MASS BETWEEN THE $Y_1^*$ AND $Y_1^{**}$ ISOBARS

W.A. Cooper, H. Filthuth, A. Fridman*, E. Malamud*
CERN, Geneva

and

E.S. Gelsera, J.C. Kuyver, and A.G. Tenner,
Zeeman Laboratorium**, Universiteit van Amsterdam, Amsterdam

8 January 1964
(Submitted to Physics Letters)

+ ) On leave of absence from the Centre de Recherches Nucléaires,
(Physique Corpusculaire), Strasbourg.

*) Visitor from the University of Heidelberg.

**) The work in Amsterdam was supported by the research programmes of
the Stichting voor Fundamenteel Onderzoek der Materie and the
Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek.
DIFFERENCE IN MASS BETWEEN THE $Y_1^{-*}$ AND $Y_1^{-*+}$ ISOBARS

by

W.A. Cooper, H. Filthuth, A. Fridman+, E. Malamud*

CERN, Geneva

and

E.S. Gelsemä, J.C. Kluhver, and A.G. Turner,
Zeeman Laboratorium**, Universiteit van Amsterdam, Amsterdam.

In the interaction of 1.455 GeV/ν K⁻ mesons in the 32 cm
CERN hydrogen bubble chamber we have identified 582 events of the
type $K⁻p → \Lambda⁰π⁺π⁻$ from a restricted fiducial volume (18 cm along
the beam direction). By examination of the Dalitz plot for these
events it appears that 60% of the interactions pass through the
intermediate state: $K⁻p → Y_1^{*-}(1385)π → \Lambda⁰ππ$. Most of the other 40% are
non-resonant: production of the reaction $K⁻p → \Lambda⁰ρ → \Lambda⁰ππ$ is
< 5%, and of the reaction $K⁻p → Y_1^{*-}(1660)π → \Lambda⁰ππ$ is ~ 1% for the
$Y_1^{*-}(1660)$ and ~ 2% for the $Y_1^{*-+}(1660)$.

A preliminary value for the mass difference of the nega-
tively and positively charged $Y_1^{*-}(1385)$ was reported at the Sienna
Conference¹, $m(Y_1^{*-}) - m(Y_1^{*-+}) = 12.9 \pm 6.5$ MeV. A more careful study
described below gives the value 17 ± 7 MeV. The values for back-
ground intensity and resolution have also been further studied and the
present values supersede those given at Sienna.

If the effective mass distributions for the ($\Lambda⁰π⁻$) and ($\Lambda⁰π⁺$)
combinations are plotted on the same graph (fig. 1), a difference in
the peaks of the negative and positive mass distributions of about
16 MeV is apparent. This difference has been investigated more fully
in the following ways:

+ ) On leave of absence from the Centre de Recherches Nucléaires
(Physique Corpusculaire), Strasbourg.

* ) Visitor from the University of Heidelberg.

** ) The work in Amsterdam was supported by the research programmes of the
Stichting voor Fundamenteel Onderzoek der Materie and the Nederlandse
Organisatie voor Zuiver Wetenschappelijk Onderzoek.

7955/p/dmn
i) **Gaussian ideograms** have been calculated for the events with \( m \) mass between 1300 and 1500 MeV (see Fig. 2). Each event is weighted in the ideogram by the error in the effective mass determination. The average value of the error is 6.5 MeV. The separation of the peaks in the ideograms is 13 MeV. The ideograms are not smooth and it is difficult to assign a value to the error in the peak; therefore the ideogram measurement is not used in obtaining the final result.

ii) **Maximum likelihood fit to a curve Breit-Wigner plus phase space.** The height of the phase-space curve as a function of effective mass changes slowly in an interval comparable to the experimental resolution (~6.5 MeV). Therefore, the experimentally measured histogram is divided by the phase-space curve and fitted to the following formula:

\[
\frac{N(m_{A/2}^{\text{to}} m_{A/2}^{\text{+}})}{\Theta(m)} = N_0 \int_{m_{A/2}^{\text{+}}}^{m_{A/2}^{\text{+}}} \left( A + \frac{\Gamma}{(m' - m_0)^2 + (\Gamma/2)^2} \right) \text{dm}'.
\]

(1)

The fit is done with four parameters, \( N_0, A, m_0, \) and \( \Gamma \), by maximum likelihood. In (1), \( m \) is the effective mass, \( m_0 \) is the effective mass of the resonance, \( \Gamma \) is the full resonance width (including experimental resolution), \( A \) is the fraction of background (non-resonant interactions), and \( N_0 \) is the normalization factor. \( \Delta \) is the interval size used in the calculations (20 MeV) which is wider than the bin size shown in fig. 1.

The fit gives the values shown in table 1.

An estimate of the error in \( m_0 \) is given by the full width \( \Gamma \) divided by \( \sqrt{N'} \), where \( N' \) is the number of events above background in the region fitted. This yields the final value of:

\[
m(X_1^{*+}) - m(X_1^{*+}) = 17 \pm 7 \text{ MeV}.
\]

(2)
Table 1

<table>
<thead>
<tr>
<th></th>
<th>($\Lambda^0\pi^+$)</th>
<th>($\Lambda^0\pi^-$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$, number of events used in the fit (between 1300 and 1500 MeV)</td>
<td>239</td>
<td>269</td>
</tr>
<tr>
<td>$m_0$</td>
<td>1375 MeV</td>
<td>1392 MeV</td>
</tr>
<tr>
<td>$\Gamma$ (full width including experimental resolution of $\sim 6.5$ MeV)</td>
<td>51 MeV</td>
<td>88 MeV</td>
</tr>
<tr>
<td>% of background at the peak of the curve</td>
<td>$\sim 6%$</td>
<td>$\sim 5%$</td>
</tr>
<tr>
<td>$N'$, number of events above background</td>
<td>$\sim 170$</td>
<td>$\sim 200$</td>
</tr>
<tr>
<td>$\Gamma / \sqrt{N'}$</td>
<td>3.9 MeV</td>
<td>6.2 MeV</td>
</tr>
<tr>
<td>$\chi^2$ (degrees of freedom = 6)</td>
<td>0.5</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Systematic errors

We have searched for possible systematic effects which could be responsible for the observed mass difference, and conclude that their effect is negligible. These are the following:

(i) Influence of the $\rho$ meson. The production cross-section and angular distribution for the $Y_{1}^{1-}$ and $Y_{1}^{1+}$ are roughly the same. Therefore there is no immediate reason why interference with the $\Lambda^0\rho$ channel would cause an apparent mass difference. Removing events with the $\pi^+\pi^-$ effective mass $> 690$ MeV does not change the mass difference observed, although its statistical significance is reduced.

The ratio of production of $Y_{1}^{0-}/Y_{1}^{0+}$ is $1.27 \pm 0.05$. Assuming no interference with the $\Lambda^0\rho$ production, a shift of the $Y_{1}^{0+}$ to heavier mass relative to the $Y_{1}^{0-}$ could result, which is opposite to the observed effect.
ii) Influence of the $Y_t^\ast$ (1660)  Any shift caused by the $Y_t^\ast$ (1660) depends on the ratio of the $Y_t^{\ast^-}$ (1660) and $Y_t^{\ast+}$ (1660) as well as the distribution of these events along their resonance bands. To eliminate such possible effects, the average value of the ($\Lambda^0\pi^-$) and ($\Lambda^0\pi^+$) combinations lying between 1300 and 1500 MeV was computed with and without the events in a crossing band of 1630 - 1670 MeV. The differences in the two averages for both $Y_t^{\ast-}$ and $Y_t^{\ast+}$ was < 0.5 MeV.

iii) Systematic errors in the bubble chamber  An error in the magnetic field could not cause a mass difference (assuming similar angular distributions for both signs), but a distortion in the liquid could. For 626 events of the type $K^-p \rightarrow \Lambda^0\pi^+\pi^0\pi^-$, and for the 582 $\Lambda^0\pi^+\pi^-$ events, the so-called "pull" quantity, i.e., the difference between fitted and measured values of 1/p divided by the error in this difference, was computed:

$$GP = \frac{(1/p)_{\text{fitted}} - (1/p)_{\text{measured}}}{\text{error in above}}$$

For these events the average values for the 1208 events are:

$$\pi^+ \overline{GP} = -0.069 \pm 0.020$$

$$\pi^- \overline{GP} = -0.107 \pm 0.020$$

These values imply that the assigned magnetic field value is about 0.5% too low (assuming a track 10 cm long), and that no appreciable distortion of a type that could cause a mass difference exists. A difference of 0.078 in $\overline{GP}$, the maximum allowed by the errors, corresponds to about 1.5 MeV difference in the ($\Lambda^0\pi^-$) and ($\Lambda^0\pi^+$) effective masses if only measured quantities are considered. Of course, the fitting procedure would reduce the difference.
Remarks on the resonance width

Assuming that the only non-uniformly populated parts of the Dalitz plot are due to the $Y_1^*(1385)$, the $Y_1^*(1660)$, and the $p$ meson, then the observed mass distributions result from different mass values for the two charged states of the $Y_1^*(1385)$. Under the same assumptions, our measurements indicate that these two isobars have different width, $\Gamma_{Y_1^{*-}} = 88$ and $\Gamma_{Y_1^{*+}} = 52$. If we estimate the error on the width as $\pm 10$ MeV, then

$$\Gamma_{Y_1^{*-}} - \Gamma_{Y_1^{*+}} = 37 \pm 15 \text{ MeV}.$$  

This difference can be explained by the following two arguments:

a) It is assumed that the charge independence violating term is mainly accounted for by the mass difference $\Delta m/m \approx 1/2 \%$. The reduced widths, $\gamma$, for the two isobars would be expected to be different by the same order of magnitude.

The observed width $\Gamma$ is related to the reduced width $\gamma$ by

$$\Gamma = \gamma \frac{p^3}{(1 + a^2 p^2)^2}, \quad (4)$$

for angular momentum $l = 1$ of the $\Lambda\pi$ system. In this expression $a$ is the "interaction distance" of the $\Lambda\pi$ final state and $p$ is the momentum of the $\Lambda$ or $\pi$ in the $Y_1^*$ rest system. Therefore, even if $\gamma$ is taken the same for the two isobars, the heavier isobar ($Y_1^{*-}$) must have a larger width than the lighter one ($Y_1^{*+}$), due to the larger phase space available to the decay products. For $m_{Y_1^{*-}} = 1392$ MeV, $p = 215$ MeV/c and for $m_{Y_1^{*+}} = 1375$ MeV, $p = 198$ MeV/c. If we take a ratio of observed widths predicted by expression (4) as 1.27, and based on the observed $\Gamma_{Y_1^{*+}} = 51$ MeV

$$\Gamma_{Y_1^{*-}} - \Gamma_{Y_1^{*+}} = 14 \text{ MeV}.$$  

b) The width is much more sensitive to background than the mass. In particular, when events in the $p$ band are removed the $\Gamma_{Y_1^{*+}}$
remains about 50 MeV and the \( Y_1^{*-} \) becomes about 65 MeV. The estimated errors are \( \pm 15 \) MeV, so the difference is:

\[
\Gamma_{Y_1^{*-}} - \Gamma_{Y_1^{*+}} \text{ (without } \rho \text{ events)} = 15 \pm 20 \text{ MeV.}
\]

The average value of the resonance width, \( \Gamma \approx 50 \text{ MeV, is in agreement with the data from other experiments} \^{2-4} \). These widths do not agree with the value found by Baltay et al. \(^6\) for \( Y_1^{*-} \)'s formed in \( \bar{p}p \) collisions, \( \Gamma_{Y_1^{*-}} = 26 \text{ MeV.} \)

Discussion

Under the unitary symmetry scheme the isobar states \( N_3^{3/2}(1238) \), \( Y_1(1385) \), \( Z_4^+(1520) \), and the predicted \( Z_0^- \) can be grouped into a 10-fold supermultiplet.

If the symmetry-breaking interaction is treated in lowest order perturbation theory, the supermultiplet splits into four equally spaced multiplets as described by the well-known mass formula\(^7,8\). Rosen\(^9\) has calculated the electromagnetic mass splitting within a multiplet assuming SU\(_3\) symmetry for the supermultiplet. If we assume that his formulas apply to the observed multiplets, one obtains the following relation:

\[
Y_1^{*-} - Y_1^{*+} = N_3^{3/2} - N_3^{5/2}.
\]

(The \( N_3^{3/2} \) and \( N_3^{5/2} \) states are the \( \pi \pi^- \) and \( \pi \pi^+ \) resonances.)

An experimental verification of Eq. (5) is not available at present. Since the \( N_3^{3/2} \) is a broad resonance, it is desirable to produce both charge states in the same experiment to minimize systematic effects. An experiment of this sort has been performed\(^10\) but the \( \pi \pi^+ \) resonance is produced less strongly than the \( \pi \pi^- \) resonance, and there are not enough events to see both \( N_3^{3/2} \) bands in the Dalitz plot.

It is with pleasure that we acknowledge helpful discussions with Dr. P. Kabir.
REFERENCES


Figure captions

Fig. 1 : Histogram of the effective masses of the combinations \( \Lambda\pi^+ \) (full line) and \( \Lambda\pi^- \) (dotted) in the region of the \( Y_1^* (1385) \) resonance.

Fig. 2 : Gaussian ideogram of the same quantities shown in Fig. 1.
MASS DISTRIBUTION FOR $K^- p \rightarrow \Lambda \pi^+ \pi^-$

$\Lambda \pi^+$

$\Lambda \pi^-$
MASS DISTRIBUTION FOR $K^- p \rightarrow \Lambda \pi^+ \pi^-$

- 1 event $\Lambda \pi^+$
- 1 event $\Lambda \pi^-$

fig. 2