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SMALL ANGLE DELBRUECK SCATTERING *)

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ABSTRACT

The Bethe-Rohrlich approximation for small angle Delbrueck scattering, which is valid for photon energies of some 100 MeV, can be easily modified so as to be also applicable at low energies of a few MeV. This allows to calculate the real and imaginary part of the scattering amplitude at low energies and scattering angles not larger than some 10 mrad. There the real part is found to be in general at most equal in magnitude with the imaginary part. Since only the real part gives evidence for the existence of the polarization of the vacuum, the possibility of its observation in experiments at a few MeV and small scattering angles is discussed.

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1. INTRODUCTION

From the early days of quantum electrodynamics one is interested in the non-linear effects predicted by this theory, because their observation can give information about the polarization of the vacuum. The scattering of light by the strong electric field of a heavy nucleus, called Delbrueck scattering ¹⁾, is one of these processes and the most likely one to be observed, because its cross-section is enhanced through the interaction with the external field ($\sigma \approx (\alpha Z)^4 r_0^2$).

Fig. 1

It is known that, even in lowest order Born approximation, to which all Feynman graphs of the type shown in Fig. 1 correspond, the calculations lead to very complicated integrals. Fortunately the higher order Born terms can be neglected since Rohrlich ²⁾ has shown that they contribute at most 10 % to the amplitude, although the convergence of the power expansion in αZ is poor for large Z . Screening of the nuclear Coulomb field by the atomic electrons also does not need to be taken into account ³⁾.

Only in the special case of forward scattering is the complete analytic integration possible ⁴⁾. Here the most elegant approach to the solution of the problem is, to start from the analytic expression for the total cross-section of pair production in an external field obtained by Jost, Luttinger and Slotnik ⁵⁾. From this one gets the imaginary part of the Delbrueck scattering amplitude via the optical theorem. A simple principal value integration leads then also to the real part. This was done first by Rohrlich and Gluckstern ⁴⁾ and by Toll and Wheeler ³⁾.

In the general case of non-vanishing scattering angle, one has to evaluate the integrals numerically. One can start from the expressions derived by Kessler ⁶⁾ for the imaginary part, but unfortunately one runs into computational difficulties at higher energies ⁷⁾ and therefore no exact values for the real part have so far been calculated from a numerical principal value integration. New efforts are made now to solve this problem but it is at present not sure whether the calculations will be successful.

At energies below the threshold of pair production, where the imaginary part is zero, the problem seems to have been solved satisfactorily by Eftimiu and Vrejoiu ⁸⁾, but at these energies the Rayleigh scattering is so dominant that the Delbrueck effect remains unobservable.

For high energies of some 100 MeV and small momentum transfer $x = k\theta/m \lesssim 1$, Bethe and Rohrlich ⁹⁾ have used the impact parameter method to derive expressions for the real and imaginary parts of the scattering amplitude, but at these energies the real part is negligibly small, so that one cannot get information about vacuum polarization.

On the other hand, it is known from the exact calculations of Rohrlich and Gluckstern for forward scattering and from the small angle approximation of Bethe and Rohrlich, that the real part is bigger than the imaginary part at photon energies less than 10 MeV and that the real part drops off faster with increasing momentum transfer x than the imaginary part. Therefore it is reasonable to assume that the real part is most likely to be observed at energies of a few MeV and very small momentum transfer $x \simeq 0.2 - 0.4$, where the real and imaginary parts can be of approximately equal magnitude, so that it should be possible to distinguish whether the real part exists or not. This should also work in spite of the fact that at these energies the Rayleigh scattering amplitude is still the dominant coherent term, since then also the interference term between Rayleigh and Delbrueck scattering will be enhanced and of measurable magnitude. Furthermore, this enhancement will take place only with the real part, since the imaginary part of the Rayleigh amplitude is known to be extremely small in close-to-forward direction.

Since the exact calculations are difficult to perform, it has been tried to extend the validity of the Bethe-Rohrlich approximation to lower energies, so that estimates can be made about the magnitude of the interference term.

2. APPROXIMATE THEORY OF SMALL ANGLE DELBRUECK SCATTERING

Bethe and Rohrlich show that in the region of high energy gamma rays and small scattering angles the Delbrueck problem can be approximately solved with the impact parameter method ^{9),10)}. They derive the following expressions for the total cross-section of pair production in an external field and the imaginary part of the Delbrueck scattering amplitude

$$\sigma(k) \approx 4\pi \int_{b_{\min}}^{b_{\max}} b db \gamma(k, b) \quad (1.a)$$

$$a_2(k, \theta) \approx k \int_{b_{\min}}^{b_{\max}} b db \gamma(k, b) J_0(bk\theta) \quad (1.b)$$

The function $\gamma(k, b)$ determines the pair production probability at the distance b from the nucleus and J_0 is the zero order Bessel function. (The units $\hbar = c = 1$ are used throughout.)

The distribution of the nuclear recoil momentum has been calculated by Bethe ¹¹⁾ for high energies. Since the approximate relation $q = 1/b$ holds ⁹⁾ between recoil momentum q and impact parameter b (which follows from the uncertainty principle of Heisenberg), Bethe and Rohrlich find, from Bethe's result, at high energies the pair production probability function to be $\gamma(k, b) = \text{const}/b^2$.

Since the expression (2) is exact, one has to get also exact values for the imaginary part of the forward scattering amplitude from the expression (3) which follows in this case ($J_0 = 1$) from (2) via the optical theorem. This has been checked by numerical comparison with results obtained from the corresponding formula of Rohrllich and Gluckstern ⁴⁾. The agreement is actually very good.

The accuracy of the results computed from Eq. (3) for non-vanishing scattering angle has been determined by comparison with the few exact data available at 2.62 and 6.14 MeV, which were obtained by numerical evaluation of the exact expressions for the imaginary part, derived by Kessler ^{6),7)}. It turns out that in the case where the gamma ray is polarized parallel to the plane of scattering, the agreement is satisfactory. Up to $x \simeq 4$ the approximate values deviate from the exact ones by at most +30 % and for the range of momentum transfer $x \lesssim 1$ the deviation is even at most +15 %. Unfortunately, when the polarization vector is perpendicular to the scattering plane, the agreement is rather poor ^{*)}. But here a very good improvement can be achieved by inserting the factor $\exp(-x^2/4Q^2)$ into the integral (3). Then the error of the approximation is also only about ± 15 % in the range of energy 1 - 10 MeV and momentum transfer $x \lesssim 1$ ^{**)}. (From the Bethe-Rohrllich theory one gets in this range of energy and momentum transfer results which are incorrect by several hundred percent.)

Because the imaginary part is not well determined by the new approximation in the case of perpendicular polarization for all energies and larger momentum transfer, also no predictions about the corresponding real part can be made. But it is known from the exact values available that the imaginary part drops down to small values much more rapidly with

*) Kessler shows that for Delbrueck scattering only the two cases of polarization need to be considered: polarization vector in the plane of scattering before and after the scattering and similar polarization vector vertical to the plane of scattering. This gives the two components of the imaginary part, which he calls

$$a_{\text{im(pol. } \parallel \text{)}}^D \quad \text{and} \quad a_{\text{im(pol. } \perp \text{)}}^D.$$

**) A similar situation is encountered in the Rayleigh scattering problem if one compares the exact theory with the form factor theory in the two relevant cases with and without polarization change ^{13),14)}.

increasing x , if the polarization is perpendicular. This will therefore also hold for the real part, so that its component parallel to the polarization plane will certainly determine the upper limit for the magnitude of the average over both polarizations, to be expected at small scattering angles.

Upon investigation of the symmetry properties of the expressions given by Kessler ⁶⁾, one finds that the imaginary part of the scattering amplitude has the following properties $a_2(K, \theta) = -a_2(-K, -\theta) = -a_2(-K, \theta)$. This allows to calculate for fixed scattering angle the real part from the principal value formula

$$a_1(K, \theta) = \frac{2K^2}{\pi} P \int_0^\infty \frac{a_2(K', \theta) dK'}{K'(K'^2 - K^2)} \quad (7)$$

which has been so far only used for forward scattering ^{3), 4)}.

3. RESULTS AND CONCLUSIONS

At small scattering angles so far only two experiments have been performed. The one by Moffat and Stringfellow ¹⁵⁾ at 87 MeV and scattering angles between 1.89 and 4.24 m rad gives essentially good agreement with the Bethe-Rohrlich theory, whereas to the second experiment by Stierlin, Scholz and Povh ¹⁶⁾ at 7 and 17 MeV this theory does not apply any more and predicts values which are by a factor 1.6 larger than the experimental ones, but these agree very well with the new approximation. So the Delbrueck effect is established by these experiments, but since in both cases the real part of its amplitude can be neglected nothing can be concluded about the polarization of the vacuum.

Whether at low energies of a few MeV and large scattering angles between 20 and 80° , the real part gives a measurable contribution to the total coherent differential cross-section, cannot be decided as long as no exact calculations are available. Experimental findings at 2.64 MeV indicate, however, that such a contribution might exist, although on the other hand later experiments at 1.33 MeV have shown that most of the deviations previously attributed to the real part of the Delbrueck amplitude (the imaginary part turns out to be negligible at large scattering angles ^{6),7)}) could be explained by improving the Rayleigh scattering calculations and the elimination of the inelastic processes ^{13),17)}. (Compton effect and secondary scattering.)

Now it is essential to point out that only the observation of the real part of the Delbrueck scattering amplitude can give evidence for the existence of the vacuum polarization. The real (or principal value) part of the scattering amplitude is closely connected to virtual, i.e., energy non-conserving intermediate states (electron pairs) and just the existence of these virtual electron loops is what brings about the polarization of the vacuum through the inhomogeneity of the field in the neighbourhood of the Coulomb source. The imaginary (or δ function) part, on the other hand, stems from the energy conserving intermediate states, i.e., real electron pairs and is therefore closely connected to the corresponding inelastic process, namely pair production. So the imaginary part does nothing more but measure the shadow of this absorptive process.

Because of this important fact, on the basis of the present calculations, an experiment is suggested at low photon energies in the range 5 to 12 MeV and small scattering angles, corresponding to a momentum transfer $x = 0.2$ to 0.4 . The underlying idea is the following.

Moffat and Stringfellow ¹⁵⁾ and Stierlin et al. ¹⁶⁾ have shown that the contribution of Delbrueck scattering to the total differential cross-section measured can be determined uniquely by taking advantage of the different dependence on the nuclear charge Z of the competing processes involved. So by fitting the experimental data obtained at

fixed momentum transfer for various heavy and medium heavy nuclei to a polynomial in Z , the coefficient of Z^4 has to be attributed to Delbrueck scattering, since at their energies nuclear Thomson scattering, which also is proportional to Z^4 , can be neglected because its cross-section does not depend on the energy, whereas the Delbrueck scattering cross-section increases about like K^2 . Furthermore at these energies and the momentum transfer chosen ($x = 0.5$ at 7 and 17 MeV) the interference term between the real part of the Rayleigh and Delbrueck amplitude can be neglected to first approximation, since the precision of these experiments is poor anyway. (The imaginary part of the Rayleigh amplitude can be neglected at small scattering angles ¹³.) But in the range of low energies and small momentum transfer, as quoted above, the interference term can give the main contribution in addition to the pure Rayleigh scattering cross-section, since one can still neglect nuclear Thomson scattering and probably also nuclear resonance scattering, so that the total elastic (coherent) differential cross-section can be written as

$$(d\sigma/d\Omega)_{coh.} \approx (a_{1R} + a_{1D})^2 + a_{2D}^2 \quad (8)$$

or more conveniently

$$(d\sigma/d\Omega)_{coh.} \approx (d\sigma/d\Omega)_{Ray.} (1 + 2r_1 + r_{12}^2) \quad (9)$$

where $r_1 = a_{1D}/a_{1R}$ and $r_{12}^2 = (a_{1D}^2 + a_{2D}^2)/a_{1R}^2$ with a_1 = real part, a_2 = imaginary part, and R and D denoting Rayleigh and Delbrueck scattering amplitudes, respectively. If one now assumes that the Rayleigh scattering amplitude at small momentum transfer closely follows a $Z^{3/2}$ law, as it is given by the form factor approximation of Franz ¹⁴) (this point would have to be investigated further), then Eq. (9) can be brought into the form

$$(d\sigma/d\Omega)_{coh.} \approx (d\sigma/d\Omega)_{Ray.} (1 + 2R_1 \lambda + R_{12}^2 \lambda^2) \quad (10)$$

where one puts $\lambda = Z^{1/2}$. So the experimental points would have to follow a parabola, as function of the variable λ , or in the case where the interference term dominates over the pure Delbrueck term, essentially a straight line. Because the coefficient R_1 contains only the real part of the Delbrueck amplitude, it would give evidence for vacuum polarization, if found to be definitely different from zero.

To get an idea about the relative magnitude, one would have to expect for the interference term $2r_1 = 2R_1 \lambda$ and the Delbrueck term $r_{12}^2 = R_{12}^2 \lambda^2$ with respect to the Rayleigh scattering cross-section, one can use the form factor formula of Franz for the Rayleigh scattering amplitude, which has at small momentum transfer x the simple form

$$a_{1R} \approx 1.5 (\alpha Z)^{3/2} r_0 \frac{1}{x^{3/2}} \quad (11)$$

and is known to give in general too large values¹³⁾. Putting $a_{1,2D} = (\alpha Z)^2 r_0 a'_{1,2D}$, one can then easily calculate the necessary coefficients from the formulae

$$\begin{aligned} R_1 &= R_1 \lambda = 0.057 x^{3/2} a'_{1D} \lambda \\ R_{12}^2 &= 0.0033 x^3 (a'_{1D}{}^2 + a'_{2D}{}^2) \lambda^2 = R_{12}^2 \lambda^2 \end{aligned} \quad (12)$$

We have calculated the values $a'_{1,2D}$ using the new Delbrueck scattering approximation. In Table I a few numerical results are presented to give insight into the order of magnitude one will have to expect for the two terms of interest. For comparison also values for the low energy $K = 2.56$ (2.64 MeV) and the higher $K = 16.63$ (17 MeV) were calculated to show that in the former case Delbrueck scattering can be neglected at small scattering angles and in the latter case the interference term is much smaller than the pure Delbrueck term, which is mainly due to the imaginary part a_{2D} . On the other hand, in the range $K = 6$ to 10 and $x = 0.2$ to 0.3 the contribution of the interference term is bigger than the one of Delbrueck scattering alone. For heavy elements ($Z = 82$) both terms can be 30 % of the Rayleigh scattering cross-section.

In order to find these contributions, the accuracy of the experiments would have to be highly improved, so as to know the total coherent differential cross-section to a few percent and Rayleigh scattering has to be investigated further, especially its exact dependence on the nuclear charge Z .

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						$\lambda = Z^{\frac{1}{2}}$		nuclear charge Z	
						5	9.05	25	82
K	x	a_{1D}'	a_{2D}'	$2R_1$	R_{12}^2	$2r_1$	$2r_1$	r_{12}^2	r_{12}^2
2.56	0.31	0.36	0.26	0.007	0.00002	0.035	0.064	0.0005	0.0015
6.0	0.18	1.42	2.20	0.012	0.00013	0.062	0.11	0.0032	0.011
"	0.36	1.00	2.03	0.025	0.00078	0.12	0.22	0.02	0.064
8.0	0.24	1.64	3.66	0.022	0.00074	0.11	0.20	0.019	0.06
"	0.48	0.86	3.16	0.033	0.0039	0.16	0.30	0.098	0.32
10.0	0.2	2.17	5.48	0.022	0.001	0.11	0.20	0.025	0.093
"	0.3	1.59	5.13	0.03	0.0026	0.15	0.27	0.065	0.21
12.0	0.24	2.10	7.15	0.028	0.0025	0.14	0.25	0.063	0.21
16.63	0.33	1.19	10.6	0.026	0.014	0.13	0.23	0.34	1.10

T A B L E I

Approximate magnitude of the interference term between real part of Rayleigh and Delbrueck scattering amplitude and of the Delbrueck scattering cross-section, expressed as fraction of the Rayleigh scattering cross-section, which is taken as unit.

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