Rapid X-Ray Variability of Neutron Stars in Low-Mass Binary Systems
Frederick K. Lamb\textsuperscript{a} M. Coleman Miller\textsuperscript{b} and Dimitrios Psaltis\textsuperscript{c}

\textsuperscript{a}University of Illinois at Urbana-Champaign, Department of Physics and Department of Astronomy, 1110 W. Green St., Urbana, IL 61801, USA
\textsuperscript{b}University of Chicago, Department of Astronomy and Astrophysics, 5640 S. Ellis Avenue, Chicago, IL 60637, USA
\textsuperscript{c}Harvard-Smithsonian Center for Astrophysics, 60 Garden St., Cambridge, MA 02138, USA

The dramatic discovery with the Rossi X-Ray Timing Explorer satellite of remarkably coherent $\sim 300$–$1200$ Hz oscillations in the X-ray brightness of some sixteen neutron stars in low-mass binary systems has spurred theoretical modeling of these oscillations and investigation of their implications for the neutron stars and accretion flows in these systems. High-frequency oscillations are observed both during thermonuclear X-ray bursts and during intervals of accretion-powered emission and appear to be a characteristic feature of disk-accreting neutron stars with weak magnetic fields. In this review we focus on the high-frequency quasi-periodic oscillations (QPOs) seen in the accretion-powered emission. We first summarize the key properties of these kilohertz QPOs and then describe briefly the models that have been proposed to explain them. The existing evidence strongly favors beat-frequency models. We mention several of the difficulties encountered in applying the magnetospheric beat-frequency model to the kilohertz QPOs. The most fully developed and successful model is the sonic-point beat-frequency model. We describe the work on this model in some detail. We then discuss observations that could help to distinguish between models. We conclude by noting some of the ways in which study of the kilohertz QPOs may advance our understanding of dense matter and strong gravitational fields.

1. INTRODUCTION

It has long been expected (see, e.g., [7,21]) that important information about the intrinsic properties of neutron stars and stellar mass black holes, as well as about the physics of accretion onto them, could be extracted from their X-ray variability at frequencies comparable to the $\sim 1$–$10$ kHz dynamical frequencies near them. The Rossi X-ray Timing Explorer (RXTE) was specifically designed [5,38] to have the large area, microsecond time resolution, and high telemetry rate needed to probe this high-frequency regime.

The value of access to these high frequencies was dramatically confirmed when, less than two months after the launch of RXTE, remarkably coherent $\sim 1000$ Hz oscillations were discovered (see [41]) in the X-ray brightness of two neutron stars in low-mass X-ray binaries (LMXBs). These are the fastest astrophysical oscillations ever discovered. It is thought that they are being generated at or near the surfaces of these neutron stars. To date, brightness oscillations with frequencies ranging from $\sim 300$ Hz to more than $1200$ Hz have been discovered in some sixteen neutron stars in LMXBs. High-frequency oscillations are observed both during thermonuclear X-ray bursts [35–37] and during intervals of accretion-powered emission [41,42]. They appear to be a characteristic feature of disk-accreting neutron stars with weak magnetic fields.

In this review we focus on the high-frequency quasi-periodic oscillations (QPOs) seen in the accretion-powered emission. The weak-field neutron stars in which these kilohertz QPOs have been discovered were studied extensively with EXOSAT and Ginga, and a detailed physical picture was developed based on their $2$–$20$ keV X-ray spectra and $1$–$100$ Hz X-ray variability (see [11,12,18,19]). In this picture, LMXBs with weak-field neutron stars fall into two classes, called the “Z” and “atoll” sources after the shapes of the paths they trace, over time, in X-ray color-color...
2. CHARACTERISTICS OF KHZ QPOs

The kilohertz QPOs observed in the neutron-star LMXBs have strikingly similar properties [41,42]. These properties include:

High frequencies.—The frequencies of the kilohertz QPOs range from \( \sim 300 \text{ Hz} \) to \( \sim 1200 \text{ Hz} \).

High maximum amplitudes.—The strongest kilohertz QPOs have fractional rms amplitudes
as high as ∼15% of the total 2–60 keV countrate. The amplitudes of the kilohertz QPOs are much larger in the atoll sources than in the Z sources, where they are typically ≤ 1%.

High coherence.—QPOs with quality factors $Q \equiv \nu_{\text{QPO}}/\Delta \nu_{\text{QPO}} \sim 50–100$ are common; in some sources, the kilohertz QPOs have $Q$ values as high as 200.

Frequency–$M$ correlation.—In the Z sources, the frequencies of the kilohertz QPOs increase steeply with $M$. In many atoll sources, the frequencies of the kilohertz QPOs increase steeply with countrate (in at least some, the frequency-countrate relation shifted significantly between observations made just a few days apart [8,23]). Although the magnetic field strengths and accretion rates of the atoll and Z sources are thought to be quite different, their frequency-countrate and frequency-accretion rate correlations are very similar.

Similar frequency ranges.—The frequency ranges of the kilohertz QPOs are similar in the Z and atoll sources, even though these two types of sources are thought to have very different magnetic field strengths and accretion rates.

QPO pairs.—Kilohertz QPOs commonly occur in pairs and, except in Sco X-1 [41] (and possibly in 4U 1608–52 [23]), maintain a constant frequency separation $\Delta \nu$ as they shift up and down in frequency by hundreds of Hertz.

Relation to spin frequency.—X-ray brightness oscillations have been observed during X-ray bursts in four sources in which kilohertz QPO pairs have been detected (see [28,41]). The frequency $\nu_{\text{burst}}$ of the burst oscillation appears to be the stellar spin frequency $\nu_{\text{spin}}$ or twice $\nu_{\text{spin}}$ [36,37]. In these four sources, the separation $\Delta \nu$ between the kilohertz QPOs appears to be $\nu_{\text{burst}}$ or half $\nu_{\text{burst}}$.

Photon-energy dependence.—The relative amplitudes of many kilohertz QPOs increase steeply with photon energy in the 2–15 keV energy band.

In order to be successful, a model of the kilohertz QPOs must explain these characteristics and be consistent with the data obtained previously on the 2–20 keV X-ray spectra and 1–100 Hz X-ray variability of the Z and atoll sources. We now review the proposed models, comparing them with these requirements.

3. KILOHERTZ QPO MODELS

3.1. Photon bubble oscillations

Photon bubble oscillations (PBOs) were first proposed as a model for X-ray brightness variations in accretion-powered pulsars (see [14] and references therein), where the strong stellar magnetic field funnels the flow onto the magnetic polar caps, producing a mass flux onto the caps that is locally super-Eddington. Under these conditions, photon “bubbles” form in the accretion column above the caps, rise, and burst, causing brightness variations. In the initial simulations [14], which considered an accretion rate of 0.2 $M_E$ and a surface dipole magnetic field of $3 \times 10^{12}$ G, brightness fluctuations occurred over a narrow range of frequencies at a few thousand Hertz.

Klein et al. [15] suggested that the same mechanism is responsible for the kilohertz QPO pairs, and predicted that additional QPOs would be found at higher frequencies. Observations of Sco X-1 performed to check this [44] showed no evidence of higher-frequency oscillations down to fairly small amplitudes. However, if there is scattering material around the neutron star magnetosphere, as inferred from EXOSAT and Ginga observations (see Fig. 1), the observed amplitudes of PBOs with frequencies above a kilohertz would be significantly reduced. In more recent simulations carried out for a magnetic field of $5 \times 10^9$ G, one or two QPOs with rms amplitudes up to 3% occurred, with frequencies that increased with increasing accretion rate, accompanied by weaker higher-frequency QPOs [16]. The relevant three-dimensional radiation-hydrodynamics is very complicated, making it hard to achieve analytic insights (but see [2]) and difficult to survey the full parameter space numerically.

The locally super-Eddington mass fluxes required for photon bubbles to form are not expected in the atoll sources, which are thought to have weak magnetic fields and total accretion rates as low as ∼ 0.003 $M_E$, but suitable conditions may exist in the Z sources. If the Z-source kilohertz QPOs are PBOs, then the atoll-source kilohertz QPOs, which appear very similar (see §2), would have to be generated by a different mechanism. It remains to be seen whether PBOs
can explain key characteristics of the kilohertz QPO pairs, such as their constant frequency separation and the steep increase in their relative amplitude with increasing photon energy.

### 3.2. Disk oscillations

Titarchuk & Muslimov [39] have proposed that the frequencies of the burst oscillation and the two kilohertz QPOs seen in many sources (see §2) are the frequencies of three specific oscillation modes of the accretion disk. In this model, as in all models in which the kilohertz QPOs are produced by disk oscillations, it is difficult to understand how QPOs with amplitudes as large as 15% of the total countrate can be generated, because typically \( \lesssim 20\% \) of the accretion luminosity is released in the accretion disk. In addition, one must explain why only three of the large number of disk oscillation modes produce large-amplitude brightness oscillations.

No proposal was made by Titarchuk & Muslimov for how the modes they discussed are excited, why only the modes at a particular radius are excited, or how they produce large-amplitude brightness oscillations. Moreover, the expression for mode frequencies that they used generally predicts that the \( \text{ratio} \) of the frequencies of two different modes will remain constant as the frequencies change, whereas it is the \( \text{separation} \) between the kilohertz QPOs in a pair that is nearly constant. For the frequency separation to be constant in this model, one must appeal either to a coincidence or to some physical principle that has not yet been elucidated. A more serious difficulty is that the method [40] used by Titarchuk & Muslimov to compute the frequencies of oscillation modes in the centrifugally supported, strongly sheared accretion disk flow is a perturbation method designed for computing the small changes in the frequencies of the nonradial oscillation modes of a spherical, gas-pressure-supported star caused by slow, uniform rotation.

### 3.3. Magnetospheric Keplerian frequency and beat-frequency oscillations

The magnetospheric beat-frequency model was developed [1,22] to explain the single, \( \sim 15-60 \) Hz “horizontal branch oscillation” (HBO) observed in the Z sources (see §1). In this model, the frequency of the HBO is the difference between the Keplerian orbital frequency \( \nu_{Km} \) at the main radius where the stellar magnetic field picks up and channels gas from the accretion disk onto the magnetic polar regions (the “main gas pick-up radius”) and the stellar spin frequency \( \nu_{\text{spin}} \). In applying the magnetospheric beat-frequency idea to the kilohertz QPO pairs, Strohmayer et al. [35] interpreted the frequency of the higher-frequency QPO in a pair as \( \nu_{Km} \) and the frequency of the lower-frequency QPO as \( \nu_{Km} - \nu_{\text{spin}} \).

An attractive feature of the magnetospheric beat-frequency interpretation of the kilohertz QPO pairs is that it explains naturally why the frequency separation between the QPOs in a pair is nearly constant in most sources and equal to the burst oscillation frequency or half this frequency. However, there are many serious difficulties with this interpretation [28]. For example, no mechanism has been found that will generate a relatively coherent QPO at the orbital frequency at the main gas pick-up radius. The high observed coherence of the kilohertz QPOs is also a serious difficulty for the magnetospheric beat-frequency interpretation. HBOs and kilohertz QPO pairs occur simultaneously in at least five of the six known Z sources (see [42]), so the magnetospheric beat-frequency model cannot explain both types of QPO. The fact (see Fig. 2) that HBOs are seen only in the Z sources, which are thought to have magnetic fields \( \sim 10^9-10^{10} \) G, and have higher amplitudes in the Z sources that have stronger magnetic fields, whereas the kilohertz QPOs are much stronger in the atoll sources, which are thought to have magnetic fields \( \sim 10^7-5 \times 10^9 \) G, is strong evidence that the HBOs are magnetospheric and that the kilohertz QPOs are not.

### 3.4. Sonic-point Keplerian frequency and beat-frequency oscillations

The sonic-point model [28] was developed specifically to explain the kilohertz QPO pairs. In this model, some accreting gas spirals inward in nearly circular Keplerian orbits until it is close to the neutron star, where radiation forces or general relativistic effects cause a sudden increase in the inward radial velocity, which becomes super-
sonic within a small radial distance. The sharp increase in the radial velocity is usually caused by the drag exerted on the accreting gas by radiation from the star, but may instead be caused by general relativistic corrections to Newtonian gravity if the gas in the Keplerian disk flow reaches the innermost stable circular orbit (ISCO) without being significantly affected by radiation. For convenience, the radius at which the radial velocity increases is referred to as the “sonic point”, even though the sonic point itself is not especially significant in this model.

In the sonic-point model, the frequency of the higher-frequency QPO in a pair is the orbital frequency \( \nu_{\text{Ks}} \) near the sonic radius. Density fluctuations at this radius are mapped onto the stellar surface, producing an X-ray beam that rotates around the star with frequency \( \nu_{\text{Ks}} \). A distant observer therefore sees quasi-periodic X-ray spectral and brightness oscillations with frequency \( \nu_{\text{Ks}} \).

The frequency of the lower-frequency QPO in a pair is the difference between this orbital frequency and the stellar spin frequency. The lower-frequency QPO is generated by weak X-ray beams produced by funneling of part of the accretion flow near the star by the star’s weak magnetic field. These beams rotate with the star and modulate the radiation drag acting on the gas at the sonic radius, modulating the inward mass flux and the luminosity at the sonic-point beat frequency \( \nu_{\text{Bs}} \) (\( \nu_{\text{Ks}} - \nu_{\text{spin}} \) or \( 2\nu_{\text{spin}} - \nu_{\text{Ks}} \)).

The sonic-point model is consistent with the accretion rates, stellar magnetic fields, and scattering optical depths inferred previously from EXOSAT and Ginga observations of the atoll and Z sources and accounts for the main features of the kilohertz QPOs (see [28]), including their high and variable frequencies, their high amplitudes and coherences, and the common occurrence of kilohertz QPOs in pairs. Like the magneto-spheric beat-frequency interpretation, the sonic-point beat-frequency model explains naturally why the frequency separation between the frequencies of a kilohertz QPO pair is nearly constant and equal to the burst oscillation frequency or half this frequency.

The sonic-point model also explains why the frequency ranges of the kilohertz QPOs are similar in sources with very different accretion rates and magnetic fields (see §4). Finally, it accounts

Figure 2. Correlation of the observed amplitudes of HBOs (left panel) and kilohertz QPOs (right panel) with the strength of the neutron star magnetic field (dashed lines) inferred from detailed radiation transport calculations of the X-ray spectrum [30–32]. The areas of the shaded circles are proportional to the measured amplitudes of the QPOs. The X’s indicate that no QPO has been detected. The three sources in the upper left are, from top to bottom, 4U 1728–34, 4U 1820–30, and 4U 1636–536. The four sources in the center are 4U 1735–444, GX 3+1, GX 9+9, and GX 13+1. The four sources in the lower right are the Z sources Sco X-1, GX 17+2, GX 5–1, and Cyg X-2. From [31].
Figure 3. View of the neutron star and disk along the rotation axis of the disk, which is rotating counterclockwise in this view. (a) The spiral trajectory followed by a single element of gas as it falls supersonically from the sonic radius to the stellar surface. (b) The spiral pattern of higher gas density formed by gas streaming inward along spiral trajectories with the shape shown in (a), from a clump orbiting near the sonic radius. The surface density of the disk flow is much smaller inside the sonic radius (lighter shaded region) than outside (darker shaded region), because of the sharp increase in the inward radial velocity at the sonic radius. The spiral trajectory and density pattern are from fully general relativistic numerical computations of the gas dynamics and radiation transport in the inner disk [28].

(see [28]) for the observed increase in QPO amplitude with increasing photon energy and the observed anti-correlation between kilohertz QPO amplitude and stellar magnetic field strength (see Fig. 2).

There are several unresolved questions related to the sonic-point model. Perhaps the most important is what fraction of the gas in the accretion disk does not couple strongly to the stellar magnetic field and therefore spirals inward in nearly circular orbits until it is close to the neutron star. This question is particularly important for the Z sources, some of which have inferred magnetic fields as high as $10^{10}$ G. Although it has been realized for more than two decades (see, e.g., [3,6,10,17]) that instabilities will allow some fraction of the accreting gas to penetrate deep inside the magnetosphere, it has not been possible to calculate this fraction reliably or to determine how much of this gas remains in nearly circular orbits. The kilohertz QPOs are strong evidence that at least some gas is orbiting close to the star, because the orbital frequencies at the radii where gas is expected to begin coupling to the stellar magnetic field are $\sim 5$–10 times lower than the frequencies of the kilohertz QPOs.

The sonic-point model is currently the most fully developed and successful explanation for the kilohertz QPOs. In the next section we describe some of the calculations that have been carried out to explore it.

4. THE SONIC-POINT MODEL

Figure 3 shows how the QPO at the sonic-point Keplerian frequency is generated in the sonic-point model. Gas in the accretion disk drifts slowly inward until it becomes exposed to radiation from the stellar surface. Once exposed, the gas loses its angular momentum to radiation drag
in a radial distance $\Delta r \lesssim 0.01 \, r$ and falls inward supersonically along spiral trajectories like that shown in Figure 3a. Gas falling inward from the sonic radius along spiral trajectories collides with the neutron star around its equator, producing an X-ray emitting equatorial ring, which is indicated by the grey ring around the star.

Magnetoturbulence, differential cooling, and radiation forces create density inhomogeneities ("clumps") in the gas in the accretion disk. Gas streaming inward from clumps orbiting near the sonic radius along trajectories with the shape shown in Figure 3a generates the density pattern shown in Figure 3b. Collision with the stellar surface of the denser gas from the clumps creates beams of brighter X-ray emission, like the beam indicated by the white dashed lines in Figure 3b, which move around the star’s equator with a frequency equal to the orbital frequency of the clumps, generating a quasi-periodic brightness oscillation with frequency $\nu_{bs}$.

The QPO at the sonic-point beat frequency is generated by weak X-ray beams that rotate with the star and modulate the radiation drag acting on the gas at the sonic radius once each beat period. As a result, the inward mass flux from each clump oscillates, causing the luminosity of the footprints to oscillate at the sonic-point beat frequency $\nu_{bs}$.

The nature of the transition to supersonic radial inflow is illustrated by the radial velocity and optical depth profiles shown in Figure 4. These profiles were calculated assuming that the azimuthal velocity of the gas in the disk is nearly Keplerian far from the star. Internal shear stress in the disk was assumed to create a constant inward radial velocity $v^r$, as measured in the local static frame, of $10^{-5}$. The half-height $h(r)$ of the disk flow at radius $r$ was assumed to be $\epsilon r$ at all radii, where $\epsilon$ is a constant and $r$ is the radius, and the kinetic energy of the gas that collides with the surface of the star was assumed to be

Figure 4. Results of fully general relativistic numerical computations [28] of the gas dynamics and radiation transport in the inner disk in the sonic-point model, for a neutron star of radius $5M$. (a) The inward radial velocity $v^r$ of the gas in the disk measured by a local static observer, as a function of the Boyer-Lindquist radial coordinate expressed in units of the stellar mass $M$. (b) The radial optical depth $\tau_r$ from the stellar surface through the disk flow to the radius shown on the horizontal axis. The four curves in each panel are labeled with the assumed accretion rate $\dot{M}$ measured in units of the accretion rate $\dot{M}_E$ that would produce an accretion luminosity at infinity equal to the Eddington critical luminosity.
converted to radiation and emitted from a band around the star’s equator with a half-height equal to $\epsilon R$. For simplicity, and to show the effects of radiation forces more clearly, any effect of the stellar magnetic field on the gas dynamics near the sonic transition was neglected.

Once the drag force exerted by the radiation from the stellar surface begins to remove angular momentum from the gas in the Keplerian disk, centrifugal support is lost and the gas falls inward, accelerating rapidly. Radiation that comes from near the star and is scattered by the gas in the disk is usually scattered out of the disk plane and hence does not interact further with the gas in the disk. Moreover, second and successive scatterings do not contribute proportionately to the azimuthal radiation drag force on the gas because the radiation field is aberrated by the first scattering and afterward carries angular momentum [25,26]. The interaction of the radiation with the gas in the disk was therefore treated by assuming that the intensity of the radiation coming from the star is attenuated as it passes through the gas in the disk, diminishing as $\exp(-\tau_r)$, where $\tau_r(r)$ is the Thomson scattering optical depth radially outward from the stellar surface to radius $r$, and that scattered radiation does not contribute to removal of angular momentum from the gas in the inner disk. In order to simply calculation of the radiation drag force, the differential scattering cross section was assumed to be isotropic in the frame comoving with the accreting gas (see [20]). The radiation field and the motion of the gas were computed in full general relativity.

Figure 4 shows that in this model, the sonic radius decreases with increasing accretion rate until it reaches the radius of the ISCO, after which it stops decreasing. Hence, the frequency of the sonic-point Keplerian frequency QPO increases steeply with increasing luminosity until the sonic radius reaches the radius of the ISCO, after which it stops increasing, as shown in Figure 5.

The sonic point must occur between the radius $R_{\text{ms}}$ of the innermost stable circular orbit and the radius where the two curves shown in Figure 6 intersect, which is $\sim 3R_{\text{ms}}$. The frequency of the sonic-point Keplerian frequency QPO is therefore restricted to a relatively small interval for all sources. For example, if the neutron stars in LMXBs all have masses $\sim 1.7 M_\odot$, then the frequencies of their sonic-point Keplerian frequency QPOs would all be confined to the range $\sim 400$–1300 Hz.

5. DISCUSSION

The discovery with RXTE of $\sim 1000$ Hz brightness oscillations from a large number of accreting neutron stars is a spectacular achievement that validates both the scientific expectations that led to the mission and the long years of hard work that were needed to bring it to fruition.

As explained in §3, the observations made to date strongly favor beat-frequency models of the kilohertz QPOs detected in the persistent emission. The magnetospheric beat-frequency interpretation of the kilohertz QPO pairs can account for the fact that their separation frequencies are approximately constant and related to the frequencies of the brightness oscillations seen during X-ray bursts, but this interpretation suffers from many serious difficulties. The sonic-point beat-frequency model also explains this fact and is the most fully developed and successful model, but it
is not yet confirmed. Therefore, it is important to consider which further observations and calculations would be particularly helpful in testing the beat-frequency hypothesis and discriminating between these two beat-frequency models.

The detection in the persistent emission of stable oscillations with frequencies harmonically related to those seen during X-ray bursts would confirm the spin-frequency interpretation of the latter and strongly support the beat-frequency interpretation of the kilohertz QPO pairs. On the other hand, detection of stable oscillations with frequencies not harmonically related to those seen during X-ray bursts would seriously undermine both models.

Beat-frequency models also predict that weak oscillations will be present at other special frequencies, such as the first overtone of the beat frequency and the sum of the spin frequency and the orbital frequency, and detection of any of these would provide strong support for these models. It is therefore important to search for QPOs at these frequencies. Even if none are detected, upper limits on their amplitudes would be valuable, because they would constrain models of the coronae around these neutron stars (see Fig. 1).

The sonic-point model predicts that the stronger the stellar magnetic field, the weaker the kilohertz QPOs will be [28]. In the magneto-spheric interpretation, one expects just the opposite. Figure 2 shows that the current data on kilohertz QPO amplitudes clearly favors the sonic-point model, but more precise and uniform amplitude measurements, as well as further progress in modeling the X-ray spectra of the Z and atoll sources, would help to clarify the situation.

Besides their intrinsic interest, the ∼1000 Hz oscillations discovered with RXTE provide a new tool with which to probe the nature of strong gravitational fields and the properties of dense matter. Modeling of general relativistic effects on the gas dynamics and radiation transport processes involved in the generation of kilohertz QPOs may provide evidence for the existence of innermost stable circular orbits [28,29] and frame-dragging [25,26,33,34], both of which are important predictions of strong-field general relativity.

If the frequencies of the highest-frequency kilohertz QPOs are orbital frequencies, as in beat-frequency models, these QPOs provide important new constraints on the masses and radii of the neutron stars in LMXBs and on the equation of state of neutron star matter [13,28,29,45]. The brightness oscillations observed during X-ray bursts may constrain the compactness of neutron stars [27,36,37].

New RXTE observations are continuing to yield significant fresh insights, and the rapid pace of important new discoveries is therefore likely to continue for many years.

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