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NUMERICAL CALCULATIONS OF THE DELBRÜCK SCATTERING AMPLITUDE

F. Ehlotzky ^{*)} and G.C. Sheppey

CERN-Geneva

A B S T R A C T

Formulae for the imaginary part of the Delbrück scattering amplitude, derived by Kessler some years ago, and a simple fixed angle dispersion relation, are used to calculate numerically the real and imaginary parts for energies between 1 and 20 MeV and scattering angles from 0 to 120° with an accuracy of at best 5 to 10%. Comparison is also made with some of the experimental data available and conclusions are drawn for future work.

^{*)} Permanent address : Institut für theoretische Physik der Universität Innsbruck, Austria.

1. INTRODUCTION

Within the last ten years several experiments have been performed ¹⁾⁻⁶⁾ to detect the Delbrück effect ⁷⁾. It is predicted by quantum electrodynamics as the elastic scattering of light by the Coulomb field of (heavy) nuclei. The observation of this non-linear process is of interest since it can furnish an independent check of the existence of the polarization of the vacuum which is so far only confirmed by the excellent agreement of the Lamb shift calculations ⁸⁾ with the experiments ⁹⁾.

But, at present, the existence of only the imaginary part of the Delbrück scattering amplitude is fairly reliably established. This has been achieved by the experiments of Stierlin et al. ⁵⁾ at 17 MeV and of Moffat and Stringfellow ⁶⁾ at 87 MeV. Both experiments were performed at small scattering angles where the imaginary part is known to dominate at these energies ^{10),11)}. Also the recent experiments of Bosch et al. ⁴⁾ at 9 MeV and larger scattering angles confirm the imaginary part. Unfortunately, the imaginary part is only due to intermediate electron states which conserve energy. Therefore, it gives exclusively information about the shadow of the corresponding inelastic process, namely pair production.

The real part, on the other hand, stems from the purely virtual intermediate states in the external field. Hence, it is closely connected to what is commonly called vacuum polarization ^{11),12)}. From this it follows that in Lamb shift experiments one is actually measuring the vacuum effect, for in all bound state phenomena the electrons of the closed loops can only be virtual. Obviously, this is not so with the Delbrück effect which can only be observed at photon energies above the threshold of pair production under the present experimental circumstances. Here the existence of the real part of its amplitude has to be verified experimentally. Certainly one can also argue that the search for the real part of the Delbrück scattering amplitude is no longer necessary to confirm

the existence of the polarization of the vacuum as soon as the imaginary part is experimentally well established since we believe today in the analytic properties of the scattering amplitudes of nature anyhow. But, it seems to us that we should not disregard the opportunity to confirm our belief through another independent check, even if it concerns electrodynamics only.

The reasons why the real part has not yet been found are twofold : first, it is difficult to perform the necessary scattering experiments. The present accuracy lies in the average at about 30% for the total measured differential cross-sections. As the Delbrück effect appears coherently and incoherently with various other scattering processes, its contribution to the cross-sections cannot be determined more accurately either. The coherent effects are nuclear Thomson scattering, nuclear resonance scattering and Rayleigh scattering, the incoherent ones are mainly Compton scattering from the atomic electrons and secondary scattering. Not all their cross-sections are theoretically well known. This means the Delbrück contribution will be hidden in many cases by the uncertainties of the background, in particular at low energies of a few MeV where it is expected to be comparatively small.

Second, there do not exist up to now exact calculations of the real part for all those energies and scattering angles at which experiments have been made. To remove at least to some extent this lack of theoretical information is the aim of the present paper.

For forward scattering exact calculations to lowest order of the Born approximation were made many years ago by Rohrlich and Gluckstern ¹⁰⁾. Only in this special case all the integrations can be done analytically and these authors presented expressions for the real and imaginary part in closed form. In a subsequent paper an approximation for high energies and small scattering angles has been given by Bethe and Rohrlich ¹¹⁾. This has been extended recently to lower energies by one of us (F. E.) ¹³⁾. Both approximations are sufficiently

accurate to lead to the confirmation of the existence of the imaginary part of the Delbrück amplitude at small angles by comparison with the experiments mentioned earlier ^{5),6)}. An approximation for high energies and large scattering angles has also been presented a short time ago by Sannikov ¹⁴⁾. This predicts that the real part is much bigger at large angles than Bethe and Rohrlich ¹¹⁾ originally expected. However, the results of Sannikov are completely incompatible with the experimental data of Bösch et al. ⁴⁾ at 9 MeV.

Formulae for the imaginary part of the Delbrück scattering amplitude valid for all scattering angles have been derived by Kessler ¹⁵⁾ some years ago in the lowest order of the Born approximation. He evaluated his expressions numerically for various angles at 2.62 MeV. By comparison with the experiments ^{2),3)} he noticed that the imaginary part gives only a very small contribution at this energy. Hence the large gap between the experimental points and the theoretical curve for the combined cross-section of nuclear Thomson scattering and Rayleigh scattering might be due to a much bigger real part of the Delbrück amplitude. However, Kessler did not calculate the real part as well. Subsequently Zernik ¹⁶⁾ checked the correctness of the formulae of Kessler and computed in addition the imaginary part at 6.14 MeV claiming for his results an accuracy of 1%. (We have compared his values for forward scattering with those easily obtained from the analytic expressions of Rohrlich and Gluckstern ¹⁰⁾ and found a deviation of 20%.)

Estimates of the contributions of the higher order Born terms and of the screening of the nuclear Coulomb field by the atomic electrons have been made at least for forward scattering by Rohrlich ¹⁷⁾ and by Toll ¹⁸⁾ respectively. The first contribution was found to be in general of the order of at most 10% of the total amplitude whereas the second can be neglected at low energies of a few MeV. If one assumes that a similar situation also holds for non-vanishing, even large scattering angles then (in view of the accuracy of the experimental data available at present) it should be sufficient to carry out computations of the Delbrück amplitude to the lowest order of the Born approximation with a precision of 5 to 10% for all scattering angles.

2. THE CALCULATIONS

We start from the expressions of Kessler¹⁵⁾ for the imaginary part which have the form of fivefold integrals and cannot be worked out by analytic calculation. Only two cases of the plane polarization of the scattered γ rays have to be considered, namely polarization parallel ($//$) to the plane of scattering and polarization perpendicular ($\perp$) to this plane before and after the scattering. (The terms with polarization flip are shown by invariance arguments not to exist.) These two amplitudes will be called $a_{2//}(K, \theta)$ and $a_{2\perp}(K, \theta)$ hereafter. $K = k/2m$ is the energy of the scattered photons measured in units of twice the rest energy, m , of the electron and θ is the scattering angle. (It should be noted that in the formulae of Kessler θ is only half the scattering angle !) The amplitudes can be expressed in the units $(\alpha Z)^2 r_0$, where α is the fine structure constant, Z the nuclear charge and $r_0 = \alpha/m$ is the classical electron radius. (For convenience the units $\hbar = c = 1$ are used.)

Zernik¹⁶⁾ mentions that it is difficult to achieve a satisfactory accuracy of the numerical calculations if one goes to higher photon energies. These troubles arise from the fact that with increasing energy the main contributions to the fivefold integrals come from those regions of the domain of integration where the momentum vectors of the intermediate electron pair are almost parallel to the momentum vector of the ingoing or outgoing photon. In other words, two conic segments of the unit sphere of polar co-ordinates are of special importance for the integration. The basis of each of these cones becomes smaller in diameter with increasing energy. The axis of each cone is determined by the direction of the momentum vector of the incident and scattered photon. For forward scattering where these two cones coincide this peaking of the integrand has been known for a long time to take place for the Bethe-Heitler formula of pair production¹⁹⁾. Evidently it is via the optical theorem a special case of the formulae of Kessler.

If one wishes to get fairly reliable values for the real part of the Delbrück scattering amplitude as well by performing a principal value integration, it is necessary to know the imaginary part also at higher energies to some accuracy, particularly at small scattering angles. There the imaginary part increases as a function of energy like $K \ln K$ so that its contributions for large K to the principal value integral cannot be neglected even if one is only interested in obtaining the real part for energies less than 20 MeV with at best 5 to 10% accuracy.

To meet these needs a special Monte-Carlo programme for the CERN IBM 7090 computer has been designed by one of the authors (G.C. S.) which takes care by itself of the special features of the integrand of the fivefold integrals for $a_{2//}$ and $a_{2\perp}$. This programme splits the domain of integration in such a way that most random points are thrown into the two cones mentioned. By this procedure, it has been possible to calculate the imaginary part sufficiently exact also for large K without undue computation time. For forward scattering the correctness of the results has been checked by comparing with the corresponding values easily obtained from the analytic expressions of Rohrlich and Gluckstern¹⁰⁾. At scattering angles different from zero the few values of Kessler¹⁵⁾ and Zernik¹⁶⁾ were available for comparison. (In the case of $a_{2//}$ the agreement of their numbers with ours is actually very good, whereas for $a_{2\perp}$ the results of Zernik at 6.14 MeV and larger scattering angles are smaller than those we obtained. However, we could find no reasons to distrust our calculations.)

This programme for the imaginary part has then been incorporated into a Monte-Carlo programme for a sixfold integration including the Cauchy principal value to compute also the real part of the Delbrück scattering amplitude. At present, it seems rather hopeless to extend these calculations to higher energies since also for $K < 20$ not all the results are of satisfactory precision.

In performing the principal value integration, advantage has been taken of the symmetry properties of the formulae for the imaginary part

$$a_{2//,\angle}(K,\theta) = -a_{2//,\angle}(-K,\theta) = -a_{2//,\angle}(-K,-\theta) \quad (1)$$

from which it follows that the real part can be found from the simple fixed angle dispersion relation

$$a_{1//,\angle}(K,\theta) = \frac{2K^2}{\pi} \mathcal{P} \int_1^{\infty} \frac{a_{2//,\angle}(K',\theta) dK'}{K'(K'^2 - K^2)} \quad (2)$$

This has been used so far only for forward scattering by Rohrlich and Gluckstern¹⁰⁾ and by Toll and Wheeler¹⁸⁾. However, it is also valid for non-vanishing scattering angles according to the considerations of Kessler¹⁵⁾ if the symmetry properties (1) are used.

The results of our calculations, mainly for energies at which experiments have been performed, are presented in Table 1. We only wish to point out one feature of the scattering amplitude which is of importance from the experimental point of view. At the same time as the amplitude is strongly peaking forward with increasing energy, there develops a large negative minimum of the real part at small momentum transfer. Here the real part can be equal or even larger in absolute magnitude than the imaginary part. Hence a contribution of the real part should be found even if the experiment is of poor precision.

Table 2 contains for comparison some values obtained from the modified Bethe-Rohrlich approximation¹³⁾. It is surprisingly good for small angles. Defining $x = 4K \sin(\theta/2)$, we find in the range of momentum transfer $x = 0$ to 3 the largest deviation to be about +25% for $a_{2//}$ and -15% for $a_{2\angle}$. This

also seems to hold for the absolute magnitude of the real parts $a_{1//}$ and $a_{1\angle}$. These deviations are in addition almost independent of K for each value of x . Therefore, this approximation can be used with advantage to extrapolate the calculations of the real and imaginary parts to higher energies in that range of momentum transfer in a fairly reliable manner. We only need to make the indicated corrections.

Finally, the values of Table 3 demonstrate that the approximation of Sannikov¹⁴⁾ for high energies and large angles,

$$a_{1//,(\angle)} = (+)(\alpha Z)^2 r_0 (\ln x)^2 / x, \quad a_{2//,\angle} \approx 0$$

predicts a much too flat angular dependence of the Delbrück scattering amplitude at least for the energies considered. (Also the imaginary part and the polarization of the photons seem to be incorrectly treated.)

3. COMPARISON WITH THE EXPERIMENTS AND CONCLUSIONS

Taking into account the accuracy of the experimental data available at present ($\pm 30\%$) and the difficulties which arise for their interpretation from the other scattering processes involved, we can draw on the basis of the results of our computations the following conclusions.

In the range below about 5 MeV probably no indication can be found for the existence of the Delbrück effect in the near future. Real and imaginary parts are too small to be of relevance compared to the other coherent effects which are mainly Rayleigh scattering and nuclear Thomson scattering. Standing and Jovanovich ¹⁾ have thoroughly analyzed the experimental results accumulated at 1.33 MeV ($K = 1.30$). They found that the magnitude of the Delbrück amplitude allowed for by the accuracy of the data and the theoretical predictions for the other processes can be in the angular range from 20° to 45° at most 1 to 5 times the corresponding nuclear Thomson amplitude. From our calculations it follows that not only the imaginary part but also the real part of the Delbrück amplitude are even smaller than the Thomson amplitude by roughly 50%. The Delbrück effect disappears in the background. A similar negative interpretation has to be given to the experiments at 2.62 MeV ($K = 2.56$) ^{2),3)}. As mentioned in the Introduction, the large gap between the experimental points and the theoretical curve for the combined cross-sections of Thomson and Rayleigh scattering in the range from 30° to 70° scattering angle, has tentatively been attributed to the real part of the Delbrück amplitude ^{2),3),15)}. Our results show, however, that this explanation has to be ruled out. According to Table 1, the real part also is only a fraction of the combined nuclear Thomson and Rayleigh amplitude, whereas the experiments would require the real part to be as much as 5 times that combined amplitude at 60° ²⁾. Furthermore, the real part adds destructively in the coherent addition of the amplitudes which only means an increase of the gap.

For energies above 5 MeV it is more likely to find a contribution from the Delbrück effect since its scattering amplitude continuously increases in particular at small angles where it is about proportional to K , whereas nuclear Thomson scattering, (nuclear resonance scattering ?) and Rayleigh scattering are rather independent of K . However, at energies less than 10 to 15 MeV and for scattering angles up to some $\theta_{\max}(K)$ the precise knowledge of the Rayleigh amplitude is still of importance for the reliable interpretation of the experimental data. Exact Rayleigh scattering calculations are very complicated at such relatively high energies and have not yet been performed. The extrapolations one can make, using the few values available from such calculations at low energies ²⁰⁾ and the form factor approximations of Franz ²¹⁾ and Bethe ²²⁾, are not very convincing. At large angles, complications for the interpretation arise from the insufficiently accurate information about nuclear resonance scattering ⁴⁾.

In spite of these difficulties the experiments of Bösch et al. ⁴⁾ at 9 MeV ($K = 8.81$) actually indicate between 20 and 60° scattering angle the existence of the Delbrück effect. The analysis of their data for lead has been repeated for the purpose of the present discussion using our values of Table 1. Now the agreement between theory and experiment has become rather good (see Fig. 1). In the range from 20 to 30° there is, for the first time, even slight evidence of the existence of the real part of the Delbrück amplitude !

For energies higher than 15 MeV and at small scattering angles, the conditions for detecting the Delbrück effect are very good. Here, as remarked before, the amplitude increases proportionally to K and hence nuclear Thomson scattering and nuclear resonance scattering can be neglected. On the other hand, incoherent effects like Compton scattering by the atomic electrons and eventually secondary scattering have to be considered. This has to be done since at small angles the separation of the total differential cross-section into its elastic

and inelastic part is, in general, difficult to be achieved experimentally. We conclude from the results of our computations (Tables 1 and 2) that for values of the momentum transfer $x < 2$ the imaginary part is dominating, whereas for $x \geq 2$ one should expect good experimental evidence for the real part of the Delbrück scattering amplitude.

The method of analyzing the dependence of the measured scattering cross-sections on the nuclear charge Z at fixed x has been successfully applied in the experiments of Stierlin et al.⁵⁾ at 17 MeV ($K = 16.6$) and of Moffat and Stringfellow⁶⁾ at 87 MeV. This method needs less theoretical knowledge of the different scattering processes involved than is necessary for the investigation of the angular dependence of the measured cross-sections for one particular heavy element. Fitting the data to an appropriate function of Z , the coefficient of Z^4 has to be pretty uniquely attributed to Delbrück scattering. Stierlin et al. found in their experiments at $x \approx 0.5$ for this coefficient the value 1.71 ($\pm 40\%$), whereas from Tables 1 and 2 follows the same theoretical value $1.86 \times 10^{-32} \text{ cm}^2/\text{sterad}$ ($\pm 10\%$). This good agreement can be taken as another confirmation of the existence of the imaginary part. Similarly, also the experiments of Moffat and Stringfellow are in fair agreement with the small angle approximations^{11),13)} of which we now definitely know that they are sufficiently accurate.

In conclusion, we may say that confirmations of the existence of the Delbrück effect have accumulated within the last years but that further experiments should be performed at energies above 5 MeV in particular to confirm also the existence of the real part of its amplitude.

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Table 1

Real and imaginary parts of the Delbrück amplitude calculated in units $(\alpha Z)^2 r_0$ from the formulae of Kessler¹⁵⁾ and the principal value integral formula (2). $x = 4K \sin(\Theta/2)$, $K = k/2m$.

K	x, Θ	0	1/2	1	3/2	2	3	10°	20°	30°	45°	60°	90°	120°
1.30	$a_{1//}$							0.073 ±0.005	0.045 ±0.002	0.030 ±0.001	0.018 ±0.001	0.0117 ±0.0004	0.0068 ±0.0003	0.0051 ±0.0002
	$a_{1\perp}$							0.058 ±0.004	0.029 ±0.002	0.015 ±0.001	0.0054 ±0.0007	0.0005 ±0.0005	0.0028 ±0.0003	0.0040 ±0.0002
	$a_{2//}$	0.0058 ±0.0001	0.0055 ±0.0001	0.0050 ±0.0001	0.0042 ±0.0001	0.0035 ±0.0001	0.0024 ±0.0001	0.0056 ±0.0001	0.0051 ±0.0001	0.0044 ±0.0001	0.0034 ±0.0001	0.0026 ±0.0001	0.0019 ±0.0001	0.0018 ±0.0001
	$a_{2\perp}$	0.0058 ±0.0002	0.0055 ±0.0001	0.0047 ±0.0001	0.0036 ±0.0001	0.0025 ±0.0001	0.00064 ±0.00003	0.0055 ±0.0001	0.0049 ±0.0001	0.0039 ±0.0001	0.0024 ±0.0001	0.0011 ±0.00004	0.00068 ±0.00003	0.0015 ±0.0000
2.56	$a_{1//}$	0.89 ±0.09	0.34 ±0.02	0.185 ±0.007	0.111 ±0.005	0.073 ±0.003	0.034 ±0.002	0.21 ±0.01	0.088 ±0.004	0.043 ±0.002	0.018 ±0.001	0.0093 ±0.0008	0.0025 ±0.0002	0.0003 ±0.0001
	$a_{1\perp}$	0.91 ±0.11	0.28 ±0.02	0.121 ±0.008	0.048 ±0.006	0.017 ±0.004	0.003 ±0.003	0.14 ±0.01	0.030 ±0.005	0.003 ±0.004	0.004 ±0.002	0.002 ±0.002	0.0004 ±0.0008	0.0003 ±0.0003
	$a_{2//}$	0.27 ±0.01	0.25 ±0.01	0.205 ±0.006	0.157 ±0.005	0.126 ±0.004	0.080 ±0.002	0.213 ±0.008	0.141 ±0.004	0.091 ±0.003	0.053 ±0.002	0.035 ±0.001	0.019 ±0.001	0.0133 ±0.0006
	$a_{2\perp}$	0.27 ±0.01	0.24 ±0.01	0.180 ±0.006	0.121 ±0.005	0.079 ±0.003	0.033 ±0.002	0.195 ±0.006	0.096 ±0.003	0.045 ±0.002	0.0111 ±0.0005	0.0015 ±0.0005	0.00085 ±0.0004	0.0098 ±0.0003
5.94	$a_{1//}$	2.9 ±0.5	0.80 ±0.07	0.21 ±0.04	0.00 ±0.03	0.09 ±0.02	0.13 ±0.01	0.11 ±0.02	0.118 ±0.009	0.082 ±0.004	0.050 ±0.002	0.034 ±0.002	0.019 ±0.001	0.0136 ±0.0005
	$a_{1\perp}$	3.3 ±0.7	0.48 ±0.09	0.02 ±0.06	0.19 ±0.04	0.19 ±0.03	0.14 ±0.01	0.18 ±0.02	0.096 ±0.009	0.041 ±0.006	0.009 ±0.003	0.002 ±0.002	0.009 ±0.001	0.0096 ±0.0007
	$a_{2//}$	2.24 ±0.08	1.90 ±0.07	1.32 ±0.06	1.05 ±0.05	0.76 ±0.04	0.47 ±0.02	0.74 ±0.04	0.30 ±0.01	0.169 ±0.007	0.083 ±0.004	0.050 ±0.002	0.022 ±0.001	0.0128 ±0.0006
	$a_{2\perp}$	2.25 ±0.11	1.78 ±0.06	1.17 ±0.04	0.73 ±0.03	0.50 ±0.02	0.23 ±0.01	0.47 ±0.02	0.117 ±0.006	0.044 ±0.003	0.010 ±0.002	0.002 ±0.002	0.0069 ±0.0006	0.0084 ±0.0004
8.81	$a_{1//}$	4.7 ±0.5	0.9 ±0.2	0.1 ±0.1	0.39 ±0.09	0.45 ±0.06	0.43 ±0.04	0.43 ±0.03	0.24 ±0.01	0.145 ±0.009	0.080 ±0.006	0.051 ±0.003	0.026 ±0.002	0.017 ±0.001
	$a_{1\perp}$	4.6 ±0.7	0.5 ±0.3	0.4 ±0.1	0.59 ±0.07	0.54 ±0.05	0.40 ±0.04	0.41 ±0.03	0.13 ±0.01	0.051 ±0.009	0.008 ±0.005	0.002 ±0.003	0.010 ±0.002	0.012 ±0.001
	$a_{2//}$	4.6 ±0.2	3.7 ±0.1	2.55 ±0.09	1.9 ±0.1	1.39 ±0.06	0.86 ±0.04	0.83 ±0.03	0.31 ±0.01	0.155 ±0.007	0.074 ±0.003	0.042 ±0.002	0.019 ±0.001	0.0104 ±0.0004
	$a_{2\perp}$	4.6 ±0.2	3.4 ±0.1	2.18 ±0.09	1.39 ±0.06	0.92 ±0.04	0.47 ±0.02	0.43 ±0.02	0.103 ±0.007	0.039 ±0.004	0.010 ±0.002	0.001 ±0.001	0.0054 ±0.0007	0.0066 ±0.0003
13.0	$a_{1//}$	7.5 ±2.2	0.25 ±0.32	0.95 ±0.21	1.2 ±0.1	1.18 ±0.09	1.01 ±0.06	0.73 ±0.04	0.35 ±0.02	0.20 ±0.01	0.103 ±0.004	0.062 ±0.002	0.030 ±0.001	0.019 ±0.001
	$a_{1\perp}$	7.1 ±1.6	0.05 ±0.3	1.3 ±0.2	1.4 ±0.1	1.24 ±0.09	0.89 ±0.06	0.52 ±0.04	0.15 ±0.02	0.06 ±0.01	0.019 ±0.007	0.002 ±0.004	0.010 ±0.002	0.013 ±0.001
	$a_{2//}$	9.2 ±0.4	7.1 ±0.3	4.5 ±0.2	3.3 ±0.1	2.3 ±0.1	1.45 ±0.06	0.83 ±0.04	0.29 ±0.01	0.140 ±0.006	0.063 ±0.003	0.034 ±0.001	0.0138 ±0.0006	0.0080 ±0.0003
	$a_{2\perp}$	9.2 ±0.4	6.4 ±0.3	4.0 ±0.2	2.5 ±0.1	1.7 ±0.1	0.81 ±0.07	0.36 ±0.04	0.087 ±0.017	0.034 ±0.008	0.010 ±0.004	0.002 ±0.002	0.003 ±0.001	0.0048 ±0.0007
16.6	$a_{1//}$	10.8 ±2.7	0.24 ±0.65	1.5 ±0.4	2.1 ±0.3	2.0 ±0.2	1.6 ±0.1	0.92 ±0.08	0.40 ±0.03	0.22 ±0.02	0.11 ±0.01	0.067 ±0.003	0.033 ±0.001	0.0204 ±0.0007
	$a_{1\perp}$	10.8 ±2.7	0.9 ±0.7	2.2 ±0.3	2.2 ±0.2	1.9 ±0.2	1.5 ±0.1	0.58 ±0.07	0.16 ±0.03	0.063 ±0.016	0.015 ±0.009	0.001 ±0.01	0.011 ±0.003	0.014 ±0.001
	$a_{2//}$	13.7 ±0.7	9.4 ±0.4	6.3 ±0.3	4.4 ±0.2	3.2 ±0.2	1.97 ±0.09	0.81 ±0.03	0.26 ±0.01	0.122 ±0.005	0.054 ±0.002	0.029 ±0.001	0.0110 ±0.0007	0.0065 ±0.0003
	$a_{2\perp}$	13.7 ±0.7	8.7 ±0.5	5.2 ±0.4	3.4 ±0.3	2.3 ±0.2	1.2 ±0.1	0.31 ±0.05	0.08 ±0.02	0.03 ±0.01	0.007 ±0.005	0.002 ±0.003	0.002 ±0.001	0.0038 ±0.0005

Table 2

Real and imaginary parts of the Delbrück amplitude calculated in units $(\alpha Z)^2 r_0$ from the small-angle approximation ¹³⁾.

K	x	0 *)	1/2	1	3/2	2	3
1.30	$a_{1//}$	0.239	0.082	---	---	---	---
	$a_{2//}$	0.0058	0.0057	0.0054	0.0049	0.0043	0.0030
	$a_{2\perp}$	0.0058	0.0053	0.0041	0.0028	0.0018	0.00063
2.56	$a_{1//}$	0.908	0.38	0.20	---	---	---
	$a_{2//}$	0.267	0.258	0.233	0.198	0.158	0.088
	$a_{2\perp}$	0.267	0.228	0.161	0.108	0.073	0.034
5.94	$a_{1//}$	3.21	0.94	0.21	- 0.05	---	---
	$a_{2//}$	2.23	2.04	1.62	1.21	0.903	0.503
	$a_{2\perp}$	2.23	1.63	1.05	0.702	0.491	0.264
8.81	$a_{1//}$	5.29	1.1	- 0.18	- 0.56	- 0.70	---
	$a_{2//}$	4.72	4.08	3.02	2.21	1.64	0.932
	$a_{2\perp}$	4.72	3.14	1.97	1.32	0.933	0.519
13.0	$a_{1//}$	8.42	0.89	- 0.73	- 1.4	- 1.5	---
	$a_{2//}$	9.18	7.35	5.19	3.76	2.79	1.61
	$a_{2\perp}$	9.18	5.54	3.41	2.29	1.63	0.928
16.6	$a_{1//}$	11.2	- 0.030	- 1.90	---	- 2.4	- 2.1
	$a_{2//}$	13.6	10.3	7.13	5.15	3.82	2.22
	$a_{2\perp}$	13.6	7.72	4.72	3.17	2.27	1.30
30.0	$a_{1//}$	21.4	- 3.9	- 6.4	---	- 6.3	- 6.0
	$a_{2//}$	32.9	21.5	14.5	10.4	7.74	4.55
	$a_{2\perp}$	32.9	16.1	9.69	6.54	4.70	2.75

*) These values are exact within 1 to 3%.

Table 3

Values of the Delbrück scattering amplitude calculated
in units $(\alpha Z)^2 r_0$ from the high-energy large-angle
approximation of Sannikov¹⁴).

K	Θ	10°	20°	30°	45°	60°	90°
8.81	$a_{1//}$ ($\perp$)	0.411	0.535	0.535	0.500	0.467	0.415
13.0	$a_{1//}$ ($\perp$)	0.505	0.537	0.50	0.45	0.408	0.354
16.6	$a_{1//}$ ($\perp$)	0.535	0.513	0.466	0.406	0.364	0.311

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FIGURE CAPTION

Experimentally determined differential cross-sections for the scattering of 9 MeV ($K = 8.81$) γ rays in lead according to Bösch et al.⁴⁾. The solid line represents the theoretically determined cross-sections for the coherent superposition of nuclear resonance scattering, nuclear Thomson scattering, Rayleigh scattering and Delbrück scattering (real and imaginary part of its amplitude). The dashed line is found for the theoretical cross-sections if the real part of the Delbrück scattering amplitude is not included.

FIG. 1