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NEUTRINO PRODUCTION OF LEPTON PAIRS

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We have calculated the high-energy asymptotic value of the total cross-section for the processes ^{*)}

$$(a) \quad \nu_\mu + Z \longrightarrow \bar{\mu} + e^+ + \nu_e + Z$$

$$(b) \quad \nu_\mu + Z \longrightarrow \bar{\mu} + \mu^+ + \nu_\mu + Z$$

$$(c) \quad \nu_e + Z \longrightarrow e^- + \mu^+ + \nu_\mu + Z$$

$$(d) \quad \nu_e + Z \longrightarrow e^- + e^+ + \nu_e + Z$$

where Z indicates coherent scattering of the charged leptons from a nucleus. The two Feynman diagrams for these processes are

Fig 1.

where the four-fermion interaction is the usual weak interaction

$$H_w = \frac{G}{\sqrt{2}} [\bar{\Psi}_{(l_1)} \gamma_\lambda (1 + \gamma_5) \Psi_{(\nu_1)}] [\bar{\Psi}_{(\nu_2)} \gamma_\lambda (1 + \gamma_5) \Psi_{(l_2)}] \quad (1)$$

$$G = (1.01 \times 10^{-5}) / M_p^2, \quad (M_p = \text{mass of proton}).$$

*) The cross-sections obtained by replacing leptons by antileptons in (a) - (d) are the same.

Several estimates of processes (b) and (d), using the Weizsacker-Williams approximation, appear in the literature ¹⁾⁻⁴⁾. Shabalin ^{1),4)} has calculated both the high-energy limit and the threshold behaviour of reaction (b) in the Weizsacker-Williams approximation. Since these processes can be calculated exactly and since recent experiments at CERN with an incident ν_μ beam indicate the presence of lepton pairs, it is very important that the exact contribution of these processes be known before one attributes the presence of lepton pairs to the creation of an intermediate vector boson. One expects that vector boson production should be the dominant process because it is proportional to G while the above process goes as G^2 . However, it is known that the mass of the boson is sufficiently high and that the high-energy neutrino tail cuts off sufficiently fast so that the incoherent "elastic" process which goes as G^2

$$\nu_\mu + n \longrightarrow \mu^- + p \quad (2)$$

where n is a neutron, predominates in the CERN experiments ⁵⁾. Coherent lepton pair production is expected to differ from this by a factor $(\alpha^2 Z_1^2)/N$ per nucleus where N is the number of neutrons in the nucleus, and this could be a fraction of a percent for heavy nuclei, which is the order of magnitude of the observed production of lepton pairs ⁵⁾.

We have calculated these processes by noting that in the total cross-section, the lepton parts of the diagrams contribute as a symmetric tensor $T_{\mu\nu}(p_1, q)$ which can only depend on p_1 , the incident neutrino four-momentum and q , the four-momentum transfer from the nucleus (see Fig. 1). Gauge invariance on the electrodynamic part of this process says $q_\mu T_{\mu\nu} = T_{\mu\nu} q_\nu = 0$. Therefore $T_{\mu\nu}$ must have the form ⁶⁾

$$T_{\mu\nu}(p_1, q) = f_1(q^2, p_1) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + f_2(q^2, p_1) \left(p_{1\mu} - \frac{p_1 \cdot q}{q^2} q_\mu \right) \left(p_{1\nu} - \frac{p_1 \cdot q}{q^2} q_\nu \right) \quad (3)$$

The functions f_1 and f_2 can be found by computing $T_{\mu\mu}$ and $p_{1\mu} T_{\mu\nu} p_{1\nu}$. This procedure simplifies the γ matrix algebra considerably. $T_{\mu\nu}$ must then be adjoined to the nuclear part of the process, and it is the nuclear form factor $|F(q^2)|^2$ (we assume a spin zero target) that limits the total cross-section¹⁾⁻⁴⁾. The lepton phase space integrals were rewritten as integrals over four invariants describing the lepton part of the process, and then the integral over final nuclear momentum becomes an integral over the invariants q^2 and $q \cdot p_1$. With the proper choice of invariants the over-all Jacobian can be made very simple. One is thus left with a six-fold integral over invariants and the matrix element squared is just a ratio of polynomials in these invariants. The limits of integration are sufficiently complicated so that exact analytical calculation appears extremely tedious. It is, however, a relatively simple matter to extract the high-energy form of the cross-sections. Neglecting $q^2/m_T^2 \ll 1$ where m_T is the mass of the target nucleus and calculating for a completely polarized incident neutrino beam as is the experimental case, one finds

$$\sigma = \frac{2Z^2\alpha^2 G^2}{9\pi^3} \epsilon_1 \int_0^\infty \frac{|F(x_1)|^2}{\sqrt{x_1}} dx_1 \left[\ln \frac{\epsilon_1 \sqrt{x_1}}{\frac{1}{2}\lambda_1} + \frac{1}{3} \ln \frac{\epsilon_1 \sqrt{x_1}}{\frac{1}{2}\lambda_2} \right] + O(\epsilon_1)(4)$$

where ϵ_1 is the incident neutrino energy in the laboratory, $x_1 = q^2$, $\alpha = \frac{1}{137}$ and λ_1 and λ_2 are given in Table 1.

Process	λ_1	λ_2	ρ_1	ρ_2
(a) $e^+ \mu^-$	x_1	m_μ^2	β^2	m_μ^2
(b) $\mu^- \mu^+$	m_μ^2	m_μ^2	m_μ^2	m_μ^2
(c) $\mu^+ e^-$	m_μ^2	x_1	m_μ^2	β^2
(d) $e^- e^+$	x_1	x_1	β^2	β^2

Table 1

The result for process (b) is a factor of $\frac{8}{3}$ larger than the asymptotic cross-section of Ref. ¹⁾. The other results are new.

Introducing nuclear form factors (where the parameters have been chosen to fit the low momentum transfer electron scattering data) one finds

Case I :

$$F(x_1) = e^{-\frac{x_1}{2\beta^2}}$$

$$\beta = \frac{\sqrt{5}}{R_0} \quad [R_0 = 1.2 \times 10^{-13} \text{ cm. } A^{1/3}]$$

$$\sigma_I = \frac{4Z^2\alpha^2G^2}{9\pi^3} \left(\frac{\sqrt{\pi}}{2}\right) \varepsilon_1 \beta \left[\ln \frac{\varepsilon_1 \beta}{\frac{1}{2} p_1} + \frac{1}{3} \ln \frac{\varepsilon_1 \beta}{\frac{1}{2} p_2} \right] + O(\varepsilon_1) \quad (5)$$

Case II : [see Ref. ⁷⁾]

$$F(x_1) = \frac{1}{\left(1 + \frac{x_1}{\beta^2}\right)^2}$$

$$\beta = \frac{\sqrt{20}}{R_0}$$

$$\sigma_{II} = \frac{4Z^2\alpha^2G^2}{9\pi^3} \left(\frac{5\pi}{32}\right) \varepsilon_1 \beta \left[\ln \frac{\varepsilon_1 \beta}{\frac{1}{2} p_1} + \frac{1}{3} \ln \frac{\varepsilon_1 \beta}{\frac{1}{2} p_2} \right] + O(\varepsilon_1) \quad (6)$$

where p_1 and p_2 are given in Table 1.

The numerical coefficient in the logarithm in Eqs. (4) - (6) has no real meaning of course since the neglected terms of order ε_1 will always effectively change it. We obtained the numerical coefficients shown in the above table by isolating the leading terms and then integrating them with the correct phase space limits. In particular m_e^2 does not appear in the denominator of the logarithms for those processes involving electrons (this comes from the fact that $x_1 \gg m_e^2$ in the region of integration) as was noted in Ref. ⁴⁾ *). Unfortunately, it is quite

*) The fact that the nuclear form factor limits x_1 to be of the order or less than m_μ^2 is enough to lead to the appearance of m_μ^2 in the denominators of the logarithms for those processes involving muons.

difficult to get the correction term $O(\varepsilon_1)$ in the above. All mass terms in the traces (i.e., those proportional to m_e^2 , m_μ^2 , and x_1) can contribute to it. It is probably negative but that is not certain.

It is not really correct therefore to apply formulae (5) and (6) to the CERN experiments since the asymptotic form can be expected to hold when

$$\ln \frac{\varepsilon_1 \beta}{\frac{1}{2} \rho^2} \gg 1 \quad (7)$$

One has

$$\left\langle \frac{\varepsilon_1}{m_\mu} \right\rangle_{n.s.} = 14.8 \quad (8)$$

$$\left\langle \frac{\varepsilon_1}{m_\mu} \ln \frac{\varepsilon_1}{m_\mu} \right\rangle_{n.s.} = 42.8 \quad (9)$$

where the average is taken over the calculated neutrino spectrum at CERN. This gives a logarithm in Eq. (7) between 3 and 4 for medium and heavy weight nuclei and the form factors I and II. If one puts numbers into the asymptotic formulae anyways, one finds

$$\langle \sigma_{II}(\mu^- \mu^+) \rangle_{n.s.} = 2.94 \times 10^{-44} \left(\frac{Z^2}{A^{1/3}} \right) \left[\ln \frac{123}{A^{1/3}} + .71 \right] \text{cm}^2 \quad (10)$$

which gives

$$\langle \sigma_{II}(\mu^- \mu^+) \rangle_{n.s.} = 1.25 \times 10^{-40} \text{cm}^2 \quad (11)$$

for $^{208}_{82}\text{Pb}$. (The results for $\sigma_I(\mu^- \mu^+)$ are 30 % smaller.) If one uses $\langle \sigma_{el}(\mu^-) \rangle_{n.s.} = 0.63 \times 10^{-38} \text{cm}^2/\text{neutron}$ which represents the cross-section for the elastic process (2) averaged over the CERN neutrino spectrum and corrected for the Pauli principle⁸⁾, then

$$\frac{\langle \sigma_{\Pi}(\bar{\mu}\mu^+) \rangle_{n.s.}}{\langle \sigma_{el.}(\bar{\mu}) \rangle_{n.s.}} = 1.58 \times 10^{-4} \quad \text{for } {}^{208}_{82}\text{Pb} \quad (12)$$

and similarly

$$\frac{\langle \sigma_{\Pi}(\bar{\mu}e^+) \rangle_{n.s.}}{\langle \sigma_{el.}(\bar{\mu}) \rangle_{n.s.}} = 1.49 \times 10^{-4} \quad \text{for } {}^{208}_{82}\text{Pb} \quad (13)$$

These numbers are too small by about a factor of 60 or so to explain the recent CERN results ^{5) *)}.

Shabalin's threshold calculation for process (b) indicates that the actual cross-section for this process probably lies below our asymptotic estimate, however, since the actual experimental conditions are neither threshold nor quite asymptotic, since all mass terms (m_e^2 , m_μ^2 , x_1) must be kept to correctly calculate $O(\epsilon_1)$, and since the cross-section can be calculated exactly, one should really have a correct numerical calculation before disregarding this process.

We have also calculated the incoherent contribution to the asymptotic cross-sections for processes (a) - (d) coming from Z protons and N neutrons. One gains something because the nucleon electromagnetic form factors die off slower than those of a heavy nucleus (note, if the nucleon form factors were to have a "core" the total cross-section for the above processes would be much larger) but of course one loses the extra power of Z compared to the coherent process. Our calculations indicate that the incoherent cross-section for (b) is about 20 % of the coherent cross-section for ${}^{208}_{82}\text{Pb}$ if the corresponding asymptotic form is averaged over the CERN neutrino spectrum.

*) The extra factor of $\frac{8}{3}$ in Eq. (4) and the use of a more realistic nuclear form factor explains why the present asymptotic estimate, Eq. (12), is one order of magnitude larger than that of Shabalin ^{1), 4)}.

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