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A PHENOMENOLOGICAL STUDY OF VERY HIGH ENERGY COLLISIONS -
ANGULAR DISTRIBUTION OF SECONDARIES IN JETS

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A B S T R A C T

The experimental data on angular distributions of secondaries in jets are examined from the point of view of a model being an idealization of multiperipheral models. We assume that pionization secondaries are emitted independently from each other but with small transverse momenta and small average inelasticity, as required by peripherality. This leads to a quasi-rectangular distribution of the $\log \tan \theta$ variable. The statistical properties of this distribution are studied using the Monte-Carlo method. A semi-quantitative analysis shows that the model explains the main features of quasi-elementary collisions ($N_h \leq 5$, $n_s \leq 20$) in the energy region $10^{12} - 10^{14}$ eV, in particular the existence of the two-centre events.

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I. INTRODUCTION

I.1 In the present development of our knowledge of very high energy phenomena, one can observe two trends. People representing the first one, mainly experimenters, introduce simple intuitive pictures and use appropriate, relatively easily measurable parameters. These pictures help very much to clear up the experimental material. They are, however, usually rather obscure from the purely theoretical point of view. Therefore, in spite of a large accumulation of empirical information, our understanding of the meaning of this information remains poor.

The second trend is more formal ; an attempt is made to build a theory of high energy interactions starting from very general principles, which turned out to be useful in investigating lower energy interactions. In this way, one can get some extremely interesting qualitative results, the meaning of which is evident, but at present it is very difficult to obtain some more or less quantitative predictions to be compared with experiment. Moreover, it is not very clear how many of the above mentioned general theoretical predictions would look when formulated in the language used by experimenters.

In this situation one has to re-examine the meaning of many experimental findings. To do this, to fill the gap between intuitive pictures and more fundamental approaches, it seems to be necessary to use some phenomenological models.

Let us visualize a high energy collision by the drawing in Fig. 1 ; as a result of their interaction, incident nucleons lose a small fraction of their energy (for instance due to a virtual emission or absorption of a pion or a "pomeron") and, with large probability, are excited. The energy lost by incident particles is transferred to a group of produced particles, mainly pions. The process symbolized by the black box in Fig. 1 will be called pionization process.

The aim of this paper is to examine the angular distribution of pionization secondaries, particularly the possible origin of the fire-balls ; as for the pionization process, we take an idealization of currently discussed multiperipheral model as our phenomenological model. This note contains a semi-quantitative discussion of this model. A detailed comparison with experiment will be done separately.

I.2 When one observes a very high energy collision in emulsion, a technique commonly used in cosmic ray physics, the angles of emission of secondaries are relatively simple to measure. It is, therefore, obvious that angular distribution of secondaries has been studied in cosmic ray physics in more detail than anything else. In this study, a particular co-ordinate $x = \log \tan \theta$ (θ is the angle of emission of a secondary) is found to be very useful. Older theories of particle production ^{*}), like for instance Fermi-Landau statistical theory, predicted a Gaussian distribution in the x co-ordinate. It is observed, however, that the actual angular distribution is quite different from the distribution expected from the Fermi-Landau theory. For laboratory energies $> 10^{12}$ eV a number of events have been found whose angular distributions show two maxima and which look as if secondaries were emitted from two centres moving in opposite directions in the c.m. system ¹⁾⁻⁴⁾. This situation has given rise to the idea of the fire-ball model. A detailed statistical analysis proves that experimental results are certainly in disagreement with a Gaussian type of distribution ^{5),6)}. The following picture of the collision seems to be compatible with experimental data.

^{*}) A systematic review of the development of multiple production theories up to 1959 can be found in Ref. ⁷⁾. A review of experimental results obtained up to 1960 is given in Ref. ⁸⁾.

In the c.m. system, two incident nucleons in the result of their interaction lose a small fraction of their energy and almost do not change their direction of motion. The energy lost by incident particles is taken away by two "fire-balls" moving in opposite directions and almost colinearly with incident particles. Fire-balls decay isotropically in their own rest frames and it is their motion which produces the collimation of secondaries.

The super-high energy collisions at lab. energies $E^{\text{lab}} > 10^{12} \text{ eV}$ are usually observed in nuclear emulsion. Of course, these are collisions with nuclei and additional selection criteria must be imposed to pick out from a set of experimental events a sample which could be considered as consisting of quasi-elementary collisions. As such criteria, one usually takes ^{3),5)} $N_h \leq 5$, $n_s \leq 20$, where N_h and n_s are the number of heavy and relativistic tracks in a star, respectively. The fire-ball model describes as well events with $N_h \leq 5$, $n_s \leq 20$ but not those with $N_h \leq 5$, $n_s > 20$. It is, however, interesting that events with $N_h > 8$ (collisions with heavy nuclei) show also a kind of double maximum structure ^{9),10)}.

One can ask whether fire-ball model is only a kinematical picture or whether fire-balls have certain dynamical meaning? No serious answer to this question exists at present. The people from the Krakow group, who were among the principal authors of the model, were always very careful in this respect. Frequently, however, fire-balls are considered as certain dynamical objects, for instance excited states of the pion field, and the two-centre structure of $\log \tan \theta$ distribution is understood as evidence for a very particular kind of correlation between secondaries.

I.3 In this section, we shall formulate the basic assumptions of this paper. First of all, we shall assume that energies $k_0^{(i)}$ of the pionization secondaries are practically limited as follows :

$$k_0^{\min} < k_0^{(i)} < k_0^{\max} \quad (1)$$

where $k_0^{\max}$ is a small fraction of the total available energy (about 7%, say). This limitation expresses the observed smallness of the average inelasticity coefficient ^{*)}. Furthermore, we shall assume that, unless $k_0^{(i)}$ are not in an immediate neighbourhood of boundaries specified by (1), pionization secondaries are emitted independently from each other remembering only the direction of flight of the incident particle. By independent emission we mean that only correlations between secondaries are those imposed by over-all conservation laws. For simplicity, constraints due to energy-momentum conservation will be neglected as being not important when the average number of secondaries is large (we shall consider in this communication only events with number of charged pionization secondaries larger than 8 and less than 21). The only form of the probability distribution which is compatible with our assumptions and with the requirement of invariance with respect to Lorentz transformations keeping the incident particle direction of flight unchanged is ^{**))}

$$dp(N_s) = \text{const.} \frac{1}{N_s!} \prod_{i=1}^{N_s} \varphi(k_T^{(i)}) \delta(k^{(i)2} - \mu^2) d^4 k^{(i)} \quad (2)$$

$$k_0^{\min} < k_0^{(i)} < k_0^{\max}$$

*) By inelasticity we shall understand the fraction of energy transferred to all secondaries, except those resulting from the excitation and decay of incident nucleons.

**) φ can depend only on k_0, k_L and k_T . k_0 and k_L are not independent due to the δ function. Thus

$$\varphi = \varphi(k_T, k_L)$$

Moreover φ cannot depend on k_L since $dp(N_s)$ must be invariant with respect to Lorentz transformations leaving the "privileged" direction in momentum space unchanged. This argument is due to K. Wilson.

$k^{(i)}$, $k_0^{(i)}$, $k_L^{(i)}$ and $k_T^{(i)}$ are 4-momentum, energy, longitudinal momentum and transverse momentum of i^{th} secondary. (2) can be written in the form

$$dp(N_s) = \text{const} \frac{1}{N_s!} \prod_{i=1}^{N_s} f(k_T^{(i)}) dk_T^{(i)2} \frac{dk_L^{(i)}}{k_0^{(i)}} \quad (3)$$

$$k_0^{\min} < k_0^{(i)} < k_0^{\max}$$

In the laboratory system almost all secondaries are very fast. Thus, due to the observed narrowness of the transverse momentum distribution, there is approximately :

$$dp(N_s) \approx \text{const} \frac{1}{N_s!} \prod_{i=1}^{N_s} f(k_T^{(i)}) dk_T^{(i)2} \frac{dk_L^{(i)}}{k_L^{(i)}} \quad (4)$$

$$k_L^{\min} < k_L^{(i)} < k_L^{\max}$$

From (3) one can easily get (neglecting constraints due to over-all energy-momentum conservation) the energy-momentum distribution for our particle

$$dp = \frac{f(k_T) dk_T^2 dk_L}{k_0 \int f(k_T) dk_T^2 \cdot \ln \left(\frac{k_L^{\max} + k_0^{\max}}{k_L^{\min} + k_0^{\min}} \right)} \quad (5)$$

Under the same condition as above, (3) leads also to a logarithmic increase of the average multiplicity with the incident energy.

$$\langle N_s \rangle = \left(\int f(k_T) dk_T^2 \right) \ln \left(\frac{k_L^{\max} + k_0^{\max}}{k_L^{\min} + k_0^{\min}} \right) \sim \log E \quad (6)$$

General hypotheses leading to (5) and (6) (as well as their relation with the Regge-type behaviour of the elastic diffraction scattering) were studied in more detail by K. Wilson ^{*)} - the reasoning presented above is in fact a simplified and slightly modified version of one of Wilson's results ¹¹⁾.

(5) and (6) are the general results of the multiperipheral model proposed by Amati, Fubini and Stanghellini ¹²⁾. Actually, (5) and (6) are more general than (2) since to get (5) and (6) it is not necessary to assume independent emission of all secondaries.

It is plausible that our model is an idealization of the mechanism found in the Amati, Fubini and Stanghellini model, as well as in other multiperipheral models related to it. All these models are characterized by a successive emission of secondaries from a single virtual line connecting incident particles. Increasing energy increases the number of elementary emission acts but the topological structure of the graph responsible for inelastic processes remains unchanged. Such a mechanism guarantees the smallness of inelasticity and transverse momenta of secondaries. On the other hand constraints on longitudinal momenta are rather weak, in the sense that the fact of production of a secondary with a definite longitudinal momentum has in general not much influence on the probability than another secondary will have a given longitudinal momentum.

The difference between our approach and multiperipheral models is that in the latter, particles are in general produced in groups so that even if these groups were emitted independently the fact of their existence introduces certain degree of correlation between individual secondaries. This type of correlation is, however, much attenuated

*) See also a very interesting paper by L. Van Hove ¹³⁾.

if one considers only charged secondaries, as will be the case in our analysis.

It is worthwhile to remark that (4) is invariant under the scale transformation applied to all longitudinal momenta

$$k_L' = c k_L$$

except that the boundaries specified by (1) change ^{*)}. This is another resemblance to multiperipheral models which are known to be asymptotically scale invariant.

I.4 Let us introduce the variables

$$x^{(i)} = \log \tan \Theta^{(i)} = \log (k_T^{(i)} / k_L^{(i)})$$

(4) can be written in the form

*) In the laboratory system one has to apply the scale transformation to $k_L^{\max}$ only, while $k_L^{\min}$ remains fixed, (see section II.). This is connected with the fact that in this system, target particle must be from definition at rest and the scale transformation does not apply to it.

$$dp(N_s) = \text{const} \frac{1}{N_s!} \prod_{i=1}^{N_s} g(k_T^{(i)}) dk_T^{(i)2} \frac{dk_L^{(i)}}{k_L^{(i)}} \mathcal{V}(k_L^{\max} - k_L^{(i)}) \mathcal{V}(k_L^{(i)} - k_L^{\min})$$

$$= \text{const} \frac{1}{N_s!} \prod_{i=1}^{N_s} g(k_T^{(i)}) dk_T^{(i)2} dx^{(i)} \mathcal{V}(k_L^{\max} \cdot 10^{x^{(i)}} - k_T^{(i)}) \mathcal{V}(k_T^{(i)} - k_L^{\min} \cdot 10^{x^{(i)}}) \quad (7)$$

where $\mathcal{V}(z)$ is the step function

$$\mathcal{V}(z) = \begin{cases} 1 & z > 0 \\ 0 & z < 0 \end{cases}$$

The $\log \tan \theta$ distribution given by (7) has for $k_L^{\max} \gg k_L^{\min}$ a long flat part. If $k_T^{(i)}$ were fixed, the $x^{(i)}$ distribution would be a rectangular one. Due to the spread of $k_T^{(i)}$'s, the rectangle boundaries are diffuse (see section II).

At first sight, it may appear that (7) is incompatible with experiment which indicates that $\log \tan \theta$ distribution has a two-maximum structure. Notice, however, that two maxima are visible only for events with sufficiently large anisotropy⁵⁾. This fact is explained in the framework of the fire-ball model in the following way.

The collimation of secondaries is a result of the motion of fire-balls; faster fire-balls yield, larger anisotropies are. Therefore, there must exist a correlation between the separation of two emitting centres and anisotropy.

The fire-ball model was proposed at the time when theoretically a Gaussian shape of the x distribution was expected. It needed a considerable effort to convince people that frequent appearance of two maximum events is a real physical effect, incompatible with the Fermi-Landau theory. Two centre events cannot be generated in an appreciable number from a Gaussian distribution; it was, however, not shown that

the same is true for a rectangular or rather quasi-rectangular distribution. We shall show that in the energy region where the overwhelming majority of jets were found, jets with sufficiently high multiplicity generated from the rectangular x distribution can imitate the picture expected on the basis of the fire-ball model.

II. ELEMENTARY CONSIDERATIONS

II.1 Integrating (5) with respect to k_T one can get the x distribution. To do this, however, one has to know the distribution of transverse momentum k_T . Taking as $\rho(k_T)$ the result of accelerator energy measurements, and approximating it for simplicity of calculation by a triangle one gets for dp/dx the curve shown in Fig. 2. dp/dx looks like a rectangle with diffuse edges. The exact form of these edges is - in our model - determined by the detailed form of $\rho(k_T)$. However, the physically most important feature of the x distribution in the model we are discussing is the existence of a long flat part in this distribution. Since the purpose of this paper is not to make a very detailed comparison with experiment but rather investigate the general consequences of a flat x distribution, we shall approximate dp/dx by a rectangle. We shall also neglect a slight asymmetry of dp/dx with respect to the point $\log \frac{1}{\gamma}$ (γ is the Lorentz factor of the transformation from laboratory to c.m. system), and we shall not consider fluctuations of the symmetry point of the x distribution, around $\log \frac{1}{\gamma}$, these fluctuations are due to not equal energy losses of incident nucleons. This last restriction corresponds to the fact that, in this paper, we are interested exclusively in the properties of the pionization process.

The tangent of the minimal angle in laboratory system is roughly ^{*)}

$$\tan \Theta_{\min} \approx \frac{\langle k_T \rangle}{k_L^{\max}} \approx \frac{\langle k_T \rangle}{k_0^{\max}} \quad (8)$$

Therefore, one half of the length of our rectangle is

$$\begin{aligned} l &= \log \frac{1}{\gamma} - \log \tan \Theta_{\min} \approx \\ &\approx \log \frac{k_0^{\max}}{\gamma \langle k_T \rangle} = \log \frac{\mathcal{K} \sqrt{2E}}{\langle k_T \rangle} \end{aligned} \quad (9)$$

where $\mathcal{K} = k_0^{\max}/E$, and E is the incident particle laboratory energy.

The x distribution for one particle will be therefore approximated by

$$dp = \frac{1}{2l} dx, \quad \log \frac{1}{\gamma} - l < x < \log \frac{1}{\gamma} + l \quad (10)$$

II.2 The parameter $\mathcal{K}$, maximum fraction of total energy, which can be taken away by one pionization secondary is, of course, closely related with the average inelasticity $\langle K \rangle$ (K is the inelasticity in laboratory system). From (5) we get the average energy of a pionization secondary

$$\langle k_0 \rangle = \frac{k_0^{\max} - k_0^{\min}}{\ln \left(\frac{k_L^{\max} + k_0^{\max}}{k_L^{\min} + k_0^{\min}} \right)} \quad (11)$$

*) Further we shall work exclusively in the laboratory system.

Therefore the average inelasticity is

$$\langle K \rangle = \frac{\langle N_s \rangle \langle k_0 \rangle}{E} \approx x \int g(k_T) dk_T^2 \quad (12)$$

$k_L^{\min}$ can be determined if $k_L^{\max}$ is known from symmetry reasons. Notice that the minimum energy in the laboratory system must transform into the maximum energy when we pass to the anti-laboratory system and that for nucleon-nucleon collisions there is a symmetry between these two reference frames. Thus

$$\begin{aligned} k_0^{\min} &= \frac{E}{m} (k_0^{\max} - \beta k_L^{\max}) \approx \frac{E}{m} (k_0^{\max} - k_L^{\max}) = \\ &= \frac{E (k_T^2 + \mu^2)}{m (k_0^{\max} + k_L^{\max})} \approx \frac{(k_T^2 + \mu^2)}{2 m x} \end{aligned} \quad (13)$$

Collecting (6), (12) and (13) we obtain

$$\langle K \rangle = \frac{x \langle N_s \rangle}{\ln \left[\frac{4 m x^2 E}{(k_T^2 + \mu^2) \left(1 + \sqrt{1 - \frac{4 m^2 x^2}{(k_T^2 + \mu^2)}} \right)} \right]} \quad (14)$$

Cosmic ray experiments for energies larger than 10^{12} eV are very strongly biased against low multiplicity events. However, at lower energies, the measurements of average multiplicity are much more reliable. For instance, at 250 GeV, Dobrotin and Slavatsky¹⁴⁾ found the average number of charged secondaries $\langle n_s \rangle = 8 \pm 1$, in an experiment where the bias was not very important. $\langle N_s \rangle$ in (14) is the pionization multiplicity of charged and neutral pions thus roughly

$$\langle N_s \rangle \approx \frac{3}{2} \langle n_s \rangle - 4$$

if one assumes as suggested by certain experimental data ¹⁵⁾ as well as by the Amati, Fubini and Stanghellini model that, besides the pionization secondaries, two excited barions are produced in a nucleon-nucleon collision. For 250 GeV, $k_T = 0.4$ MeV, and $\mathcal{K} \approx 0.07$ one gets a reasonable estimate $\langle K \rangle \approx 0.2$. The above reasoning is very rough ; in fact, it is possible that we have slightly over-estimated the inelasticity - k_T in (14) is the transverse momentum of fastest pionization secondaries in the c.m. system and it may be smaller than the average transverse momentum. For secondaries whose energies are close to the boundaries specified by (1), a certain amount of correlation between k_T and k_L has to be expected. Such a correlation, however, would have a very local effect only, changing the shape of the edges of the x distribution without much influence on our main results. Let us notice also that a relation between inelasticity, multiplicity and anisotropy very similar to that obtained on the basis of fire-ball model, can be derived from our model too.

II.3 The second moment of the x distribution (10) with respect to the point $x = \log \frac{1}{8}$ is

$$\langle (x - \log \frac{1}{8})^2 \rangle = \frac{\ell^2}{3} \quad (15)$$

The anisotropy σ of a jet is defined by ^{*)}

*) In the following, N_s will denote the number of charged pionization secondaries.

$$\sigma^2 = \frac{1}{N_S - 1} \sum_{i=1}^{N_S} (x_i - \langle x \rangle)^2$$

$$\langle x \rangle \equiv \log \frac{1}{\gamma_c} = \frac{1}{N_S} \sum_{i=1}^{N_S} x_i \quad (16)$$

Since γ_c fluctuates around γ^7 and $N_S \gg 1$, we can approximately put

$$\langle \sigma \rangle \approx \frac{1}{\sqrt{3}} = \frac{1}{2\sqrt{3}} \log \frac{2\pi^2 E}{\langle k_T \rangle^2} \quad (17)$$

We see that in the model discussed, average anisotropy increases in the asymptotic limit logarithmically with energy.

Another quantity will be used which was introduced as a measure of the separation of fire-balls ⁵⁾

$$D = \frac{N_e - N_i}{N_e + N_i} \quad (18)$$

$N_e(N_i)$ is the number of particles for which $|x - \langle x \rangle|$ is larger (smaller) than 0.674σ . For a Gaussian x distribution $\langle D \rangle = 0$. Fire-ball model predicts $\langle D \rangle > 0$.

III. SOME GENERAL FEATURES OF THE MODEL

III.1 In order to study statistical features of the rectangular distribution, the Monte-Carlo method was used. Calculations were done with the help of the Ferranti-Mercury computer for three samples of jets with multiplicities 8, 14 and 18 respectively.

In this section, we shall be interested only in those general characteristics of high-energy collisions which can be discussed without detailed knowledge of the energy spectrum of the primary cosmic radiation and without an estimation of experimental biases ^{*)}. For this purpose it will be convenient to use particularly normalized co-ordinates, independent of the length of the basic rectangle.

$$\frac{x - \langle x \rangle}{\langle \sigma \rangle} \quad , \quad \frac{x - \langle x \rangle}{\sigma}$$

can be chosen ^{**)} as such co-ordinates. The second one of these co-ordinates was used to prove the statistical significance of the existence of two-centre jets ⁵⁾.

^{*)} The constraints resulting from over-all energy conservation are ignored in our Monte-Carlo calculation, as being unimportant for considered multiplicities due to the smallness of $\mathcal{H}$; the results of this section are, however, independent of the actual value of $\mathcal{H}$, provided $\mathcal{H}$ is sufficiently small.

^{**)} We call the reader's attention to the difference between $\langle \sigma \rangle$ the average anisotropy and σ , the anisotropy of a particular jet. $\langle x \rangle$, $\langle \sigma \rangle$ and σ were defined in the preceding section.

The general idea of this section is to apply to Monte-Carlo jets the same investigation procedure one uses with respect to experimental events and to see what will come out of it.

III.2 First of all, to become more familiar with the properties of the model, we sampled 300 Monte-Carlo jets, 100 for each multiplicity. We found that dividing our sample of jets according to the condition

$$\langle \sigma \rangle < \sigma \quad \text{or} \quad \langle \sigma \rangle > \sigma \quad (19)$$

we get roughly two-centre and Gaussian jets, respectively. To show this we use the co-ordinate $(x - \langle x \rangle)/\sigma$ and a non-linear scale in which a normal distribution looks like a straight horizontal line. The resulting angular distributions, for each multiplicity, are presented in Figs. 3, 4 and 5. More accurate calculation shows that jets with $\sigma > \langle \sigma \rangle$ represent about 48% of all jets, almost independently of multiplicity - this is not evident from Figs. 3, 4 and 5 due to fluctuations. Jets with $\sigma < \langle \sigma \rangle$ are almost Gaussian, with small deviation for large $(x - \langle x \rangle)/\sigma$ - it is amusing, although not very significant, that this deviation is also present in backward cones of experimental jets with small anisotropy, i.e., for angles which are more accurately measured.

An additional criterion

$$\begin{aligned} \sigma > \langle \sigma \rangle, \quad D > 0, \quad & \text{for} \quad N_S = 8 \\ \sigma > \langle \sigma \rangle, \quad D > 0.14, \quad & \text{for} \quad N_S = 14 \\ \sigma > \langle \sigma \rangle, \quad D > 0.11, \quad & \text{for} \quad N_S = 18 \end{aligned} \quad (20)$$

picks out the most two-centre jets ; their angular distributions are also illustrated in Figs. 3, 4 and 5. Among jets selected according to (20) almost all have very well separated backward and forward "cones". These jets will be later used to make a Duller-Walker plot (see section III.4), they represent about 20-25% of all Monte-Carlo jets.

For the sake of completeness, Figs. 3, 4 and 5 contain also angular distribution of all jets, without specifying the anisotropy. To make a rough comparison with experiment, we have taken 47 jets, satisfying the criteria $N_h \leq 5$, $9 \leq n_s \leq 20$, $\gamma_c \gg 23$ *) ; jets with $n_s < 8$ were not used since they are frequently asymmetric 14), 16) and do not show any two-centre structure ; in fact small multiplicity jets have to be discussed separately. **). The experimental jets we used have an average multiplicity 14. We made a plot of their composite angular distribution in $\frac{|x - \langle x \rangle|}{\sigma}$ co-ordinate ***)) and compared with

*) We are very grateful to Professor M. Miesowicz for sending us the list of experimental jets in possession of the Krakow laboratory.

**) To discuss asymmetry effects, so important among low multiplicity jets, one has to take into account fluctuations of the angular distribution as well as fluctuations of the centre of the basic x distribution around $\log \frac{1}{x}$. Asymmetries are not a pure pionization effect, they are given mainly by the mechanism of energy-momentum transfer between interacting nucleons and the black box in Fig. 1. This problem will be discussed separately.

***)) In order to decrease the fluctuations, we used $\left| \frac{x - \langle x \rangle}{\sigma} \right|$ co-ordinate instead of $\frac{(x - \langle x \rangle)}{\sigma}$.

the composite angular distribution for 3000 Monte-Carlo jets with multiplicity 14 - the result is shown in Fig. 6. The χ^2 test gives the confidence level 0.58 ; in fact, however, the agreement is even better since the slight discrepancy in the last two columns of the histogram is due to the neglect of the smearing of the edges of the basic rectangle.

In order to illustrate in more details the dependence of the angular distribution on anisotropy we sampled 3000 jets for each multiplicity. Resulting angular distributions, plotted in the $\frac{x - \langle x \rangle}{\langle \sigma \rangle}$ co-ordinate are shown in Figs. 7, 8 and 9. Jets with small $\sigma / \langle \sigma \rangle$ ratio are almost Gaussian. For $\frac{\sigma}{\langle \sigma \rangle} \approx 1$ the two-centre character of the angular distribution begins to come out and becomes apparent for larger values of $\sigma / \langle \sigma \rangle$.

III.3 It was found experimentally ⁵⁾ that there is a correlation between σ (anisotropy of a jet) and D (measure of the separation of two centres - see section II.3). The existence of this correlation can be easily explained in the framework of the fire-ball model (see section I.4) and was one of the important arguments in favour of this model.

We found in our model also a rather strong correlation between σ and D . Fig. 10 shows the dependence of the average D on $\sigma / \langle \sigma \rangle$. This regression curve was calculated for a sample of 3000 jets for each multiplicity ; we did not observe any significant dependence of the shape of the regression curve on multiplicity. The type of σ - D correlation is essentially of the same nature as observed experimentally. (The agreement with experiment is better than for the two-centre model.)

III.4 We mentioned already in section III.2 that selecting jets according to the criteria (20) one gets a sample of events with well separated two "fire-balls". In order to see whether it is possible to get, in the framework of our model, the experimentally observed isotropy

of emission of particles in fire-ball's rest system, we applied to our Monte-Carlo jets the procedure used by cosmic ray physicists in their actual work ¹⁾.

Among jets selected by the criteria (20), from the original sample of 100 jets for each multiplicity, we picked out those with better separation of "fire-balls" - about 20-25% of all jets are of this type - and for each jet we divided its $\log \tan \theta$ plot into two parts ^{*)}. For each part we calculated the mean x

$$\langle x \rangle_F = \log \frac{1}{\gamma_F} = \frac{1}{m} \sum_{i=1}^m \log \tan \theta_{(i)} \quad (21)$$

where m is the number of particles emitted by the "fire-ball" and γ_F is the Lorentz factor of the "fire-ball" rest system. Further we introduced the co-ordinate $\log \frac{1}{\gamma_F} \tan \theta_{(i)}$ and we found the composite angular distribution of secondaries "emitted" from a "fire-ball". Finally, we present the composite angular distribution in the well-known form of Duller-Walker integral distribution ¹⁷⁾. The straight line of slope 2 corresponds in this plot to the isotropic decay of the fire-ball. For Monte-Carlo jets with $\langle \sigma \rangle = 0.8$ and $N_s = 14$ we get the Duller-Walker plot shown in Fig. 11 - an almost identical result was obtained for $N_s = 18$ - the dispersion σ' in $\log \frac{1}{\gamma_F} \tan \theta_{(i)}$ co-ordinate of the composite angular distribution of particles "emitted" from one "fire-ball" was found to be $\sigma' = 0.34 \pm 0.02$.

*) The decision, as to which are the jets where there is a "sufficient" separation of fire-balls and how to divide a particular jet into two parts, contains certain amount of subjectivity. We hope that this amount of subjectivity is roughly the same in our analysis and in authentic experiments.

In our model σ' is not constant as is the case in the fire-ball model; with increasing energy the length of the basis rectangle increases logarithmically, and clearly so does σ' , since σ' is roughly proportional to $\langle\sigma\rangle$. The average anisotropy of experimental jets found in the energy region $10^{12} - 10^{14}$ eV is only slightly smaller than 0.8, the value of $\langle\sigma\rangle$ for which we found isotropic decay of Monte-Carlo "fire-balls" ^{*)}; we consider this fact as significant. It is interesting that Duller-Walker plots, given by Hasegawa ¹⁸⁾, for two groups of jets with different energies indicate a slow increase of σ' with energy as expected from our model. It is, maybe, worthwhile to mention that for a given $\langle\sigma\rangle$ we did not find any significant correlation between σ and σ' .

IV. CONCLUSION AND FINAL REMARKS

IV.1 The most important conclusion of this paper is that jets generated out of a quasi-rectangular $\log \tan \theta$ distribution obtained from a model being an idealization of multiperipheral models, can imitate in many respects the picture expected on the basis of the fire-ball model in the energy region $10^{12} - 10^{14}$ eV (that is just in the region where two-centre jets were found).

On the basis of our model one can understand why two-centre jets begin to come out at energies about 10^{12} eV; at this energy the basic $\log \tan \theta$ distribution becomes sufficiently lengthened to be replaced by a rectangle. For $E = 10^{12}$ eV and $\mathcal{H} = 0.07$ the

^{*)} Due to the neglect of the smearing of edges of the x distribution our σ' is slightly underestimated, thus isotropic decay of fire-balls corresponds to $\langle\sigma\rangle$ slightly smaller than 0.8

formula (17) of section II. gives $\langle \sigma \rangle \approx 0.52$; notice that the condition $\sigma > 0.6$ was usually used to pick out two-centre jets.

Of course, there are important asymptotic differences between our model and fire-ball model ; for instance, in our case "fire-balls" decay more and more anisotropically and the multiplicity increases logarithmically with increasing energy. This is not true in the fire-ball model ^{*)}. It is, however, very difficult to distinguish between the two pictures by going to higher energies, since for $E > 10^{14}$ eV the statistics of jets becomes poor.

From the point of view of our paper, the discovery by the Polish-Czech group^{1),3)} of the two-centre structure of jets does not lose its importance, it remains one of the most important discoveries in experimental cosmic ray physics, and it has stimulated many experimental and theoretical investigations. However, the interpretation of this effect should be, in our opinion, different from the conventional one.

*) Consider for simplicity a collision symmetric in the c.m. system and assume two "true" fire-balls are produced. Let K' and M denote inelasticity in the c.m. system and mass of the fire-ball respectively. Of course

$$n_s \sim M$$

An elementary calculation gives for $E \rightarrow \infty$

$$M \sim (K'\Delta)^{1/2} (mE)^{1/4}$$

where Δ is the invariant momentum transfer between backward and forward cones. It seems to be reasonable to think that $(K'\Delta)$ does not decrease with increasing energy according to a power law with the power $< -1/2$. Thus within the framework of the fire-ball model it is plausible that the average multiplicity increases with a power of energy.

We have seen that a two-centre structure of jets can be obtained, assuming that particles are produced independently but with small transverse momenta and small average inelasticity, as required by peripherality - our model is in fact a kind of statistical model but of a rather peculiar type ; it is, so to say, a statistical approach to peripheral interactions. Of course, we do not claim that correlations between pionization secondaries do not exist at all. It seems to us only that they are weak ^{*)}. To get more information on the physics of the high energy interactions, one has to look for deviations from our model ; in a certain sense such deviations could be more interesting than the close agreement with the model. We are doing now a much more detailed comparison with experiment, taking into account the energy spectrum of primary cosmic radiation, experimental biases, etc.

IV.2 Till now we reported about things, of which we are rather sure. Let us make now few general remarks which are suggested by our present experience but which are still under investigation.

As was mentioned in the Introduction, a two-centre structure was also observed among jets corresponding to collisions with heavy nuclei ($N_h > 8$). This fact seems to be rather puzzling from the point of view of the fire-ball model - why a rather particular kind of correlation between secondaries, observed in elementary collisions, is preserved in a collision with a composite system, like heavy nucleus ? If, however, two-centre structure is a result of lack of correlations, it is not very astonishing that one finds two-centre jets among events

*) More accurately speaking, present experimental data (at least those on jets with $N_h \leq 5$, $8 < n_s \leq 20$, which were at the basis of the two-centre model) do not indicate any strong correlation ; it may be that some new kind of experimental analysis will permit one to find these strong correlations.

with $N_h > 8$ *).

This paper is only a first attempt to examine the existing cosmic ray data from the point of view related to multiperipheral models. We have started a further investigation in order to answer the following questions :

How is our present picture changed if groups of particles (for instance quasi-particles) and not secondaries themselves are produced independently ?

Is the above interpretation of data on collisions with heavy nuclei correct ?

Are the fluctuations of the basic $\log \tan \theta$ distribution as well as of the angular distribution itself sufficient to understand the properties of asymmetric events ?

*) It may be, that in this way, it will be possible to explain why jets with $n_s > 20$, $N_h > 8$ show a two-centre structure while those with $n_s > 20$, but $N_h \leq 5$ are of different nature. Perhaps in quasi-elementary collisions with very large multiplicities some correlations are important - these correlations could be destroyed in collisions with composite systems.

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FIGURE CAPTIONS

- Fig. 1 Diagram representing a high-energy collision. The black box symbolizes the pionization process.
- Fig. 2 Distribution of the $x (\log \tan \theta)$ variable. The dashed line shows the rectangle as used in our Monte-Carlo calculation. The continuous line is the x distribution calculated taking into account the distribution of transverse momentum (the values of parameters are $E = 10000 \text{ GeV}$, $\mathcal{K} = 0.05$).
- Figs. 3, 4 and 5 Composite angular distributions for three multiplicities : 8, 14 and 18. The $\frac{x - \langle x \rangle}{\sigma}$ co-ordinate is used, as well as the non-linear scale of Ref. 5). Two upper pictures show the angular distributions for jets with $\sigma > \langle \sigma \rangle$ and $\sigma < \langle \sigma \rangle$. Two lower pictures show angular distributions for all jets and for the most two-centre jets selected according to (20).
- Fig. 6 Comparison of composite angular distributions for Monte-Carlo and experimental jets. $\left| \frac{x - \langle x \rangle}{\sigma} \right|$ co-ordinate and the non-linear scale of Ref. 5) are used.
- Figs. 7, 8 and 9 Composite angular distributions in the $\frac{(x - \langle x \rangle)}{\langle \sigma \rangle}$ co-ordinate using linear scale, but for different values of anisotropy.
- Fig. 10 The curve shows the average D corresponding to a given $\sigma / \langle \sigma \rangle$.
- Fig. 11 The composite Duller-Walker plot for "fire-balls" found in Monte-Carlo jets with well separated two groups of particles.

Fig.1

Fig. 2

$N_s = 8$

Fig. 3

$N_S = 14$

Fig. 4

$N_S = 18$

Fig.5

Fig.6

$$N_S = 8$$

Fig. 7

$$N_S = 14$$

Fig.8

$$N_S = 18$$

Fig. 9

Fig.10

Fig.11