Physics at a Muon Collider ¹

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Abstract. I discuss the exciting prospects for exploring a wide range of new physics at a low-energy muon collider.

The physics possibilities for muon colliders (µC’s) are enormous. An incomplete list includes: front-end physics; Z physics; Higgs physics, especially s-channel factory production; precision $m_W$, $m_t$ measurements; deep-inelastic physics, including lepto-quarks and contact interactions; supersymmetry, including s-channel sneutrino production in R-parity violating models; strong-WW sector physics; light and heavy technicolor resonances; and new $Z'$s. No matter what physics lies beyond the Standard Model, the muon collider will be a very exciting machine. In this talk, I will emphasize those topics that are relevant to a first ‘low’-energy muon collider ($E_{\text{beam}} \sim 50 - 250$ GeV), paying special attention to s-channel resonance probes of new physics.

The instantaneous luminosity, $L$, possible for $\mu^+\mu^-$ collisions depends on $E_{\text{beam}}$ and the percentage Gaussian spread in the beam energy, denoted by $R$. The small level of bremsstrahlung and absence of beamstrahlung implies that very small $R$ can be achieved. The (conservative) luminosity assumptions for

this workshop were:  
• \( L \sim (0.5, 1, 6) \cdot 10^{31} \text{cm}^{-2} \text{s}^{-1} \) for \( R = (0.003, 0.01, 0.1)\% \) at \( \sqrt{s} \sim 100 \text{GeV} \);  
• \( L \sim (1, 3, 7) \cdot 10^{32} \text{cm}^{-2} \text{s}^{-1} \), at \( \sqrt{s} \sim (200, 350, 400) \text{GeV} \), \( R \sim 0.1\% \).

With modest success in the collider design, at least a factor of 2 better can be anticipated. Note that for \( R \sim 0.003\% \) the Gaussian spread in \( \sqrt{s} \), given by \( \sigma_\sqrt{s} \sim 2 \text{MeV} \left( \frac{R}{0.003\%} \right) \left( \frac{\sqrt{s}}{100 \text{ GeV}} \right) \), can be comparable to the few MeV widths of very narrow resonances such as a light SM-like Higgs boson, sneutrino resonance, or technicolor boson. This is critical since the effective resonance cross section \( \sigma \) is obtained by convoluting a Gaussian \( \sqrt{s} \) distribution of width \( \sigma_\sqrt{s} \) with the standard s-channel Breit Wigner resonance cross section \( \sigma(\sqrt{s}) = 4\pi \Gamma(\mu\mu) \Gamma(X) / (\sqrt{s} - M^2 + [\Gamma_{\text{tot}}]^2) \). For \( \sqrt{s} = M \), the result,  

\[ \sigma \simeq \frac{\pi \sqrt{2\pi} \Gamma(\mu\mu) B(X)}{M^2 \sigma_\sqrt{s}} \times \left( 1 + \frac{\pi}{8} \left[ \frac{\Gamma_{\text{tot}}}{\sigma_\sqrt{s}} \right]^2 \right)^{-1/2}, \]

will be maximal if \( \Gamma_{\text{tot}} \) is small and \( \sigma_\sqrt{s} \sim \Gamma_{\text{tot}} \). Also critical to scanning a narrow resonance and for precision \( m_W \) and \( m_t \) measurements is the ability [?] to tune the beam energy to one part in 10^6. Finally, by constructing the muon collider at a facility (such as Fermilab) with a high energy proton beam one opens up the possibility of having a \( \mu p \) collider option. The luminosity expected for 200 GeV \( \mu^+ \) and \( \mu^- \) beams in collision with the 1 TeV proton beam of the Tevatron (yielding \( \sqrt{s} = 894 \text{ GeV} \)) is \( L \sim 1.3 \cdot 10^{33} \text{cm}^{-2} \text{s}^{-1} \).

**PHYSICS**

• Front-End and \( \mu \) Beam Physics

A proton driver and intense cooled low-energy muon beam will be the first components of the muon collider to be constructed. These alone will yield a large program of “front-end” physics. In particular, low-energy hadronic

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2) For yearly integrated luminosities, we use the standard convention of \( L = 10^{32} \text{cm}^{-2} \text{s}^{-1} \Rightarrow L = 1 \text{ fb}^{-1} / \text{yr} \).

3) In actual numerical calculations, bremsstrahlung smearing is also included (see Ref. [?]).

4) Although smaller \( \sigma_\sqrt{s} \) (i.e. smaller \( R \)) implies smaller \( L \), the \( L \)'s given earlier are such that when \( \Gamma_{\text{tot}} \) is in the MeV range it is best to use the smallest \( R \) that can be achieved.
physics \((p, \bar{p}, K, \pi)\) \cite{ref1} and low-energy neutrino physics (analogous to the LSND and BOONE experiments) can be explored with much improved statistics \cite{ref2}. Great strides in stopped/slow intense muon beam physics (e.g. \(g_\mu - 2\), \(\mu N \rightarrow eN\) conversion, \(\mu \rightarrow eee\), \(\mu \rightarrow e\gamma\)) will also be possible \cite{ref3, ref4}. The search for \(\mu N \rightarrow eN\) deserves special mention as it would probe for lepton-flavor violation at a level that is generically expected from any one of several sources present in supersymmetric models and other extensions of the SM \cite{ref5, ref6}. 

- **Z Physics**

A low-energy muon collider could be run as a Z factory that would quickly exceed statistical levels achieved at LEP and SLC/SLD. Using \(\sigma(Z)_{\text{peak}} \sim 6 \times 10^7 \text{ fb} \) \((\Gamma^{\text{peak}}_Z \gg \sigma_Z)\) and \(\mathcal{L} \sim 10^{32} \text{ cm}^{-2} \text{ s}^{-1}\) for the \(\mu C\) (assuming \(R \gtrsim 0.1\%\) as is perfectly acceptable for Z physics) leads to \(\sim 6 \times 10^7 Z\)'s per year (about four times the best yearly rate achieved at LEP); partial (\(\sim 20\%)\) polarization for both beams would be automatic. \(^5\)

The many important physics topics include the following. (a) \(B_s - \bar{B}_s\) mixing. (b) An improved measurement of \(\sin^2 \theta^\text{eff}_W\), as probed via \(A_{LR}\) or \(A_{FB}\), to resolve the LEP/SLD disagreement. (c) Improved \(\alpha_s\) determination. (d) \(CP\) Violation, as probed e.g. by \(Z \rightarrow B_d \bar{B}_d\) \((B_d, \bar{B}_d \rightarrow \psi K_S)\) decays. (e) \(\tau\) Michel parameters using \(Z \rightarrow \tau^+\tau^-\) decays. (f) Separation of color-octet from color-singlet \(J/\psi\) production; detailed distributions in the final state would allow this, but LEP statistics have proved inadequate. (g) Improved limits (or actual observation) of flavor-changing-neutral-current (FCNC) rare decays; the current limits on \(Z \rightarrow e\mu, Z \rightarrow e\tau, Z \rightarrow \mu\tau\) from the PDG \cite{ref7} are \(1.7 \times 10^{-6}, 9.8 \times 10^{-6}, 1.7 \times 10^{-5}\), respectively. Some types of new physics would predict such decays at levels just below this. (h) Improved limits on or observation of \(Z \rightarrow \gamma X\) decays, which probe many kinds of new physics.

Of these, (a) (b) and (c) received attention during the workshop \cite{ref8}. With the expected \(L \sim 1 \text{ fb}^{-1}/\text{yr} \) (20% polarization for the beams being acceptable) one can achieve \(\Delta \alpha_s \sim 0.001\) (vs. the current \(\sim 0.003\)) and an actual measurement of the \(x_s\) parameter of \(B_s - \bar{B}_s\) mixing (for which LEP provides only an upper bound). Using \(\Delta A_{LR} = (P\sqrt{N})^{-1}\), where \(P = \frac{P^+ - P^-}{1 - P^+ P^-}\), and \(\Delta \sin^2 \theta^\text{lept}_{\text{eff}} \sim \Delta A_{LR}/7.9\), one finds \(\Delta \sin^2 \theta^\text{lept}_{\text{eff}} \sim 0.0001\) (current error

\(^5\) At the \(\mu C\), substantial polarization (\(\gtrsim 50\%)\) for both beams can be achieved only with a significant sacrifice in luminosity.
being $\lesssim 0.00025$ from combined LEP data) for a sample of $\sim 10^7$ $Z$’s with $P^\pm \sim \pm 30\%$ polarization for the $\mu^\pm$ beams. This would take at most a few years of operation for current $\mu$C designs.

The list of new physics probed by $Z \to \gamma X$ decays is impressive. The factor of ten improvement in sensitivity to such decays, coming from the $\gtrsim 10^8$ $Z$’s produced after a few years at a muon collider $Z$ factory, would be very valuable. (i) An anomalous $ZZ\gamma$ CP-conserving and/or CP-violating coupling that might arise beyond the SM would lead to $Z \to \gamma Z^* \to \gamma \nu \bar{\nu}$ events; current limits from LEP [?] and D0 [?] are already constraining on SM extensions. (ii) Anomalous trilinear and quartic couplings can lead to $Z \to \gamma \gamma \gamma$ events. The SM prediction is $B(\gamma \gamma \gamma) \sim 10^{-9}$ while the current limit is $\lesssim 10^{-5}$; many SM extensions predict branching ratios of this latter size [?]. (iii) The magnitude of the $\nu_{\tau}$ magnetic moment is very relevant to understanding basic neutrino properties and can have a large impact on predictions for this source of dark matter. Non-zero $\mu_{\nu}$ leads to $\gamma$ radiation from the final $\nu$ and $\bar{\nu}$ in $Z \to \nu \bar{\nu}$. Current LEP data yields [?] $\mu_{\nu} \lesssim 3.3 \times 10^{-6} \mu_B$ (90\% CL). Limits from elsewhere are competitive. (iv) Improved limits on axions would be possible from searches for $Z \to \gamma A$, where $A$ decays invisibly. Current limits on this branching ratio from LEP are [?] few $\times 10^{-6}$. If axions exist, $Z \to \gamma A$ decays might be observed with improved sensitivity. Stronger limits would significantly constrain many models. (v) Also of interest are decays of the type $Z \to \gamma + \text{meson}$, e.g. $Z \to \gamma \pi^0, \gamma \eta, \gamma J/\psi, \ldots$. Current limits on such branching ratios are $\lesssim 10^{-5}$ [?]. Not only has there been much dispute about the SM predictions, but also new physics could enter. Surprises could emerge with any increase in sensitivity. (vi) A particularly important probe of technicolor theories is the $Z \to \gamma \gamma \gamma, \gamma \ell^+\ell^-, \gamma E_T, \gamma q\bar{q}, \gamma gg$ class of decays expected from $Z \to \gamma P_0$, where $P_0$ is an electrically neutral pseudo-Nambu-Goldstone boson (PNGB) that can decay to one or more of the indicated channels [?]. The predicted branching ratio for $Z \to \gamma P_0$ is $B(Z \to \gamma P_0) \sim 10^{-5} \left( \frac{123 \text{ GeV}}{f} \right)^2 (N_{TC} A_{Z\gamma})^2 \beta^3$, where the anomaly factor $A_{Z\gamma}$ is $O(.05-1)$ and $f$ is the technipion decay constant. Improving limits in the above channels by a factor of ten would rule out light $P_0$’s in many technicolor models, whereas currently the light PNGB’s of most models would have escaped detection. (vii) Finally, we note that many of the above exotic decays could have large branching ratio if the particles involved are composite.
Overall, a muon collider $Z$ factory would have the luminosity needed to resolve some important outstanding $Z$ physics and would provide increased sensitivity to very important rare processes that probe new physics.

- **Higgs Physics**

The potential of the muon collider for Higgs physics is truly outstanding. First, it should be emphasized that away from the $s$-channel Higgs pole, $\mu^+\mu^-$ and $e^+e^-$ colliders have similar capabilities for the same $\sqrt{s}$ and $L$ (barring unexpected detector backgrounds at the muon collider). At $\sqrt{s} = 500$ GeV, the design goal for a $e^+e^-$ linear collider ($eC$) is $L = 50$ fb$^{-1}$ per year. The conservative $L$ estimates given earlier suggest that at $\sqrt{s} = 500$ GeV the $\mu C$ will accumulate at least $L = 10$ fb$^{-1}$ per year. If this can be improved somewhat, the $\mu C$ would be fully competitive with the $eC$. We will use the notation of $\ell C$ for either a $eC$ or $\mu C$ operating at moderate to high $\sqrt{s}$.

Of course, the totally unique feature of the $\mu C$ is the very large cross section expected for production of a Higgs boson in the $s$-channel when $\sqrt{s} = m_h$, see Fig. 1 [?]. Small $R$ is crucial as it leads to dramatically increased peaking of $\sigma_h$ [Eq. (1)] at $\sqrt{s} \sim m_h$, as illustrated in Fig. 2 for a SM Higgs ($h_{SM}$) with $m_{h_{SM}} = 110$ GeV ($\Gamma_{h_{SM}}^{\text{tot}} \sim 3$ MeV).

### FIGURE 1. Feynman diagram for $s$-channel production of a Higgs boson.

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**A Standard Model-Like Higgs Boson**

For SM-like $h \to WW, ZZ$ couplings, $\Gamma_h^{\text{tot}}$ becomes big if $m_h \gtrsim 2m_W$, and $\sigma_h \propto B(h \to \mu^+\mu^-)$ [Eq. (1)] will be small; $s$-channel production will not be useful. But, as shown in Fig. 2, $\sigma_h$ is enormous for small $R$ when the $h$ is
light, as is very relevant in supersymmetric models where the light SM-like $h^0$ has $m_{h^0} \lesssim 150$ GeV. In order to make use of this large cross section, we must first center on $\sqrt{s} \sim m_h$. Once this is done we proceed to the precision measurement of the Higgs boson’s properties.

For a SM-like Higgs with $m_h \lesssim 2m_W$ one expects $|\Delta m_h| \sim 100$ MeV from LHC data ($L = 300$ fb$^{-1}$) (smaller if $\ell$C data is available). Thus, a final ring that is fully optimized for $\sqrt{s} \sim m_h$ can be built. Once it is operating, we scan over the appropriate $\Delta m_h$ interval so as to center on $\sqrt{s} \simeq m_h$ within a
fraction of $\sigma_{\sqrt{s}}$. For $m_h$ of order 100 GeV, $R = 0.003\%$ implies $\sigma_{\sqrt{s}} \sim 2$ MeV. The luminosity required for a $5\sigma$ observation of the SM Higgs boson with $\sqrt{s} = m_{h_{SM}}$ is plotted (along with individual signal and background rates) in Fig. 3. In the “typical” $m_h \sim 110$ GeV case, $\Delta m_h \sim 100$ MeV implies that $\Delta m_h/\sigma_{\sqrt{s}} \sim 50$ points are needed to center within $\lesssim \sigma_{\sqrt{s}}$. From Fig. 3 we find that each point requires $L \sim 0.0015$ fb$^{-1}$ in order to observe or eliminate the $h$ at the $3\sigma$ level, implying a total of $L_{\text{tot}} \leq 0.075$ fb$^{-1}$ is needed for centering. Thus, for the anticipated $L \sim 0.05 - 0.1$ fb$^{-1}/$yr, centering would take no more than a year. However, for $m_h \simeq m_Z$ a factor of 50 more $L_{\text{tot}}$ is required just for centering because of the large $Z \to b\bar{b}$ background. Thus, for the anticipated $L$ the $\mu$C is not useful if the Higgs boson mass is too close to $m_Z$.

Once centered, we will wish to measure with precision: (i) the very tiny Higgs width — $\Gamma_{h}^{\tau} = 1 - 10$ MeV for a SM-like Higgs with $m_h \lesssim 140$ GeV; (ii) $\sigma(\mu^+\mu^- \to h \to X)$ for $X = \tau^+\tau^-, b\bar{b}, c\bar{c}, WW^*, ZZ^*$. The accuracy achievable was studied in Ref. [?]. The three-point scan of the Higgs resonance described there is the optimal procedure for performing both measurements simultaneously. We summarize the resulting statistical errors in the case of a SM-like $h$ with $m_h = 110$ GeV, assuming $R = 0.003\%$ and an integrated (4 to 5 year) $L_{\text{tot}} = 0.4$ fb$^{-1}$. $^{6}$ One finds $1\sigma$ errors for $\sigma B(X)$ of 8, 3, 22, 15, 190% for the $X = \tau^+\tau^-, b\bar{b}, c\bar{c}, WW^*, ZZ^*$ channels, respectively, and a $\Gamma_{h}^{\text{tot}}$ error of 16%. These results assume the $\tau, b, c$ tagging efficiencies described in Ref. [?]. We now consider how useful measurements at these accuracy levels will be.

If only $s$-channel Higgs factory $\mu$C data are available (i.e. no $Zh$ data from an $eC$ or $\mu C$), then the $\sigma B$ ratios (equivalently squared-coupling ratios $^{7}$) that will be most effective for discriminating between the SM Higgs boson and a SM-like Higgs boson such as the $h^0$ of supersymmetry are $\frac{(WW^*h)^2}{(b\bar{b}h)^2}$, $\frac{(c\bar{c}h)^2}{(b\bar{b}h)^2}$, $\frac{(c\bar{c}h)^2}{(\tau\tau h)^2}$, and $\frac{(c\bar{c}h)^2}{(\tau\tau h)^2}$. The $1\sigma$ errors (assuming $L_{\text{tot}} = 0.4$ fb$^{-1}$ at $m_h = 110$ GeV) for these four ratios are $15\%$, $20\%$, $18\%$ and $22\%$, respectively. Systematic errors for $(c\bar{c}h)^2$ and $(b\bar{b}h)^2$ of order $5\% - 10\%$ from uncertainty in the $c$ and $b$ quark mass will also enter. In order to interpret these errors one must compute the amount by which the above ratios differ in the minimal

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$^{6}$ For $\sigma B$ measurements, $L_{\text{tot}}$ devoted to the optimized three-point scan is equivalent to $\sim L_{\text{tot}}/2$ at the $\sqrt{s} = m_h$ peak.

$^{7}$ From Eq. (1), $\sigma(\mu^+\mu^- \to h \to X)$ provides a determination of $\Gamma(h \to \mu^+\mu^-)B(h \to X)$ (which is proportional to the $(Xh)^2$ squared coupling) when $\sigma_{\sqrt{s}} \gtrsim \Gamma_{h}^{\text{tot}}$, as is the case.
supersymmetric model (MSSM) vs. the SM for \( m_{h^0} = m_{h_{SM}} \). The percentage difference turns out to be essentially identical for all the above ratios and is a function almost only of the MSSM Higgs sector parameter \( m_{A^0} \), with very little dependence on \( \tan \beta \) or top-squark mixing. At \( m_{A^0} = 250 \text{ GeV} \) (420 GeV) one finds \( \text{MSSM}/\text{SM} \sim 0.5 \) (\( \sim 0.8 \)). Combining the four independent ratio measurements and including the systematic errors, one concludes that a \( > 2\sigma \) deviation from the SM predictions would be found if the observed Higgs is the MSSM \( h^0 \) and \( m_{A^0} < 400 \text{ GeV} \). Note that the magnitude of the deviation would provide a determination of \( m_{A^0} \).

\[ \text{FIGURE 4.} \text{ We give } (m_{A^0}, \tan \beta) \text{ parameter space contours for } \frac{\Gamma(h \rightarrow \mu^+ \mu^-)}{\Gamma(h_{SM} \rightarrow \mu^+ \mu^-)}; \text{ no-squark-mixing, } m_{h^0}, m_{h_{SM}} = 110 \text{ GeV.} \]

If, in addition to the \( s \)-channel measurements we also have \( \ell C \sqrt{s} = 500 \text{ GeV}, L_{\text{tot}} = 200 \text{ fb}^{-1} \) data, it will be possible to discriminate at an even more accurate level between the \( h^0 \) and the \( h_{SM} \). The most powerful technique for doing so employs the four determinations of \( \Gamma(h \rightarrow \mu^+ \mu^-) \) below:

\[
\begin{align*}
\frac{\Gamma(h \rightarrow b\bar{b})}{B(h \rightarrow b\bar{b})} &; \quad \frac{\Gamma(h \rightarrow ZZ^*)}{\Gamma(h \rightarrow ZZ^*)} \mu C, \\
\frac{\Gamma(h \rightarrow \mu^+ \mu^-) B(h \rightarrow b\bar{b})_{\mu C}}{\Gamma(h \rightarrow b\bar{b})_{\mu C}} &; \quad \frac{\Gamma(h \rightarrow \mu^+ \mu^-) B(h \rightarrow ZZ^*)_{\mu C}}{\Gamma(h \rightarrow ZZ^*)_{\mu C}}, \\
\frac{\Gamma(h \rightarrow \mu^+ \mu^-) B(h \rightarrow WW^*)_{\mu C}}{\Gamma(h \rightarrow WW^*)_{\mu C}} &; \quad \frac{\Gamma(h \rightarrow \mu^+ \mu^-) B(h \rightarrow ZZ^*)_{\mu C}}{\Gamma(h \rightarrow ZZ^*)_{\mu C}}.
\end{align*}
\]
The resulting 1σ error for Γ(h → µ⁺µ⁻) is ≲ 5%. Fig. 4, which plots the ratio of the h⁰ to h⁰_SM partial width in (m_{A⁰}, tan β) parameter space for m_{h⁰} = m_{h_SM} = 110 GeV, shows that this level of error allows one to distinguish between the h⁰ and h⁰_SM at the 3σ level out to m_{A⁰} ≳ 600 GeV. This result holds for all m_h ≲ 2m_W (m_h ≠ m_Z). Additional advantages of a Γ(h → µ⁺µ⁻) measurement are: (i) there are no systematic uncertainties arising from uncertainty in the muon mass; (ii) the error on Γ(h → µ⁺µ⁻) increases only very slowly as the s-channel L_{tot} decreases, in contrast to the errors for the previously discussed ratios of branching ratios from the μC s-channel data which scale as 1/√L_{tot}. Finally, we note that Γ_{tot}^h alone cannot be used to distinguish between the MSSM and SM in model-independent way. Not only is the error substantial (≈ 12% if we combine μC, L = 0.4 fb⁻¹ s-channel data with ℓC, L = 200 fb⁻¹ data) but also Γ_{tot}^h depends on many things, including (in the MSSM) the squark-mixing model. Still, deviations from SM predictions are generally substantial if m_{A⁰} ≲ 500 GeV.

Precise measurements of the couplings of the SM-like Higgs boson could reveal many other types of new physics. For example, if a significant fraction of a fermion’s mass is generated radiatively (as opposed to arising at tree-level), then the hff coupling and associated partial width will deviate from SM expectations [?]. Deviations of order 5% to 10% (or more) in Γ(h → µ⁺µ⁻) are quite possible and, as discussed above, potentially detectable.

**The MSSM H⁰, A⁰ and H±**

We begin by recalling [?] that the possibilities for H⁰, A⁰ discovery are limited at other machines. (i) Discovery of H⁰, A⁰ is not possible at LHC for all (m_{A⁰}, tan β): e.g. if m_{t̃} = 1 TeV, consistency with the observed value of B(b → sγ) requires m_{A⁰} > 350 GeV, in which case the LHC will not detect the H⁰, A⁰ if tan β ≳ 3 (and below a much higher m_{A⁰}-dependent value). (ii) At √s = 500 GeV, e⁺e⁻ → H⁰A⁰ pair production probes only to m_{A⁰} ≈ m_{H⁰} ≲ 230 – 240 GeV. (iii) A γγ collider could potentially probe up to m_{A⁰} ≈ m_{H⁰} ≈ 0.8√s ≈ 400 GeV, but only for L_{tot} ≳ 150 – 200 fb⁻¹.

Thus, it is noteworthy that μ⁺μ⁻ → H⁰, A⁰ in the s-channel potentially allows production and study of the H⁰, A⁰ up to m_{A⁰} ≈ m_{H⁰} ≲ √s. To assess the potential, let us (optimistically) assume that a total of L_{tot} = 50 fb⁻¹ (5 yrs

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8) This is because the Γ(h → µ⁺µ⁻) error is dominated by the √s = 500 GeV measurement errors.
running at \( < \mathcal{L} > = 1 \times 10^{33} \) can be accumulated for \( \sqrt{s} \) in the \( 250 - 500 \) GeV range. (We note that \( \Gamma_{\text{tot}}^{A^0} \) and \( \Gamma_{\text{tot}}^{H^0} \), although not big, are of a size such that resolution of \( R \gtrsim 0.1\% \) will be adequate to maximize the \( s \)-channel cross section, thus allowing for substantial \( L \).)

![Graph](image-url)

**FIGURE 5.** \( N(b\bar{b}) \) in the \( m_{b\bar{b}} \pm 5 \) GeV interval vs. \( m_{b\bar{b}} \) for \( \sqrt{s} = 500 \) GeV, \( L_{\text{tot}} = 50 \) fb\(^{-1} \), and \( R = 0.1\% \); peaks are shown for \( m_{A^0} = 120, 300 \) or \( 480 \) GeV, with \( \tan \beta = 5 \) and 20 in each case.

There are then several possible scenarios. (a) If we have some preknowledge or restrictions on \( m_{A^0} \) from LHC discovery or from \( s \)-channel measurements of \( h^0 \) properties, then \( \mu^+\mu^- \rightarrow H^0 \) and \( \mu^+\mu^- \rightarrow A^0 \) can be studied with precision for all \( \tan \beta \gtrsim 1 - 2 \). (b) If we have no knowledge of \( m_{A^0} \) other than \( m_{A^0} \gtrsim 250 - 300 \) GeV from LHC, then we might wish to search for the \( A^0, H^0 \) in \( \mu^+\mu^- \rightarrow H^0, A^0 \) by scanning over \( \sqrt{s} = 250 - 500 \) GeV. If their masses lie in this mass range, then their discovery by scanning will be possible for most of \( (m_{A^0}, \tan \beta) \) parameter space such that they cannot be discovered at the LHC (in particular, if \( m_{A^0} \gtrsim 250 \) GeV and \( \tan \beta \gtrsim 4 - 5 \)). (c) Alternatively, if the \( \mu C \) is simply run at \( \sqrt{s} = 500 \) GeV and \( L_{\text{tot}} \sim 50 \) fb\(^{-1} \) is accumulated, then \( H^0, A^0 \) in the \( 250 - 500 \) GeV mass range can be discovered in the \( \sqrt{s} \) bremsstrahlung tail if the \( b\bar{b} \) mass resolution (either by direct reconstruction or hard photon recoil) is of order \( \pm 5 \) GeV and if \( \tan \beta \gtrsim 6 - 7 \) (depending on \( m_{A^0} \)). Typical peaks are illustrated in Fig. 5.  

\(^9\) SUSY decays are assumed to be absent in this and the following figure.
Finally, once the closely degenerate $A^0, H^0$ are discovered, it will be extremely interesting to be able to separate the resonance peaks. This will probably only be possible at a muon collider with small $R < 0.01\%$ if $\tan\beta$ is large, as illustrated in Fig. 6.

We end with just a few remarks on the possibilities for production of $H^0 A^0$ and $H^+ H^-$ pairs at a high energy $\mu C$ (or $e C$). Since $m_{A^0} \gtrsim 1 \text{ TeV}$ cannot be ruled out simply on the basis of hierarchy and naturalness (although fine-tuning is stretched), it is possible that energies of $\sqrt{s} > 2 \text{ TeV}$ could be required for pair production. If available, then it has been shown [? ,?] that discovery of $H^0 A^0$ in their $b\bar{b}$ or $t\bar{t}$ decay modes and $H^+ H^-$ in their $t\bar{b}$ and $b\bar{t}$ decays will be easy for expected luminosities, even if SUSY decays are present. As a by-product, the masses will be measured with reasonable accuracy.

Regardless of whether we see the $H^0, A^0$ in $s$-channel production or via pair production, one can measure branching ratios to other channels, including supersymmetric pair decay channels with good accuracy. In fact, the ratios of branching ratios and the value of $m_{A^0} \sim m_{H^0} \sim m_{H^\pm}$ will be measured with sufficient accuracy that, in combination with one gaugino mass, say the chargino mass (which will also presumably be well-measured) it will be possible [?] to discriminate with incredible statistical significance between different
closely similar GUT scenarios for the GUT-scale soft-supersymmetry-breaking masses. Thus, Higgs pair production could be very valuable in the ultimate goal of determining all the soft-SUSY-breaking parameters.

Finally, entirely unexpected decays of the heavy Higgs bosons of SUSY (or other extended Higgs sector) could be present. For example, non-negligible branching ratios for $H^0, A^0 \rightarrow t\bar{t} + c\bar{t}$ FCNC decays are not inconsistent with current theoretical model-building ideas and existing constraints \[^7\]. The muon collider $s$-channel $\mu^+\mu^- \rightarrow H^0, A^0$ event rate is sufficient to probe rather small values for such FCNC branching ratios.

**Exotic Higgs Bosons**

If there are doubly-charged Higgs bosons, $e^-e^- \rightarrow \Delta^{--}$ probes $\lambda_{ee}$ and $\mu^-\mu^- \rightarrow \Delta^{--}$ probes $\lambda_{\mu\mu}$, where the $\lambda$’s are the strengths of the Majorana-like couplings \[^7\]. Current $\lambda_{ee,\mu\mu}$ limits are such that factory-like production of a $\Delta^{--}$ is possible if $\Gamma^{tot}_{\Delta^{--}}$ is small. Further, a $\Delta^{--}$ with $m_{\Delta^{--}} \lesssim 500-1000$ GeV will be seen previously at the LHC (for $m_{\Delta^{--}} \lesssim 200-250$ GeV at TeV33) \[^7\]. For small $\lambda_{ee,\mu\mu,\tau\tau}$ in the range that would be appropriate, for example, for the $\Delta_L^{--}$ in the left-right symmetric model see-saw neutrino mass generation context, it may be that $\Gamma^{tot}_{\Delta^{--}} \ll \sigma_{\ell\ell \rightarrow \Delta^{--}} \propto \lambda_{\ell\ell}^2/\sigma_{\ell\ell}$. Note that the absolute rate for $\ell^-\ell^- \rightarrow \Delta^{--}$ yields a direct determination of $\lambda_{\ell\ell}^2$, which, for a $\Delta^{--}$ with very small $\Gamma^{tot}_{\Delta^{--}}$, will be impossible to determine by any other means. The relative branching ratios for $\Delta^{--} \rightarrow e^-e^-, \mu^-\mu^-, \tau^-\tau^-$ will then yield values for the remaining $\lambda_{\ell\ell}$’s. Because of the very small $R = 0.003\% - 0.01\%$ achievable at a muon collider, $\mu^-\mu^-$ collisions will probe much weaker $\lambda_{\mu\mu}$ coupling than the $\lambda_{ee}$ coupling that can be probed in $e^-e^-$ collisions. In addition, it is natural to anticipate that $\lambda_{\mu\mu}^2 \gg \lambda_{ee}^2$.

**• Precision Measurements of $m_W$ and $m_t$**

Let us consider the extent to which the muon collider could contribute to precision measurements of $m_W$ and $m_t$. Current expectations for the Tevatron, LHC and $eC$ for various benchmark accumulated luminosities appear in Table ?? \[^7\]. Note that more than $L_{tot} = 50$ fb$^{-1}$ is not useful for these measurements at an electron collider since errors become systematics dominated.

\[^7\] For small $\lambda_{ee,\mu\mu,\tau\tau}$, $\Gamma^{tot}_{\Delta^{--}}$ is very small if the $\Delta^{--} \rightarrow W^-W^-$ coupling strength is very small or zero, as required to avoid naturalness problems for $\rho = m_W^2/\cos^2\theta_W m_Z^2$. 