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PERIPHERAL ANALYSIS OF DOUBLE PION PRODUCTION
IN PROTON-PROTON COLLISIONS ^{+))}

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A B S T R A C T

Different reactions of double pion production in proton - proton collisions at 2.00 and 2.85 GeV are discussed from the point of view of the peripheral mechanism of interaction. It is found that the two types of peripheral diagrams which can occur (where the two pions come out from different vertices and from the same vertex respectively) are both important for the description of these reactions. It is also shown that the off-shell corrections cannot be neglected. The comparison with experiment, and the discussion of the points of agreement and discrepancy are carried out in detail.

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1. Qualitative Evidence for a Peripheral Mechanism of Interaction

It has recently been started a systematic experimental analysis of the processes of double pion production in proton-proton collisions at not too high energies ($\lesssim 3$ GeV). The experimental data available at present refer to incident proton energies of 2.00 GeV¹⁾ and 2.85 GeV²⁾: the good statistics obtained allowed the experimentalists to separate the various possible reactions and, for each of them, to determine the spectra of the final products. The analyses performed so far^{1),2)} refer mainly to the formation of the first πN isobar (the Δ -resonance) shown by the c.m. energy distributions. Another characteristic feature observed in these reactions is the strong forward-backward peaking of the c.m. angular distributions of the nucleons (integrated over all energies), which decrease steeply as one goes off the preferred directions. It may be interesting to recall that this characteristic behaviour is shown also by the nucleon angular spectra in the processes of single pion production in pp collisions in the same interval of energy³⁾; for the latter case, an explanation has been proposed in terms of the model considering the process as mediated by the exchange of one single virtual pion between the initial nucleons, i.e., by the "peripheral" model⁴⁾. The predictions of this model agree with the experimental data to a surprising degree of accuracy^{5),6)}. The best agreement with experiment, however, has been obtained only after taking fully into account the fact that the intermediate pion is off the mass shell: this can be achieved by introducing the form factors describing pion emission and propagation⁷⁾ in the usual peripheral formulae where the pion is considered everywhere as a real particle (pole approximation)⁸⁾. It is then straightforward to ask the question whether the peripheral mechanism of interaction can be considered as responsible for the observed forward-backward peaking also in the case of double pion production, and to work out the predictions of the model for an exhaustive comparison with experiment. Such is the aim of the present paper. However, because of the presence of four particles in the final state, the calculation of the spectra is not so straightforward as it was in the case of single pion production. It will be shown that, in order to get something more than an order of magnitude evaluation, several approximations

and assumptions must be introduced in the calculation. In particular, it will be shown that a satisfactory evaluation of double pion production in proton-proton collisions implies the knowledge of the energy behaviour of the reactions of single pion production in pion-proton collisions, including the ones initiated by π^0 's (today completely unknown).

In Section 2 we shall give a general formulation of the problem, in Section 3 we shall calculate the contribution of one of the types of diagrams occurring, in Section 4 we shall discuss the processes $\pi + N \rightarrow 2\pi + N$, in Section 5 we shall calculate the contribution of another type of diagram occurring. Finally, Section 6 will contain the comparison with the experiment and the discussion of the results obtained.

2. Peripheral Contributions to Double Pion Production

It can be easily seen that all the peripheral diagrams describing double pion production in nucleon-nucleon collisions can be grouped into two classes : i.e., the ones where the two final pions come out from different vertices (Fig. 1a) and the ones where the two final pions come out from the same vertex (Fig. 1b). We shall call the diagrams of the type shown in Fig. 1a as "double isobar diagrams", and the diagrams of the type shown in Fig. 1b as "Drell diagrams" (since they were first considered in a paper by Drell)⁹⁾.

In each diagram the exchange of the initial or/and final nucleons is allowed; thus each reaction is contributed by four diagrams of one type, and four diagrams of the other. For the c.m. angular distribution in proton-proton collisions, however, the exchange of the initial particles does simply give a reflection of the spectrum around 90° : then half of the graphs occurring need no additional calculations. Furthermore for the double isobar diagrams, the exchange of the final nucleons may only alter the isotopic spin coefficients relevant to the processes of πN scattering at the vertices.

In the calculation of the contributions of the eight diagrams in question, all the interferences between them will be systematically neglected. This will be done mainly for simplicity, but also on the basis of some crude qualitative considerations valid for the single pion production case and discussed in a previous paper [Ref. ¹⁰⁾, Sect. II.5]. From the same paper [Ref. ¹⁰⁾, Sect. III] we will take the formulae giving the contributions of the square moduli of the various diagrams. In this work we will not report such formulae for the sake of simplicity : we will only briefly summarize the approximations used for their numerical evaluation.

An important point to be discussed is the introduction of the off-shell corrections due to the virtuality of the intermediate pion. For the double isobar diagrams, such corrections are unambiguously expressed in terms of the ones valid for the single pion production reactions. For the Drell diagrams, no evaluation of the off-shell effects exists at present : however, it should look reasonable to approximate such effects by one momentum-transfer-dependent cut-off function, of the same type as the one working in the peripheral diagrams describing single pion production. We think that this procedure, though lacking a convincing justification, is the most reasonable one in the present situation ¹¹⁾. In any case, the need for a correction in the Drell diagrams will be apparent from the analysis of the experimental data, as it will be discussed in Section 6.

Before discussing further details of the calculation in the following Section, we want to summarize here the present status of the experimental information about these reactions. At 2.85 GeV ²⁾ only the reaction

$$p+p \rightarrow p+p+\pi^+\pi^- \quad (1)$$

has been fully analyzed. At 2.00 GeV ¹⁾ analysis has been made of reaction (1) and of the following ones

The c.m. proton spectra have been given for reactions (1) to (3) : only the total cross-section has been determined for reaction (4). The measured total cross-sections are the following

Reaction	2.00 GeV	2.85 GeV
(1)	(2.51±0.14) mb	(2.67±0.13) mb
(2)	(3.88±0.21) mb	
(3)	(0.89±0.10) mb	
(4)	(0.60±0.08) mb	

We anticipate here that the nearly constant value of the production rate for reaction (1) is in a certain sense surprising, because of the large increase of phase space, and does not admit a "peripheral" interpretation of the mechanism of interaction. The other features of these processes can be satisfactorily accounted for by the model.

3. The Evaluation of the Double Isobar Diagrams

The double isobar diagrams (Fig. 1a) describing reactions (1) to (4) can be easily constructed. Due to the small influence of the higher ($T=1/2$) πN resonances (suppressed by phase-space, especially at 2 GeV) the ratios among the rates for the different reactions are substantially

determined by the isospin coefficients of the $T=3/2$ resonance. The formulae used for the calculation of the spectra are the ones given in Section III.2 of Ref. ¹⁰⁾, but in order to reduce the huge computer work required by their numerical evaluation, some approximations have been introduced. We are going to list them in the following.

i) Isobar approximation

The πN system outgoing from that vertex where full integration over the nucleon variables is performed is replaced by an "isobar" of zero width. This can be achieved by writing the πN resonant scattering cross-section entering in the formula in the following approximate form

$$\sigma(v) \simeq \alpha \cdot \frac{2\pi^2}{q^2} \Gamma \delta(v_r - v) \quad (5)$$

Here v is the total c.m. energy of the πN system, v_r is the resonance value, q is the c.m. momentum, Γ is the actual width (these two quantities, because of the δ function, will be calculated at the resonance); α is a coefficient including angular momentum and isospin numerical factors: e.g., for the $\pi^+ p$ system in the 33 -resonant state $\alpha = 2$.

In this way the four-fold integration required for the calculation of the nucleon spectra is reduced to a three-fold integration. In order not to get too drastic approximations, it is necessary to maintain the actual structure of the πN cross-section in the other vertex.

The effect of such an approximation can be estimated by calculating the total cross-section, for which a simpler formula can be used ¹²⁾, by retaining the actual width of the πN scattering in both vertices. By comparison it is seen that the isobar approximation increases the total contribution of a graph by about 20-30 %.

ii) Approximate cut-off function

The off-shell corrections are introduced in the spectra through the approximate form of the cut-off function given in Ref. 5) instead of the more rigorous formula given in Ref. 6). If we call ω and ν the total c.m. energies at the two vertices, and Δ^2 the squared 4-momentum of the intermediate pion, the matrix element for the double scattering diagram is proportional to (μ = pion mass)

$$\frac{K'(\Delta^2)}{\Delta^2 + \mu^2} M(\omega, \Delta^2) M(\nu, \Delta^2) \quad (6)$$

where M is the matrix element for off-shell pion-nucleon scattering and K' is the form factor of the pion propagator. In Ref. 7) it has been shown that $M(\omega, \Delta^2)$ is proportional to the form factor $K(\Delta^2)$ of the proper pion-nucleon vertex :

$$M(\omega, \Delta^2) = K(\Delta^2) M_0(\omega, \Delta^2) \quad (7)$$

In the same reference it has also been given a form of $M_0(\omega, \Delta^2)$ valid in the 33-resonance region; this form, under certain approximations, can be simplified through the relation 5)

$$G(\Delta^2) M_0(\omega, \Delta^2) \sim \phi(\Delta^2) M(\omega) \quad (8)$$

where $G(\Delta^2) = K^2(\Delta^2) K'(\Delta^2)$ and $M(\omega)$ is the on-shell, physical ~~πN~~ scattering matrix element. From Eqs. (6), (7) and (8) one can see that the final contribution corrected for the off-shell effects can be obtained by multiplying the contribution calculated in the pole approximation $\propto (\Delta^2 + \mu^2)^{-1} M(\omega) M(\nu)$ by a cut-off function $\Lambda(\Delta^2)$ given by

$$\Lambda(\Delta^2) = \phi^2(\Delta^2) / G(\Delta^2) \quad (9)$$

The cut-off function (9) is now completely determined, since $\phi(\Delta^2)$ can be taken from Ref. ⁵⁾ and $G(\Delta^2)$ has been obtained in Ref. ⁶⁾. It may be worth noting that, for $\Delta^2 \lesssim 20 \mu^2$, $\Lambda(\Delta^2) > 1$, so that the off-shell corrections enhance the contribution of the diagram with respect to the pole approximation. For higher Δ^2 , on the other hand, $\Lambda(\Delta^2)$ is < 1 and then acts as an effective, though smooth, cut-off function. Also in this case the effect of the approximation induced by the use of (8) can be evaluated by inspection of the total cross-section calculated correctly. It is found that this approximation reduces the total contribution of a graph by about 20-25 %.

It can then be hoped that the effects of the two approximations i) and ii) performed together compensate each other to a certain extent, so that the final spectra can be considered as acceptable ^{13), 14)}.

4. The Reactions of Single Pion Production in Pion-Proton Collisions

For the calculation of the Drell diagrams (Fig. 1b) it is necessary to know the energy behaviour of the total cross-section for the reaction of the type $\pi + N \rightarrow 2\pi + N$ occurring in the lower vertex of the graph. The knowledge of such a behaviour is required from threshold to a maximum energy value depending on the incident proton energy. This maximum at 2.85 GeV is ~ 1600 MeV of incident pion energy, but the main contribution to the Drell diagram is provided by the energy region below 1100 MeV. In this region, however, the present experimental information on single pion

production by pions is far from being complete : we have summarized the experimental points known today ¹⁵⁾ for the various reactions in Fig. 2, where the energy scale is given by the total c.m. energy ω measured in units of the nucleon mass M . Such reactions are :

$$\pi^+ + p \rightarrow \pi^+ + \pi^0 + p \quad (10)$$

$$\pi^+ + p \rightarrow \pi^+ + \pi^+ + n \quad (11)$$

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n \quad (12)$$

$$\pi^- + p \rightarrow \pi^- + \pi^0 + p \quad (13)$$

$$\pi^- + p \rightarrow \pi^0 + \pi^0 + n \quad (14)$$

In the following we will denote the total cross-section for reaction (10) by $\sigma(10)$, and so on.

In the plots of Fig. 2 some empirical curves used for fitting the data are reported. Unfortunately these "fits" allow a large uncertainty and many other curves can be drawn, which are compatible with the experimental data. However, since these cross-sections will always be integrated over the energy, the uncertainty on the final result should be smaller.

In spite of the meagre information available about these reactions, one can attempt a sort of isospin analysis of the processes. In the case of a three-particle final state, one must add a further quantum number to the total isospin and its third component, in order to describe a physical state completely. In the present case it is convenient to choose the total isospin of the system of the two final pions.

By calling T, T_3 the total isospin and its third component, and t the isospin of the two final pions, the matrix element for each reaction will be described as a linear combination of quantities of the type $M(T, T_3; t)$ which, for the principle of charge independence, will be independent of T_3 . By working out the formulae explicitly and by remarking that amplitudes with even and odd t do not interfere, all cross-sections for the reactions of interest can be expressed as combinations of six functions of energy : such a procedure is worked out in the Appendix. By comparison with the experimental data, one can obtain the following qualitative picture of the phenomenon ¹⁶⁾ :

- a) at very low energies ($\lesssim 300$ MeV) the $T = 3/2$ cross-section is practically zero, and all the effects are due to the $T = 1/2$ interaction (presumably in the state $t=0$).
- b) as the energy increases, those reactions where the contribution of the states $t=1$ is possible show a rapid increase both for $T = 1/2$ and $T = 3/2$. This effect is particularly marked in the region from 500 to 800 MeV, where unfortunately we were not able to find experimental data. Above 800 MeV, though the experimental points are affected by large errors, the total cross-sections seem to stay approximately constant. It has been conjectured ¹⁶⁾ that such an interaction is mainly of peripheral nature, at least for small momentum transfers to the nucleon.
- c) from comparison of the rates of reactions (10) and (11) there seems to be an indication that the contribution of the states $t=2$ is small with respect to the contributions of other type. This might be explained by the fact that the present experimental information on $\pi\pi$ interaction is consistent with the absence of strong effects or resonances for isospin 2 in the considered interval of energy.

We pass now to the discussion of the most difficult problem, i.e., the evaluation of the cross-sections for incident π^0 's. It is easily seen that the Drell diagrams for reaction (1) do not contain any of the

reactions (10) to (14), but only the following one

and similarly the other processes require the knowledge of the other two reactions

It is essential for our procedure to give a criterion for evaluating at least an approximate form of the behaviour of reactions (15) to (17). A sort of "experimental" determination of the energy behaviour of σ (15) has been made in Ref. ²⁾. The authors assume that the whole of the contributions to the angular distribution in the region of the peaks ($|\cos\theta_{c.m.}| \gtrsim 0.8$) is provided by the Drell diagram calculated in the pole approximation. They analyze the experimental distribution for reaction (1) and, after subtraction of a uniform "background" (see next Section), they obtain the points reported in Fig. 3.

Although the order of magnitude obtained is satisfactory, we think that this method is subjected to some criticism. It can be shown that in the pole approximation the Drell diagram must be dominant over the double isobar diagram (see Section 6); but we shall also see that for these reactions the pole approximation is not to be trusted also in the region of the peaks, since the momentum transfers involved there are not small enough, and one has to deal with sensitive off-shell corrections.

A more general criterion for evaluating the cross-sections for reactions (15) to (17) is obtained by bringing to its "logical" consequence the qualitative evidence listed under c) above : i.e., by neglecting all amplitudes having $t=2$ when also other values of t are possible.

Under this hypothesis one can express $\sigma(15)$, $\sigma(16)$ and $\sigma(17)$ in terms of the cross-sections for reactions (10) to (14); for the latter two processes the expressions obtained are particularly simple. One has

$$\begin{aligned}\sigma(15) &= \frac{1}{2} [\sigma(10) - \sigma(12) + \sigma(13) + 4\sigma(14)] \\ \sigma(16) &= \frac{1}{2} \sigma(14) \\ \sigma(17) &= \sigma(12) - 2\sigma(14)\end{aligned}\tag{18}$$

In line of principle, this criterion gives an unambiguous evaluation of the unknown cross-sections. Such an evaluation is anyway rather safe at low energy ($\omega \lesssim 1.5$ M) : if we exploit the fact that the $T = 3/2$ matrix elements are negligible in this region, we find that the cross-section for each π^0 reaction is one half of the cross-section for another corresponding reaction initiated by π^- (see Appendix). Furthermore we can assume that at such energies only the interaction with $t=0$ gives an important contribution; this fact implies $\sigma(12) \gg \sigma(13)$, and this relation is confirmed by experiment. Then from the knowledge of the behaviour of $\sigma(12)$ (fairly well known at low energy), and of $\sigma(13)$, one can determine the behaviour of all π^- and π^0 reactions.

As the energy increases, the situation becomes worse even if we use the approximation leading to Eqs. (18). In fact, if we combine several experimental cross-sections the experimental errors are enhanced, and also in the most optimistic view the values obtained from (18) for $\sigma(15)$ and $\sigma(17)$ can always be varied by 1-2 mb for $\omega \gtrsim 1.55$ M. However, the order of magnitude and the trend obtained for such cross-sections are not much changed by changing the fits of Fig. 2 in a reasonable way. We can then accept the curves drawn in Fig. 3, obtained from Eqs. (18) by using the curves drawn in Fig. 2. We would like to remark that the behaviour obtained for $\sigma(\pi^+\pi^-p)$ is consistent with the "experimental" points deduced in Ref. ²⁾ : one may believe this fact to

be something more than a pure coincidence. Also for these reactions the consideration holds that, after integration over the energy, the influence of the described uncertainties over the final result should be reduced. Needless to say, there are no means of evaluating the actual amount of the uncertainty due to this cause; therefore, it is of no use to evaluate the contributions of other type (e.g., the double isobar diagrams) too accurately.

5. The Evaluation of the Drell Diagrams

After the discussion of how to evaluate the cross-sections to be inserted into the Drell diagrams, only a few further points need to be explained. As usually, in the case of a Drell diagram the final spectrum of the nucleons will consist of two parts : one contribution from the graph where the nucleon in question comes out alone, and another contribution from the graph where the outgoing nucleon is accompanied by the two pions. The first case is easy to deal, see e.g., again Section III.2 of Ref. ¹⁰⁾. The second case will provide a sort of "background term" which cannot be evaluated correctly without knowing the structure of the pion production cross-section. This background can by no means be neglected : e.g., for two protons in the final state the background is of the same magnitude as the contribution from the other term. However, one can expect such a background to be nearly isotropic. Such was the hypothesis made in Ref. ²⁾ for the evaluation of ϵ (15). We have improved this approximation by postulating an isotropic cross-section for the $\pi + N \rightarrow 2\pi + N$ reaction; under this hypothesis, after suitable averages over the internal degrees of freedom of the $(2\pi + N)$ system, one can express the background term as a function only of the energy behaviour of the total cross-section. In such a way we have obtained the following formula [notation of Ref. ¹⁰⁾], M = nucleon mass, μ = pion mass, $\Gamma(\Delta^2)$ cut-off function (see later)] :

$$\frac{d\sigma_{\text{back}}}{d\cos\theta^B} = \frac{1}{8\pi^2} \frac{1}{k^B W^2} \left(\frac{G^2}{4\pi} \right) \int_{M+2\mu}^{W-M} q_1^B du \int_{2\mu}^{u-M} q^P v dv \cdot \int_{-1}^1 d\cos\theta_2^P \cdot$$

$$\frac{\Delta^2}{(\Delta^2 + \mu^2)^2} \Gamma^2(\Delta^2) \int_0^{2\pi} d\varphi_2^P \omega k_1^Q \sigma_1(\omega) \frac{4\omega^2 \log \frac{\omega}{M}}{(\omega - M)^4} \quad (19)$$

Because of the exchange of the initial protons, one must add to Eq. (19) the same spectrum reflected around 90° . The spectrum obtained from Eq. (19) satisfies the correct normalization to the expected total cross-section (the latter can be calculated from the other diagram). Eq. (19) provides again a forward-backward peak, which is much less pronounced than the peak due to the other term.

For the off-shell corrections, in Section 2 we have already stated that it is not unreasonable to introduce them through a multiplicative cut-off function $\Gamma(\Delta^2)$ of the "mass" Δ^2 of the exchanged pion. By analogy with the single pion production case ⁵⁾ we conjecture the following form for $\Gamma(\Delta^2)$

$$\Gamma(\Delta^2) = \left(1 + \frac{\Delta^2 + \mu^2}{\alpha} \right)^{-1} \quad (20)$$

and we treat α as an adjustable parameter.

We want again to stress that the effects of the approximations introduced by the use of formulae (18) and (19) cannot be evaluated at present. We think, however, that such effects are much less important than the ones due to the uncertainty about the production cross-sections. In the next Section, when comparing to experiment, we shall no longer recall this point; all the approximations discussed will be used without further comments.

6. Comparison with Experiment

In order to get a first insight into the mechanism of interaction, let us first compare theory and experiment for the reaction having the best statistics, i.e., reaction (1). We start by evaluating the contribution of the double isobar diagram in the pole approximation : at 2.85 GeV, we find a cross-section much smaller ($\sim 1/5$) than the experimental one, and whose forward-backward peaking is quite irrelevant. The situation improves by introducing the form factors (see Fig. 4); it is clear, however, that if the interaction is of peripheral nature, some other process besides the double isobar formation must be present. This additional contribution is likely to be provided by the Drell diagrams. If we calculate the Drell diagrams again in the pole approximation, and insert the π^0 cross-section derived in Section 4, we see that this extra contribution is just of the order of magnitude required in order to obtain the right height of the peaks. However in the high momentum transfer region (around $\Theta = 90^\circ$), this contribution is definitely too large. This situation occurs also in the pole approximation calculations of the single pion production [cf. Da Prato, Ref. ⁸⁾] and strongly suggests the presence of a cut-off on the high values of the momentum transfer. By using Eq. (20) and by choosing $\alpha = 90 \mu^2$ (of the same order of magnitude as in single pion production), we get the total spectrum plotted in Fig. 5. In the same figure we have plotted also the pole approximation spectrum for comparison.

On the basis of these data, we calculate the spectra for the reactions observed at 2 GeV; they are plotted in Fig. 6. The corresponding total cross-sections are listed in Table I, where we indicate also the partial double isobar and Drell contributions. We see that the relative importance of the double isobar and Drell contributions is different for different reactions. However, in general the final results agree satisfactorily with the order of magnitude of the experimental cross-sections and the behaviour of the angular distributions. The agreement can probably still be improved by the choice of a slightly larger value of the cut-off α in Eq. (20).

However, one of the experimental cross-sections is not fitted by this analysis, i.e., the cross-section for reaction (1) at 2 GeV. It is clear that once all the free parameters have been determined at 2.85 GeV, no additional assumptions can be made at other energies. It can be still observed that the form factors give a better shape of the spectrum than the pole approximation, but the total cross-section is definitely too small. We could rather say that the peripheral model does not explain the ratio between the rates at 2.00 and 2.85 GeV for reaction (1); in fact, should we manage to get in agreement with experiment at 2 GeV by a different choice of the parameters, then the cross-section predicted at 2.85 GeV would be definitely too large. In fact, the peripheral predictions for both the double isobar and the Drell diagrams show a strongly increasing behaviour in the region of $1 \div 4$ GeV, in the pole approximation, as well as with the off-shell corrections: this situation is shown in Fig. 7 (the formulae used cannot be extended to the region beyond 5 GeV). It is our opinion that such a behaviour is due mainly to the increase of phase space, and not specifically to the structure of the interaction; therefore, the absence of such an increase in the experimental behaviour of $\sigma(1)$ seems to us to be rather puzzling also outside of the peripheral philosophy.

This point of disagreement is a serious one, since it contradicts the criterion assumed in Ref. ⁶⁾ as a test for the peripheral nature of the interaction. We point out, however, that the rapid increase of all the total cross-sections in this region has some resemblance with the behaviour of the peripheral cross-sections for single pion production in the "low energy region" (i.e., for incident energies below 800 MeV). In such a region the increase of phase space dominates the process, and gives the same trend of the cross-sections in the Mandelstam picture ¹⁷⁾ based on πN and $N N$ final state interactions, as well as in the peripheral approach which is conceptually quite different. If we examine the predictions of the peripheral model in this region, we see that they are sensitive to certain effects which rapidly become negligible as the energy increases. These characteristic features of the low energy

peripheral interaction are the following :

- a) the interference terms are very significant;
- b) final state nucleon-nucleon interactions are probably not negligible;
- c) the detailed form of the structure of the off-shell corrections is important and the use of the approximate formula (8) is not good.

In the present work, the approximations relevant to a), b) and c) have been assumed to be good without discussion, according to a "medium energy region" philosophy; but it might happen that, if our energies are still in the "low energy region" for the peripheral production of two pions, such approximations (in particular the neglecting of the interference terms) may strongly influence the energy behaviour of the cross-sections. Only a more accurate calculation of the peripheral diagrams can make one sure about this point; however, the theoretical work and the experimental information available at present do not allow this programme to be carried out. Needless to say, we can always ascribe the observed discrepancies to the presence of important non-peripheral contributions ¹⁸⁾.

We would attempt to conclude that the analysis of our reactions from the point of view of the peripheral model is anyway instructive. Many questions and uncertainties are still left, but also some definite answers have been given. We think that a detailed experimental and theoretical study of these reactions at higher energies may be of great interest for a better understanding of the mechanism of the inelastic processes.

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T A B L E I

Observed and calculated total cross-sections for the reactions studied in this paper. The theoretical values are split into double isobar and Drell contributions. The double isobar contributions are calculated exactly, i.e., without the approximations i) and ii) explained in Section 3.

Energy	Reaction	Cross-sections (mb)			
		Experim.	Theor.	Double Isobar	Drell
2.85 GeV	$pp \rightarrow pp\pi^+\pi^-$	2.67 ± 0.13	2.59	0.89	1.70
2.00 GeV	$pp \rightarrow pp\pi^+\pi^-$	2.51 ± 0.14	1.39	0.39	1.00
	$pp \rightarrow pp\pi^0\pi^0$	0.89 ± 0.10	0.61	0.32	0.29
	$pp \rightarrow pn\pi^+\pi^0$	3.88 ± 0.21	3.37	1.01	2.36
	$pp \rightarrow nn\pi^+\pi^+$	0.60 ± 0.08	0.58	0.08	0.50

A P P E N D I X

Given a state of two pions and one nucleon, we denote by T the total isotopic spin, by T_3 its third component and by t the total isotopic spin of the two-pion system. For a system of two unlike pions, we choose as basic orthogonal states the ones which are even or odd under the exchange of the two pions, since they correspond to physical states with even or odd angular momentum respectively. We introduce the notation

$$|P^{+-}\rangle = \frac{1}{\sqrt{2}} (|\pi^+\pi^-\rangle + |\pi^-\pi^+\rangle); |D^{+-}\rangle = \frac{1}{\sqrt{2}} (|\pi^+\pi^-\rangle - |\pi^-\pi^+\rangle) \quad (A.1)$$

We will use also analogous expressions obtained by a straightforward generalization of (A.1).

We then get the decomposition of the physical $\pi\pi\mathcal{N}$ states in terms of isospin eigenstates $|T, T_3; t\rangle$

$$\begin{aligned} |\pi^+\pi^+p\rangle &= |\frac{5}{2}, \frac{5}{2}; 2\rangle \\ |\pi^+\pi^+n\rangle &= \frac{1}{\sqrt{5}} |\frac{5}{2}, \frac{3}{2}; 2\rangle + \frac{2}{\sqrt{5}} |\frac{3}{2}, \frac{3}{2}; 2\rangle \\ |P^{+0}, p\rangle &= \frac{2}{\sqrt{5}} |\frac{5}{2}, \frac{3}{2}; 2\rangle - \frac{1}{\sqrt{5}} |\frac{3}{2}, \frac{3}{2}; 2\rangle \\ |D^{+0}, p\rangle &= |\frac{3}{2}, \frac{3}{2}; 1\rangle \\ |P^{+0}, n\rangle &= \sqrt{\frac{2}{5}} |\frac{5}{2}, \frac{1}{2}; 2\rangle + \sqrt{\frac{3}{5}} |\frac{3}{2}, \frac{1}{2}; 2\rangle \\ |D^{+0}, n\rangle &= \frac{1}{\sqrt{3}} |\frac{3}{2}, \frac{1}{2}; 1\rangle + \sqrt{\frac{2}{3}} |\frac{1}{2}, \frac{1}{2}; 1\rangle \\ |P^{+-}, p\rangle &= \frac{1}{\sqrt{5}} |\frac{5}{2}, \frac{1}{2}; 2\rangle - \sqrt{\frac{2}{15}} |\frac{3}{2}, \frac{1}{2}; 2\rangle + \sqrt{\frac{2}{3}} |\frac{1}{2}, \frac{1}{2}; 0\rangle \\ |D^{+-}, p\rangle &= \sqrt{\frac{2}{3}} |\frac{3}{2}, \frac{1}{2}; 1\rangle - \frac{1}{\sqrt{3}} |\frac{1}{2}, \frac{1}{2}; 1\rangle \\ |\pi^0\pi^0p\rangle &= \sqrt{\frac{2}{5}} |\frac{5}{2}, \frac{1}{2}; 2\rangle - \frac{2}{\sqrt{15}} |\frac{3}{2}, \frac{1}{2}; 2\rangle - \frac{1}{\sqrt{3}} |\frac{1}{2}, \frac{1}{2}; 0\rangle \end{aligned} \quad (A.2)$$

The states with negative T_3 can easily be obtained from (A.2).

We can now calculate the transition rates from any state πN to any state $\pi\pi N$. We shall assume the validity of charge independence, and drop the dependence on T_3 in the matrix elements, which will be denoted by $M(T;t)$. We need listing only the transitions where the initial nucleon is a proton. The others can be obtained from charge symmetry.

It is obvious to remark that the transition rate to an observed state, e.g., $|\pi^+\pi^-p\rangle$ will be the sum of the transition rates to a state $|P^{+-},p\rangle$ and to a state $|D^{+-},p\rangle$. From Eqs. (A.2) it is easily seen that elements with even and odd t do not interfere. Any other choice of the basic isospin vectors would lead to the same conclusion. Here is the list of the transition rates :

$$\begin{aligned}
 \sigma(\pi^+p \rightarrow \pi^+\pi^+n) &= \frac{4}{5} \left| M\left(\frac{3}{2}; 2\right) \right|^2 \\
 \sigma(\pi^+p \rightarrow \pi^+\pi^0p) &= \frac{1}{5} \left| M\left(\frac{3}{2}; 2\right) \right|^2 + \left| M\left(\frac{3}{2}; 1\right) \right|^2 \\
 \sigma(\pi^0p \rightarrow \pi^0\pi^0p) &= \frac{1}{9} \left| 2\sqrt{\frac{2}{5}} M\left(\frac{3}{2}; 2\right) - M\left(\frac{1}{2}; 0\right) \right|^2 \\
 \sigma(\pi^0p \rightarrow \pi^+\pi^-p) &= \frac{2}{9} \left| \sqrt{\frac{2}{5}} M\left(\frac{3}{2}; 2\right) + M\left(\frac{1}{2}; 0\right) \right|^2 + \frac{1}{9} \left| 2M\left(\frac{3}{2}; 1\right) + M\left(\frac{1}{2}; 1\right) \right|^2 \\
 \sigma(\pi^0p \rightarrow \pi^0\pi^+n) &= \frac{2}{5} \left| M\left(\frac{3}{2}; 2\right) \right|^2 + \frac{2}{9} \left| M\left(\frac{3}{2}; 1\right) - M\left(\frac{1}{2}; 1\right) \right|^2 \quad (A.3) \\
 \sigma(\pi^-p \rightarrow \pi^0\pi^0n) &= \frac{2}{9} \left| \sqrt{\frac{2}{5}} M\left(\frac{3}{2}; 2\right) + M\left(\frac{1}{2}; 0\right) \right|^2 \\
 \sigma(\pi^-p \rightarrow \pi^-\pi^+n) &= \frac{2}{9} \left| \frac{1}{\sqrt{5}} M\left(\frac{3}{2}; 2\right) - \sqrt{2} M\left(\frac{1}{2}; 0\right) \right|^2 + \frac{2}{9} \left| M\left(\frac{3}{2}; 1\right) - M\left(\frac{1}{2}; 1\right) \right|^2 \\
 \sigma(\pi^-p \rightarrow \pi^-\pi^0p) &= \frac{1}{5} \left| M\left(\frac{3}{2}; 2\right) \right|^2 + \frac{1}{9} \left| M\left(\frac{3}{2}; 1\right) + 2M\left(\frac{1}{2}; 1\right) \right|^2
 \end{aligned}$$

In the "very low energy approximation" we can retain only the amplitude $M(\frac{1}{2}; 0)$ and neglect all others. The only important reactions are then (12), (14), (15) and (16) and one gets :

$$\sigma(12) : \sigma(14) : \sigma(15) : \sigma(16) \cong 4 : 2 : 2 : 1 .$$

In particular, this approximation implies $\sigma(12) \gg \sigma(13)$.

If we allow also $t=1$ states to be present, but still neglect $T = 3/2$ states (according to the experimental evidence in the low energy region), we have :

$$\sigma(12) \simeq \frac{4}{9} |M(\frac{1}{2}; 0)|^2 + \frac{2}{9} |M(\frac{1}{2}; 1)|^2 \quad \sigma(15) \simeq \frac{2}{9} |M(\frac{1}{2}; 0)|^2 + \frac{1}{9} |M(\frac{1}{2}; 1)|^2$$

$$\sigma(13) \simeq \frac{4}{9} |M(\frac{1}{2}; 1)|^2 \quad \sigma(17) \simeq \frac{2}{9} |M(\frac{1}{2}; 1)|^2$$

$$\sigma(14) \simeq \frac{2}{9} |M(\frac{1}{2}; 0)|^2 \quad \sigma(16) \simeq \frac{1}{9} |M(\frac{1}{2}; 0)|^2$$

We see that the rates from the π^- reaction and the π^0 reaction lying on the same line differ by a factor of 2. Furthermore, $\sigma(12) \simeq 2\sigma(14) + \frac{1}{2}\sigma(13)$. At higher energies we can still neglect $t=2$ states for all reactions different from (11). In this case we get :

$$\sigma(10) \simeq |M(\frac{3}{2}; 1)|^2$$

$$\sigma(12) \simeq \frac{4}{9} |M(\frac{1}{2}; 0)|^2 + \frac{2}{9} |M(\frac{3}{2}; 1) - M(\frac{1}{2}; 1)|^2; \quad \sigma(15) \simeq \frac{2}{9} |M(\frac{1}{2}; 0)|^2 + \frac{1}{9} |2M(\frac{3}{2}; 1) + M(\frac{1}{2}; 1)|^2$$

$$\sigma(13) \simeq \frac{1}{9} |M(\frac{3}{2}; 1) + 2M(\frac{1}{2}; 1)|^2; \quad \sigma(17) \simeq \frac{2}{9} |M(\frac{3}{2}; 1) - M(\frac{1}{2}; 1)|^2 \quad (A.4)$$

$$\sigma(14) \simeq \frac{2}{9} |M(\frac{1}{2}; 0)|^2; \quad \sigma(16) \simeq \frac{1}{9} |M(\frac{1}{2}; 0)|^2$$

From this hypothesis one can get some interesting inequalities about the measured cross-sections, i.e., $\sigma(12) > 2\sigma(14)$. By an elementary manipulation of Eq. (A.4) one obtains Eqs. (18).

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- 11) If we look at the structure of the diagram of Fig. 1b and of the single pion production diagram, we see that in both graphs the same elements occur, namely one pion-nucleon vertex, one pion propagator and one pion-nucleon cross-section (for scattering in one case, for production in the other). One can then suppose that the off-shell corrections in both cases have roughly the same behaviour. The case of the diagram of Fig. 1a is different, since two cross-sections occur, and no coupling constant.
- 12) Cf., e.g., Eq. (3.33) of Ref. ¹⁰⁾ (in the pole approximation).
- 13) We will discuss later the fact that the double isobar diagrams are never dominant with respect to the Drell diagrams (of much more uncertain evaluation). Therefore there is no use calculating these spectra more accurately. Only the total cross-sections, obtainable by a simpler procedure, have been worked out exactly.

- 14) As a final remark, we point out that when the two final pions are identical an additional factor $1/2$ must be introduced, since, by integrating over the two pions independently, one counts twice the number of final states. For the same reason, when the two final nucleons are identical, the spectrum obtained is normalized to twice the total cross-section.
- 15) For a survey of the situation about single pion production by pions (with emphasis on the region from 500 to 1200 MeV) and for extensive experimental bibliography, see : R. Omnès and G. Valladas, Proceedings of the 1961 Aix-en-Provence Conference, p. 467. Almost all results above 600 MeV have been taken from Table I of this paper, where reference to the original work is contained. Further reference for the other data used :
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FIGURE CAPTIONS

- Fig. 1 The two possible types of peripheral diagrams describing double pion production : a) double isobar diagram; b) Drell diagram. Full lines represent nucleons, wavy lines represent pions.
- Fig. 2 Experimental points and eye-fits for reactions (10) to (14) in the energy region below 1300 MeV. For literature on the experiments, see Ref. ¹⁵⁾. For the energy scale, see text. The arrow in Fig. 2a indicates the position of a very precise low energy measurement of $(\pi^+\pi^+n)$ [Kirz et al. : $\sigma = (0.12 \pm 0.01) \text{mb}$ at $\omega/M = 1.443 \pm 0.003$].
- Fig. 3 Total cross-sections for reactions (15) and (17) calculated from Eqs. (13). The points shown have been taken from Ref. ²⁾.
- Fig. 4 Experimental c.m. angular distribution of the protons from reaction (1) at 2.85 GeV [from Ref. ²⁾], compared with the predictions of the peripheral model. In this figure the double isobar and Drell contributions are plotted separately. Dashed lines : pole approximation. Full lines : calculation taking into account the off-shell effects.
- Fig. 5 This figure refers to the same situation as in Fig. 4, but here the double isobar and Drell contributions are summed together in order to give the final theoretical curve. Dashed line : pole approximation. Full line : calculation taking into account the off-shell effects.
- Fig. 6 Experimental c.m. angular distributions of the protons from reactions (1) to (3) at 2.00 GeV [from Ref. ¹⁾], compared with the predictions of the peripheral model : a) reaction (1); b) reaction (2); c) reaction (3). Dashed lines : pole approximation. Full lines : calculation taking into account the off-shell effects.

Fig. 7 Energy behaviour of the contributions to the total cross-section for reaction (1) : a) of the double isobar diagrams; b) of the Drell diagrams. Dashed line : pole approximation. Full line : calculation taking into account the off-shell effects. The total cross-section σ is given by $\sigma_1 + \sigma_2$. T is the incident lab. kinetic energy.

Fig. 1

Fig. 3

Fig 2

Fig.4

Fig.5

Fig. 6

Fig. 7