CM-P00056878

SOME MYRIOTIC PARABOSON FIELDS

A. Galindo ⁺⁾

CERN, Geneva

⁺⁾ On leave of absence from the Junta de Energía Nuclear.

The author has given ¹⁾ explicit irreducible realizations (with non-degenerated vacuum) of the Green ²⁾ and Volkov ³⁾ commutation relations for paraboson and parafermion fields of multiplicity two. These are essentially obtained by taking as destruction and creation operators a_k, a_ℓ^+ the following expressions :

$$\begin{aligned} a_k &= \varepsilon \alpha_k \varepsilon \\ a_\ell^+ &= \varepsilon \alpha_\ell^+ \varepsilon \end{aligned} \quad (1)$$

where ε is the symmetrizer (anti) of particles occupying even places times the symmetrizer (anti) of those at odd places, and α_k, α_ℓ^+ are the Fock unsymmetrized destruction, creation operators. The Hilbert space of state vectors is the subspace $\varepsilon \mathcal{F}$, where $\mathcal{F}$ is the Fock general space $\bigoplus_{n=0}^{\infty} \mathcal{F}_1^{\otimes n}$.

The question naturally arises whether there exist other irreducible representations inequivalent to the previous ones, as it is the case for ordinary fields ⁴⁾. By using Haag's trick ⁵⁾ we exhibit, in this note, a continuous infinity of inequivalent irreducible representations for the paraboson commutation relations of rank 2. All but the standard one share the well-known property of the myriotic realizations : non-existence of a vacuum state and of the total particle number operator.

Let us write down the commutation relations for parabosons of rank 2 :

$$\begin{aligned} [a_k a_\ell a_m]_- &= 0 \\ [a_k a_\ell a_m^+]_- &= \lambda \delta_{\ell m} a_k \\ [a_k a_\ell^+ a_m]_- &= \lambda \delta_{k\ell} a_m - \lambda \delta_{\ell m} a_k \end{aligned} \quad (2)$$

2.

where $[ABC] \equiv ABC-CBA$ and $\lambda > 0$, in Kamefuchi-Takahashi's notation ⁽¹⁾.
By defining

$$p_k = 2^{-1/2} (a_k^+ + a_k), \quad q_k = i 2^{-1/2} (a_k^+ - a_k), \quad A_k = i \lambda^{-1} (a_k^+ a_k^+ - a_k a_k)$$

standard algebraic manipulations show that

$$a_m(\alpha; k) \equiv e^{i A_k \alpha} a_m e^{-i A_k \alpha} = a_m - \delta_{km} a_k + \delta_{km} (\cosh \alpha a_k + \sinh \alpha a_k^+) \quad (3)$$

Hence

$$p_m(\alpha; k) = p_m - \delta_{km} p_k + \delta_{km} e^{\alpha} p_k \quad (4)$$

$$q_m(\alpha; k) = q_m - \delta_{km} q_k + \delta_{km} e^{-\alpha} q_k$$

Consider now the "unitary" operator $U(\alpha) = \prod_{k=1}^{\infty} e^{i \alpha A_k}$. It follows formally from the above relations that

$$p_m(\alpha) \equiv U(\alpha) p_m U^{-1}(\alpha) = e^{\alpha} p_m \quad (5)$$

$$q_m(\alpha) \equiv U(\alpha) q_m U^{-1}(\alpha) = e^{-\alpha} q_m$$

$$a_m(\alpha) = \cosh \alpha a_m + \sinh \alpha a_m^+$$

$$a_m^+(\alpha) = \sinh \alpha a_m + \cosh \alpha a_m^+ \quad (6)$$

It is rather obvious that if a_m, a_n^+ are canonically realized, then the operators $a_m(\alpha), a_n^+(\alpha)$ are densely defined and closeable operators which satisfy (2), and the algebra generated by $a_m(\alpha), a_n^+(\alpha)$ ($m, n = 1, 2, \dots$) is irreducible as well.

Let us finally prove that

- 1) There is no (vacuum) state vector $|\alpha\rangle \neq 0$ such that $a_m(\alpha) |\alpha\rangle = 0$ ($m = 1, 2, \dots$). In fact, let

$$|\alpha\rangle = c_0 |0\rangle + c_1 |1\rangle + \dots + c_n |n\rangle + \dots$$

where $|0\rangle$ is the standard vacuum, and $|n\rangle$ is some normalized n -particle ket. Then $a_m(\alpha) |\alpha\rangle = 0$ evidently implies, by using (6)

$$c_1 a_m |1\rangle = 0 \quad (m=1, 2, \dots) \quad (7.1)$$

$$c_2 \cosh \alpha a_m |2\rangle + c_0 \sinh \alpha a_m^+ |0\rangle = 0 \quad (m=1, 2, \dots) \quad (7.2)$$

$$\begin{aligned} & \dots \dots \dots \quad (7) \\ & c_n \cosh \alpha a_m |n\rangle + c_{n-2} \sinh \alpha a_m^+ |n-2\rangle = 0 \quad (7.n) \\ & (n=2, 3, \dots; \quad m=1, 2, \dots) \end{aligned}$$

(7.1) implies $c_1 = 0$; if $c_0 = 0$, then (7.2) would imply $c_2 = 0$, and thus from (7.3) $c_3 = 0$, and so on. Suppose on the contrary that $c_0 \neq 0$; from (7.2) one gets $c_2 \neq 0$, and

$$a_k a_m |2\rangle = -\frac{c_0}{c_2} \tanh \alpha a_k a_m^+ |0\rangle$$

which shows that $\langle 2|2 \rangle = \infty (!)$. Therefore, either $\langle \alpha | \alpha \rangle = 0$ or $\langle \alpha | \alpha \rangle = \infty$, whence one can conclude that there does not exist a normalizable vacuum state.

- 2) The operator for the total number of particles $N(\alpha) = \sum_k^\infty N_k(\alpha)$, where

$$N_k(\alpha) = \lambda^{-1} [a_k^+(\alpha) a_k(\alpha) + a_k(\alpha) a_k^+(\alpha)] - 1 \quad (8)$$

is not defined anywhere in $\mathcal{E}\tilde{\mathcal{H}}$.

In fact, suppose that $\exists |\psi\rangle$ such that $\lim_{M \rightarrow \infty} \sum_1^M N_k(\alpha) |\psi\rangle$ exists in the strong sense. Let

$$|\psi\rangle = x_r |r\rangle + x_{r+1} |r+1\rangle + \dots + x_n |n\rangle + \dots, \quad (x_r \neq 0)$$

where $|n\rangle \in \mathcal{E}\tilde{\mathcal{H}}_n$.

As

$$N_k(\alpha) = \cosh 2\alpha N_k + 2 \sinh^2 \alpha + \lambda^{-1} \sinh 2\alpha [a_k a_{k+1} + a_{k+1}^+ a_k^+] \quad (9)$$

it follows from the above assumption that

$$\lim_{M \rightarrow \infty} \left[x_r \langle r | \cosh 2\alpha \sum_1^M N_k + 2 \sinh^2 \alpha M | r \rangle + \lambda^{-1} x_{r+2} \langle r | \sinh 2\alpha \sum_1^M a_k a_{k+1} | r+2 \rangle \right] \quad (10)$$

has to be finite ; hence

$$\lim_{M \rightarrow \infty} \frac{x_{r+2}}{x_r} \coth \alpha \langle r | \frac{\sum_1^M a_k a_k}{M} | r+2 \rangle = -\lambda \quad (11)$$

That this relation is impossible follows from the remark that if $\{z_k\}_1^\infty$ is a sequence of complex numbers such that :

$$i) \quad \sum |z_k|^2 < \infty$$

$$ii) \quad \lim_{M \rightarrow \infty} \frac{\sum_1^M z_k}{M} = \gamma$$

then $\gamma = 0$.

3) The algebras

$$\{a_m(\alpha), a_\ell^+(\alpha)\} \quad \text{and} \quad \{a_m(\beta), a_\ell^+(\beta)\}$$

are inequivalent provided that $\alpha \neq \beta$.

For if this were not true, then $\exists$ a unitary operator V such that

$$\begin{aligned} V p_k(\alpha) V^{-1} &= p_k(\beta) \\ V q_k(\alpha) V^{-1} &= q_k(\beta) \end{aligned} \quad (12)$$

But looking at (5), one sees that (12) is equivalent to

$$\begin{aligned} V p_k V^{-1} &= p_k(\beta - \alpha) \\ V q_k V^{-1} &= q_k(\beta - \alpha) \end{aligned} \quad (13)$$

This relation contradicts the facts already proved in 1) and 2),
q.e.d.

The author wishes to thank Profesor V. Glaser and Dr. H. Epstein
for their critical remarks.

R E F E R E N C E S

- 1) A. Galindo, "Paraestadística : Análisis y Representaciones Irreducibles de Paracampos", (to be published).
- 2) H.S. Green, Phys.Rev. 90, 270 (1953).
- 3) D.V. Volkov, JETP 9, 1107 (1959), 11, 375 (1960).
- 4) K.O. Friedrichs, "Mathematical Aspects of the Quantum Theory of Fields", Interscience, New York 1953.
L. Van Hove, Physica 18, 145 (1952).
L. Garding and A.S. Wightman, Proc.Nat.Acad.Sci. 40, 617 (1954).
- 5) R. Haag, Dan.Mat.Fys.Medd. 29, No. 12, (1955).
- 6) S. Kamefuchi and Y. Takahashi, Nucl.Phys. 36, 177 (1962).