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ON THE PERATIZATION OF NUCLEON-LEPTON INTERACTIONS

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The peratization method has been extensively developed by Feinberg and Pais (FP) for the purely leptonic weak interactions ¹⁾. Some consequences have been derived by the same authors and by Bouchiat et al. ²⁾ for the case of semi-leptonic interactions. The peratization in this case gives the prescription for summing the most divergent contributions of the ladder graphs in the approximation of neglecting the following points :

- a) the baryon mass in the baryon propagator for the leading term,
- b) the influence of strong interactions on the baryon propagators and on the baryon W vertices.

In this note, we consider possible effects of the nucleon structure when the FP method is applied to semi-leptonic processes (we consider here only the case of nucleons). As in the hypothesis of CVC theory and G invariance, we describe the nucleon W vertex by

$$G_V \left[\gamma_\mu (1 + \lambda \gamma_5) F_1(q^2) - \frac{\mu}{2M} \sigma_{\mu\nu} q_\nu F_2(q^2) \right] \quad (1)$$

where F_1 and F_2 are the usual electromagnetic form factors ; we have assumed that the axial vector and vector form factor F_A, F_V are equal. We do not consider here the induced pseudoscalar form factor F_P . We are well aware of the fact that also the nucleons are off the mass shell ; therefore, one should replace the expression (1) by the complete vertex function corresponding to all virtual particles. As a matter of fact, the most important contributions of the ladder graphs correspond to infinite values of the virtual masses. However, since one does not know the behaviour of the vertex function when more than one particle is infinitely off the mass shell, we will limit ourselves to a model

where the structure of the vertex function is still given by (1). Following the FP procedure, one gets two coupled integral equations for the amplitudes $M_{\text{odd}}, M_{\text{even}}$ which correspond to the exchange of an odd and even number of intermediate W mesons, in the process represented in Fig. 1. Decoupled equations can be written for the amplitudes $M_{\pm} = M_{\text{odd}} \pm M_{\text{even}}$, as follows

$$\begin{aligned}
 M_{\pm}(p'_1, p'_2, p_1, p_2) = & \mathcal{M}(q) \pm \frac{\alpha}{(2\pi)^4} \int [\gamma_{\mu}^{(1)} (1 + \lambda \gamma_5^{(1)}) F_1(q'^2) - \\
 & - \frac{\mu}{2M} \tilde{\sigma}_{\mu \times}^{(1)} q'_x F_2(q'^2)] \cdot \frac{1}{p_1''} \left(\delta_{\mu \nu} + \frac{q_{\mu} q_{\nu}}{m^2} \right) \left\langle \frac{1}{q'^2 + m^2} \right\rangle \cdot \\
 & \cdot \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}) \cdot \frac{1}{p_2''} M_{\pm}(p_1'', p_2'', p_1, p_2) \cdot d^4 q'
 \end{aligned} \quad (2)$$

where

$$\alpha = - \frac{igG_V}{\sqrt{2}}$$

$$q = p_1 - p'_1 = p'_2 - p_2$$

$$q' = p_1'' - p'_1 = p'_2 - p_2''$$

$$p_2'' = p_1 + p_2 - p_1''$$

The Born term $\mathcal{M}(q)$ is given by

$$\mathcal{M}(q) = \alpha \mathcal{M}_1(q) F_1(q^2) + \alpha \mathcal{M}_2(q) F_2(q^2)$$

$$\mathcal{M}_1(q) = \gamma_{\mu}^{(1)} (1 + \lambda \gamma_5^{(1)}) \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}) \left(\delta_{\mu \nu} + \frac{q_{\mu} q_{\nu}}{m^2} \right) \left\langle \frac{1}{q^2 + m^2} \right\rangle$$

$$\mathcal{M}_2(q) = - \frac{\mu}{2M} \tilde{\sigma}_{\mu \times}^{(1)} q_x \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}) \left(\delta_{\mu \nu} + \frac{q_{\mu} q_{\nu}}{m^2} \right) \left\langle \frac{1}{q^2 + m^2} \right\rangle \quad (3)$$

The W propagator is regularized as in FP ; the superscripts 1, 2 refer to the nucleonic and leptonic lines, respectively.

The Eq. (2) can be easily rewritten as

$$M_{\pm}(p'_1, p'_2, p_1, p_2) = \alpha M_1(q) F_1(q^2) + \alpha M_2(q) F_2(q^2) \pm$$

$$\pm \frac{\alpha}{(2\pi)^4} \left\{ \left[K_1^{(0)} + K_1^{(1)} \right] F_1(q'^2) + K_2 F_2(q'^2) \right\} M_{\pm}(p''_1, p''_2, p_1, p_2) d^4 q' \quad (4)$$

where

$$K_1^{(0)} = (1 - \lambda \gamma_5^{(1)})(1 - \gamma_5^{(2)}) \left\langle \frac{1}{q'^2 + m^2} \right\rangle$$

$$K_1^{(1)} = \gamma_{\mu}^{(1)} (1 + \lambda \gamma_5^{(1)}) \frac{1}{\not{p}_1''} \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}) \frac{1}{\not{p}_2''} \left(\delta_{\mu\nu} + \right.$$

$$\left. + \frac{p'_{1\mu} p'_{2\nu} + p'_{1\nu} p'_{2\mu} - p'_{1\mu} p'_{2\nu}}{m^2} \right) \left\langle \frac{1}{q'^2 + m^2} \right\rangle \quad (5)$$

$$K_2 = -\frac{\mu}{2M} \bar{\sigma}_{\mu\alpha} q'_x \frac{1}{\not{p}_1''} \delta_{\mu\nu} \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}) \frac{1}{\not{p}_2''} \left\langle \frac{1}{q'^2 + m^2} \right\rangle.$$

The kernel $K_1^{(0)}$ contains the most divergent part of the vector and axial vector terms ; $K_1^{(1)}$ contains the remaining terms which are, at least, one degree less divergent. We remark that the leading singularities coming from the magnetic term in Eq. (2) drop out, because of the antisymmetry of $\bar{\sigma}_{\mu\alpha}$ and the fact that it is contracted with the symmetric tensor $q'_\mu q'_\nu$ the remaining terms are given in K_2 , which has the same degree of divergence as $K_1^{(1)}$.

To solve the general Eq. (4), we start from a subsidiary equation ³⁾

$$\tilde{M}_{\pm}(p_1', p_2', p_1, p_2) = \alpha Z_2 m_1(q) \pm \frac{\alpha}{(2\pi)^4} Z_2 \int [K_1^{(0)} + K_1^{(1)}] \tilde{M}_{\pm}(p_1'', p_2'', p_1, p_2) d^4 q' \quad (6)$$

where Z_2 , if one takes literally the expression (1), would represent the wave function renormalization constant of the nucleon introduced in (6) as the limit of $F_1(q^2)$ for $q^2 \rightarrow \infty$ when only the W meson is off the mass shell. However, as we remarked earlier, the kernel of the Eq. (4) should contain the complete vertex function, in which case Z_2 should be replaced by the asymptotic limit of the vertex function when both q'^2 and $p_1''^2$ (see Fig. 1) go to infinity. The Eq. (6) is a trivial generalization of the equation studied by FP; in the case $\lambda = 1$ the solution is just the FP solution multiplied by the factor Z_2 ; in the case $\lambda \neq 1$, the equation can be split into two similar equations for scalar amplitudes, as done by Bouchiat et al.

The difference between (4) and (6) gives

$$\begin{aligned} M_{\pm} = & \tilde{M}_{\pm} + \alpha m_1(q) [F_1(q^2) - Z_2] + \alpha m_2(q) F_2(q^2) \pm \\ & \pm \frac{\alpha}{(2\pi)^4} \left\{ [K_1^{(0)} + K_1^{(1)}] [F_1(q'^2) - Z_2] M_{\pm} + K_2 F_2(q'^2) M_{\pm} + \right. \quad (7) \\ & \left. + [K_1^{(0)} + K_1^{(1)}] Z_2 [M_{\pm} - \tilde{M}_{\pm}] \right\} d^4 q' \end{aligned}$$

We assume now that $F_2(q^2)$ goes to zero as $1/q^2$ (see Ref. ⁴⁾), and that $F_1(q^2) - Z_2$ goes to zero faster than $1/q^2$ ⁵⁾. Then for $M_{\pm} = \tilde{M}_{\pm}$ (first order approximation) the integral will be convergent even with unregularized

propagators. The usual perturbation expansion gives then finite results, and all the corrections to $M_{\perp}$ are at least of order α^2 .

From the above arguments, we can write the "allowed" and "forbidden" amplitudes $M_{\text{odd}}, M_{\text{even}}$ as follows

$$\begin{aligned} M_{\text{odd}} &= \tilde{M}_{\text{odd}} + \alpha m_1(q) [F_1(q^2) - Z_2] + \alpha m_2(q) F_2(q^2) + O_{\text{odd}}(\alpha^2) \\ M_{\text{even}} &= \tilde{M}_{\text{even}} + O_{\text{even}}(\alpha^2) \end{aligned} \quad (8)$$

where M_{even} is of the same order of O_{even} , and $\tilde{M}_{\text{odd}}$ is given by

$$\begin{aligned} \tilde{M}_{\text{odd}} &= M_{\mu\nu}^L \gamma_{\mu}^{(1)} (1 + \gamma_5^{(1)}) \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}) + M_{\mu\nu}^R \gamma_{\mu}^{(1)} (1 - \gamma_5^{(1)}) \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}) \\ M_{\mu\nu}^{L,R} &= Z_2 \alpha \frac{1 \pm \lambda}{2} \left[\frac{3}{4} \delta_{\mu\nu} + \frac{1}{m^2} (q_{\mu} q_{\nu} - \frac{\delta_{\mu\nu}}{4} q^2) \right] \frac{1}{q^2 + m^2} + O_{\mu\nu}^{L,R}(\alpha^2) \end{aligned} \quad (9)$$

We write for completeness the explicit expression for M_{odd} (7) keeping only the terms of order α

$$\begin{aligned} M_{\text{odd}}(q) &= \alpha m_1(q) [F_1(q^2) - Z_2] + \alpha m_2(q) F_2(q^2) + \\ &+ \alpha Z_2 \left[\frac{3}{4} \delta_{\mu\nu} + \frac{1}{m^2} (q_{\mu} q_{\nu} - \frac{\delta_{\mu\nu}}{4} q^2) \right] \frac{1}{q^2 + m^2} \gamma_{\mu}^{(1)} (1 + \lambda \gamma_5^{(1)}) \cdot \\ &\cdot \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}). \end{aligned}$$

In conclusion, we see that keeping only the terms of lowest order, we do not obtain any correction to the Born result for the term containing F_2 ; for the terms containing F_1 , even at low momentum transfer q , there is a correction depending on the constant Z_2 . In fact, the coupling constant (at $q = 0$) is renormalized by $(1 - \frac{1}{4} Z_2)$, instead of $\frac{3}{4}$ as in the purely leptonic case. In this way, one would get different renormalization for the phenomenological couplings of the μ decay and β decay. We think that this last remark is rather interesting since, unless $Z_2 = 1$, it seems that the FP peratization method might have some difficulties in explaining the experimental fact of the almost exact equality of the μ decay and β decay couplings.

R E F E R E N C E S

- 1) G. Feinberg and A. Pais, "A field theory of weak interactions", I, Princeton preprint, 1963.
- 2) C. Bouchiat, B. d'Espagnat and J. Prentki, "The selection rules in the Feinberg and Pais theory of weak interactions", CERN preprint, 1963, (to be published in *Nuovo Cimento*).
- 3) The solution $\tilde{M}_+$ of Eq. (6) will depend simply on q if one keeps only the most divergent part $K_1^{(0)}$ (see Ref. ¹).
- 4) R.G. Sachs, Proceedings of the International Conference on High Energy Physics at CERN, 1962, p. 197.
- 5) Even though a weaker hypothesis might be sufficient, we assume this stronger behaviour for simplicity.

Fig. 1