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BOOTSTRAPPING OF VECTOR MESONS ?

K. Dietz and G. Domokos ^{*)}

CERN-Geneva

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^{*)} On leave from the Joint Institute of Nuclear Research, Dubna.
Permanent address : Central Research Institute for Physics,
Budapest.

Recently much effort has been devoted to the study of the generation of strong interaction symmetries in so-called "bootstrap" calculations¹⁾⁻³⁾. Part of the calculations aim at finding the symmetry of a given number of interacting particles (usually vector mesons) while more optimistic ones find masses, coupling constants and symmetries at the same time from the requirement of self-consistency. In doing the calculations, one usually assumes that having written down the matrix amplitude A in the form ND^{-1} , the nominator function can be approximated by a Born term, i.e., just one particle exchange.

In the present note, we want to investigate this procedure more carefully by looking into the manifold of the solutions of the system of integral equations for the matrix functions N and D . We write down the equation for the N function

$$N(v) = a F(v) + \frac{1}{\pi} \int_P dv' \frac{F(v) - F(v')}{v - v'} \rho(v') N(v') \quad (1)$$

Then

$$D(v) = -a + \frac{1}{\pi} \int_P dv' \frac{\rho(v') N(v')}{v' - v} \quad (2)$$

In the equation above, $\int_P$ means an integral taken along the physical cut, a is a constant matrix to be determined from the threshold behaviour of the amplitude; $F(v)$ is the value of the amplitude taken along the unphysical cut :

2.

$$F(v) = \frac{1}{\pi} \int_u \frac{dv'}{v' - v} A(v')$$

v stands for the CM momentum squared and $\rho(v)$ is the phase space matrix.

To be specific, we consider, assuming the existence of a symmetry group, the exchange of a vector meson multiplet between spinless particles; other cases can be treated analogously. We therefore have to treat only one equation instead of a matrix equation. In order to determine the parameters of the vector meson from a self-consistency requirement, we have to consider Eqs. (1) and (2) for the P wave.

a) By approximating $F(v)$ just by the Born term (exchange of one vector meson), one obtains :

$$F(v) = \frac{\lambda g^2}{4\pi} \frac{8v + \mu^2 + 2}{v} Q_1\left(1 + \frac{\mu^2}{v}\right) \quad (3)$$

$m^2 = 2\mu^2$ is the mass of the vector meson in units of the spinless one, λ is the crossing matrix element.

For $v \rightarrow \infty$

$$F(v) = \frac{\lambda g^2}{\pi} \log v + O(1)$$

Thus (1) is not an equation of the Fredholm type, nevertheless, contrary to some statements made in the literature, it can be solved with the help of a Mellin transform. We write

$$\frac{F(v) - F(v')}{v - v'} = \frac{\lambda g^2}{\pi} \frac{\log v - \log v'}{v - v'} + K(v, v')$$

where $K(v, v')$ is a Fredholm kernel which does not change the manifold of solutions.

The equation

$$\psi(v) = \frac{a\lambda g^2}{\pi} \log v + \frac{\lambda g^2}{\pi^2} \int_1^\infty \frac{\log v - \log v'}{v - v'} \psi(v') dv' \quad (4)$$

[the "asymptotic part" of Eq. (1)] after the application of a Mellin transform has the form

$$\tilde{\Psi}(s) = \Phi(s) + V(s)\tilde{\Psi}(s) \quad (5)$$

with

$$\begin{aligned} \tilde{\Psi}(s) &= \int_1^\infty dv v^{s-1} \psi(v) \equiv \mathcal{M}_s[\psi(v)] \\ \Phi(s) &= \mathcal{M}_s\left[\frac{a\lambda g^2}{\pi} \log v\right] = -\frac{a\lambda g^2}{\pi s^2} \quad (\operatorname{Re} s < 0) \\ V(s) &= \mathcal{M}_s\left[\frac{\lambda g^2}{\pi^2} \frac{\log v/v'}{v/v' - 1}\right] = \frac{\lambda g^2}{\sin^2 \pi s} \quad (0 < \operatorname{Re} s < 1) \end{aligned} \quad (6)$$

We see from Eq. (6) that the Mellin transform of ψ

$$\tilde{\psi}(s) = \frac{\phi(s)}{1 - V(s)} \quad (7)$$

cannot be inverted because the two strips given in Eq. (6) do not overlap, therefore, Eq. (4) does not possess a solution in the ordinary sense. A possible cure is to introduce a CDD pole in the denominator function which modifies the inhomogeneity of Eq. (1). In our "asymptotic" equation we get $\frac{a\lambda g^2}{\pi} \frac{\log v}{v}$ instead of $\frac{a\lambda g^2}{\pi} \log v$. The Mellin transform of the modified inhomogeneity is :

$$\bar{\phi}(s) = \frac{a\lambda g^2}{\pi} (s-1)^2 \quad (\text{Re } s < 1)$$

So, Eq. (7) can be now inverted in the strip $0 < \text{Re } s < 1$.

After inversion of Eq. (7) we have for $\lambda < 0$ ("repulsive interaction") :

$$\psi(x) = a_1 \frac{\sin(\lambda \log x)}{x} + a_2 \frac{\cos(\lambda \log x)}{x} \quad (8)$$

$$\lambda = \pi^{-1} A \text{sh}(-\lambda g^2)^{1/2}$$

where a_1 and a_2 are arbitrary constants. They arise from adding the solution of the homogeneous counterpart of Eq. (4) to $\tilde{\psi}(v) = \pi^{-1} \frac{\bar{\phi}}{1-V}$ (the Fredholm alternative does not hold for our equation). For the "attractive case" ($\lambda > 0$), we have similarly :

$$\psi(x) = \begin{cases} a \exp \tilde{\lambda} \log x & (\lambda g^2 < 1) \\ a_1 \frac{\sin(\beta_1 \log x)}{x^{1/2}} + a_2 \frac{\sin(\beta_2 \log x)}{x^{1/2}} & (\lambda g^2 > 1) \end{cases} \quad (9)$$

Here a , a_1 , a_2 are arbitrary constants,

$$\tilde{\lambda} = \pi^{-1} \arcsin (\lambda g^2)^{1/2} - 1$$

$$\beta_{1,2} = \frac{1}{\pi} \log [(\lambda g^2)^{1/2} \pm (\lambda g^2 - 1)^{1/2}]$$

Notice that the solution has a branch point in g^2 at $\lambda g^2 = 0$ and $\lambda g^2 = 1$. A similar behaviour in g^2 is obtained by considering the same physical problem in the Bethe-Salpeter formalism ⁴⁾.

Hence if our equations possess solutions at all, they give a continuous infinity of functions. In order to get rid of this apparently unphysical situation, we have to modify the expression for $F(v)$.

- b) A possible model leading to a Fredholm kernel is to consider a - unitary - Breit-Wigner amplitude in the crossed channels. This means that instead of a simple pole term in the t - and u - channels we write :

$$A(s, t) \sim \frac{g^2 P_1 \left(1 + \frac{2s}{t-4}\right) (t-4)}{t - m^2 + i \Gamma (t-4)^{3/2}} \quad (10)$$

Projecting out the P wave in the S channel, we find that for $\nu \rightarrow \infty$

$$F(\nu) = O(\nu^{-1/2}).$$

Therefore, the kernel $\frac{F(\nu) - F(\nu')}{\nu - \nu'}$ is already square integrable; however, in order to assure $\int_P F(\nu)^2 d\nu < \infty$, one has to introduce again a CDD pole or a decreasing form factor $g(t)$ in Eq. (10).

- c) Another possibility is to consider the exchange of "reggeized" particles. Then, the asymptotic behaviour of $\text{Im}F(\nu)$ is ⁵⁾

$$\text{Im}F(\nu) = O(\nu^{\alpha(0)-1}) \quad \text{for } \alpha(0) < 1$$

or

$$\text{Im}F(\nu) = O\left(\frac{1}{\log \nu}\right)$$

for the exchange of the Pomeranchuk pole with $\alpha(0) = 1$. The latter surely intervenes at least in some channels.

Inclusion of inelastic contributions on the physical cut in the form of Regge pole exchange does not improve the situation.

Summarizing, we can say that the possibility of generating vector mesons in a N/D calculation depends critically on the knowledge of the distant singularities.

For the models we have considered, there are the following cases:

Model a) is meaningless without at least one CDD pole, i.e., without introducing bare fields. Once we introduce a bare field, however, the physical content of the problem is changed and even with that modification it seems to be impossible to obtain a physically meaningful answer.

Model b) with a form factor can give an acceptable solution; however, the knowledge of the form factor must be presupposed. With a CDD pole, model b) corresponds once again to a problem of "dressing" a bare particle rather than finding a self-maintaining bound state.

A similar situation is found in model c) where, at least, the Pomeranchuk pole has to be fed in from the beginning.

The somewhat mysterious success of some attempts to generate symmetries from bootstrap calculations seems to be explained by the circumstance that the final expressions obtained can be as well interpreted in terms of a field theory. While a pure S matrix theory together with the assumption of the predominance of the nearest singularities does not seem to be self-consistent, it has been demonstrated ^{2),6)}, even without recourse to perturbation theory ⁷⁾, that a certain type of Lagrangian field theory of vector mesons implies the desired symmetry properties.

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