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ON THE CONNECTION BETWEEN PRODUCTION MECHANISM
AND DECAY OF RESONANCES AT HIGH ENERGIES

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A B S T R A C T

The production of unstable particles in peripheral collisions at high energies is considered, with emphasis on the connection between the production mechanism and the angular correlations in the decay of the unstable particles. By means of the crossing relations for helicity amplitudes a direct relation (10) is established between the t channel helicity amplitudes and the spin space density matrix (relative to a suitable system of co-ordinate axes) of one of the reaction products. The extent to which the decay angular distributions determine the density matrix is fully discussed. As an example, various quasi-two-body channels of KN collisions are considered, especially $KN \rightarrow K^*N$. It is shown that, while the decay correlations of K^* at 3 GeV/c are consistent with dominantly vector meson exchange, they are also consistent with the exchange of systems with any $J \geq 1$ provided the parity is $(-1)^J$. Advantage is taken of this observation to generalize the one meson exchange model by means of Regge pole exchange in a way which gives a natural explanation of the very peripheral nature of the production process, while maintaining agreement with the decay data.

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1. INTRODUCTION

A striking feature of high energy collision processes with multiparticle final states is the presence, and often the dominance, of quasi-two-body reactions,

$$a + b \rightarrow c + d, \quad (1)$$

where one (or both) of the final systems is dynamically unstable. Examples are ρ production in $\pi^- p \rightarrow \pi^- \pi^0 p$ ¹⁾, ρ and N^* production in $\pi^+ p \rightarrow \pi^+ p \pi^+ \pi^-$ ²⁾, ω and N^* production in $\pi^+ p \rightarrow \pi^+ p \pi^+ \pi^- \pi^0$ ²⁾, K^* or N^* production in $K^+ p \rightarrow K^0 \pi^+ p$ ^{3),4),5)}, and K^* and N^* production in $K^+ p \rightarrow K^+ \pi^- \pi^+ p$ ^{5),6)}. Generally these reactions occur with relatively small momentum transfers, indicating that the long range part of the interaction plays a dominant role. Consequently there have been numerous attempts to interpret such peripheral collisions in terms of a model wherein a small number of light particles (pseudoscalar or vector mesons) are exchanged by the participants in the collision ^{7),8),9)}. To be somewhat more precise, these models approximate (1) by the scheme,

$$a + \bar{c} \rightarrow Z \rightarrow \bar{b} + d \quad (2)$$

in the crossed channel, where Z is the exchanged particle. The reaction amplitude therefore contains the propagator of Z , coupling constants for the two vertices $(a\bar{c}Z)$, $(\bar{b}dZ)$, and usually some empirical form factors to include off-energy shell effects.

As long as the differential cross-section for production is the only available data, it is frequently impossible to discriminate between models which employ Z 's that do not differ appreciably in mass. In particular, the spin and parity of the object Z often cannot be determined uniquely. If, however,

one or both of the production products have spin and are unstable ^{*)}, the decay angular correlation may yield more specific clues regarding the nature of the exchanged system Z . Thus, for example, comparison of the K^* decay angular distributions in the reactions $KN \rightarrow K^*N$ and $KN \rightarrow K^*N^*$ at $3\text{GeV}/c$ ⁵⁾ with the predictions of the one meson exchange model leads to the conclusion that the former is mediated almost entirely by a spin one, and the latter predominantly by a spin zero, meson. On the other hand, the decrease of the production cross-section with increasing momentum transfer is much more rapid than expected on the basis of one-meson exchange, especially when the exchanged particle is required to be one of the vector mesons. The peripheral model must therefore be modified in a fairly drastic fashion. The simplest alteration is the inclusion of form factors, but their momentum transfer dependence must be so violent that comparison of theory and experiment is almost reduced to empirical curve fitting. The necessity of very strong form factors in the peripheral model is not particular to the KN processes at $3\text{GeV}/c$, but is a general feature of reactions in this energy range ^{7),10)}.

The uncertainties and limitations associated with the one meson exchange model and its drastic form factors argue for an analysis which will clarify the connection between the production mechanism and the angular correlations in the decay of the unstable particles. In the $KN \rightarrow K^*N$ reaction, for instance, one would like to know whether the decay data really require the system Z to have spin and parity 1^- , as the study of some simple diagrams appears to indicate, or whether the data only lead to a weaker statement. We shall give quite general answers to this type of question in what follows - thus for the

) In our general discussion the unstable particles will be considered as resonances (ρ , K^ , N^* , etc.). But the analysis can be applied equally to particles (Λ , Σ , etc.) that are unstable because of weak interactions, provided the decay distributions are generalized to allow for the lack of parity conservation.

reaction $KN \rightarrow K^*N$ we shall show that the observed decay correlations at 3 GeV/c imply only that the important states in the crossed channel have $J \geq 1$ and parity $(-1)^J$.

The observation that vector meson exchange is just a special case of a large class of exchanges which can give agreement with the K^* decay data in $KN \rightarrow K^*N$ allows us to contemplate generalizations of the one meson exchange model. As a prototype of such a calculation we consider a Regge pole exchange model with the important trajectories having spin parity $J, (-1)^J$, and find that reasonable agreement with the production cross-section can easily be obtained.

The plan of the paper is as follows : in section 2 we show that the spin space density matrix of either of the reaction products can be expressed directly in terms of helicity amplitudes in the crossed (t) channel, provided the co-ordinate system is chosen judiciously. As a consequence, one can learn about the relative interaction strengths in the various helicity states from the decay distribution of the resonance. The explicit formulae relating the spin space density matrix and the angular distribution for two-body decay are presented in section 3, where the important examples of K^* and N^* decay are discussed. In sections 4 and 5 the reaction $KN \rightarrow K^*N$ is treated in detail, first by a partial wave analysis in the t channel where we establish the connections between helicity populations and the spin parity of the intermediate states, and then in terms of the Regge pole exchange model. The other channels ($KN \rightarrow KN^*$ and $KN \rightarrow K^*N^*$) of the KN system are discussed briefly in section 6.

A short account of some aspects of this work has already been published ¹¹⁾.

2. DIFFERENTIAL CROSS SECTION AND SPIN SPACE DENSITY MATRIX
IN TERMS OF t CHANNEL HELICITY AMPLITUDES ^{*)}

We shall adhere to the spirit of "peripheralism" by analyzing the collision in terms of amplitudes for the corresponding reaction in the t channel :

$$a + \bar{c} \rightarrow \bar{b} + d. \quad (3)$$

At the outset we shall not make any specific assumptions concerning the form of these amplitudes, and to that extent our treatment is not confined to the domain of large s and small t ^{**)}. On the other hand, a description in the t channel is simpler than a straightforward analysis in the s channel only for such values of s and t .

Trueman and Wick ¹²⁾ and Muzinich ¹³⁾ have recently shown that the crossing relations are especially elegant and almost visualizable in terms of helicity amplitudes, and we shall therefore use this representation throughout. Let $\langle \lambda_d \lambda_c | G(p_d p_c ; p_b p_a) | \lambda_b \lambda_a \rangle$ be the reaction amplitude for the s channel in an arbitrary Lorentz frame, where λ_i are the helicities and p_i the 4 momenta of the various particles ; the analogous quantity in the t channel is $\langle \lambda_d \bar{\lambda}_b | F(p_d \bar{p}_b ; \bar{p}_c p_a) | \bar{\lambda}_c \lambda_a \rangle$. In their respective centre of mass frames the operators will be written as $G(s,t)$ and $F(s,t)$. Isospin indices will be suppressed for simplicity. It should be noted that the helicity amplitudes defined here differ in normalization from those of Jacob and Wick ¹⁴⁾.

*) Those not interested in technical details may, after reading the definitions, go directly to (10), and from there to (17).

***) We use the conventional nomenclature : s is the square of the total centre of mass energy in (1), and t is the same quantity for (3).

The present amplitudes are the helicity analogues of the Lorentz invariant Feynman amplitude; the precise normalization can be inferred from the cross-section expressions (5) and (5') below.

The crossing relation derived by Trueman and Wick is ^{*)}

$$\begin{aligned} \langle \lambda_d \lambda_c | G(s, t) | \lambda_b \lambda_a \rangle &= \sum_{\lambda'_a \lambda'_b \lambda'_c \lambda'_d} \langle \lambda'_d \lambda'_b | F(s, t) | \lambda'_c \lambda'_a \rangle \\ &\times d_{\lambda'_a \lambda_a}^{s_a}(\psi_a) d_{\lambda'_b \lambda_b}^{s_b}(\psi_b) d_{\lambda'_c \lambda_c}^{s_c}(\psi_c) d_{\lambda'_d \lambda_d}^{s_d}(\psi_d) \end{aligned} \quad (4)$$

where s_i is the spin of particle i , and $d_{\lambda' \lambda}^s(\psi)$ are the elements of the $(2s + 1)$ - dimensional representation of the rotation group for a rotation through ψ about the normal to the scattering plane, (the y axis). The angles ψ_i appearing in (4) are defined in Fig. 1, and explicit expressions for them will be found in Trueman and Wick. As we shall soon see, we only require the geometric definitions of these angles, and not their numerical values.

When the incident beam is unpolarized and no spin measurements are made, the centre of mass differential cross-section for reaction (1) is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2s_a+1)(2s_b+1)} \frac{p'}{sp} \sum_{\lambda_a \dots \lambda_d} \left| \langle \lambda_d \lambda_c | G(s, t) | \lambda_b \lambda_a \rangle \right|^2 \quad (5)$$

*) Our definition of particle "2" has been chosen to eliminate the phase factor $(-1)^{\eta}$ from (TW-41).

where p and p' are the magnitudes of the initial and final 3 momenta in the centre of mass system. According to (4), however, the crossing relation is an orthogonal transformation, and we therefore have

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2s_a+1)(2s_b+1)} \frac{p'}{p} \sum_{\lambda_a \lambda_b \lambda_c \lambda_d} \left| \langle \lambda_d \lambda_c | F(s, t) | \lambda_b \lambda_a \rangle \right|^2. \quad (5')$$

Needless to say, the value of $F(s, t)$ which appears in (5') must be obtained by analytic continuation from the physical region in the t channel. Equation 5 merely shows that a computation of the crossing matrices is not necessary if one already knows the amplitudes in the t channel *).

The decay correlations of one of the product particles, say d , depend on the population of its magnetic sub-states. Consequently we wish to evaluate the spin space density matrix of d in its rest frame (designated by O_d), under the assumption that the initial state is unpolarized and λ_c , the helicity of c , is undetected. The Lorentz transformation needed to bring d to rest is parallel to $\underline{p}_d$ (and antiparallel to $\underline{p}_c$). Hence the helicities λ_c and λ_d are unaffected. But the helicity states of a and b will undergo a linear transformation [see (TW-25)]:

$$\begin{aligned} & \langle \lambda_d \lambda_c | G(\underline{p}_d=0; s, t) | \lambda_b \lambda_a \rangle \\ &= \sum_{\lambda'_b \lambda'_a} \langle \lambda_d \lambda_c | G(s, t) | \lambda'_b \lambda'_a \rangle d_{\lambda'_b \lambda_b}^{s_b}(\beta_b) d_{\lambda'_a \lambda_a}^{s_a}(\beta_a) \end{aligned} \quad (6)$$

*) One must still carry out the crossing calculation in isotopic spin space.

The values of the angles $\beta_{a,b}$ need not concern us because they will disappear from the final result. Inserting (4) into (6), and using the group property, we have

$$\begin{aligned} \langle \lambda_d \lambda_c | G(\underline{p}_d = 0; 1, t) | \lambda_b \lambda_a \rangle &= \sum_{\lambda'_a \lambda'_d \bar{\lambda}_b \bar{\lambda}_c} \langle \lambda'_d \bar{\lambda}_b | F(1, t) | \bar{\lambda}_c \lambda'_a \rangle \\ &\times d_{\lambda'_a \lambda_a}^{s_a}(\psi_a + \beta_a) d_{\bar{\lambda}_b \lambda_b}^{s_b}(\psi_b + \beta_b) d_{\bar{\lambda}_c \lambda_c}^{s_c}(\psi_c) d_{\lambda'_d \lambda_d}^{s_d}(\psi_d). \end{aligned} \quad (7)$$

To repeat, this expression is proportional to the probability amplitude for finding the spin projection λ_d along the direction $-\underline{p}_c$ in the rest frame O_d . The study of specific diagrams has, however, taught us that the description of the spin state takes on an especially simple form if one uses the direction of $\underline{p}_b$ as seen in O_d as the quantization axis. That such a simplification occurs in this co-ordinate system is actually well known in the theory of direct nuclear reactions. Consider the transition $|n_1\rangle \rightarrow |n_2\rangle$ between two nuclear states in the Born approximation. The matrix element is proportional to $\langle n_2 | e^{i\underline{q} \cdot \underline{r}} | n_1 \rangle$, where $\underline{q}$ is the momentum transferred by the projectile. Since the perturbation is cylindrically symmetric about $\underline{\hat{q}}$, the component of angular momentum $\underline{J} \cdot \underline{\hat{q}}$ does not change in the collision, and the excited system is found to be in a non-random state whose description is, needless to say, most concise when $\underline{\hat{q}}$ is chosen as the quantization axis. Now the direction of the negative of the momentum transfer $\underline{\Delta} = \underline{p}_a - \underline{p}_c$ is precisely $\underline{\hat{p}}_b$ in O_d , and we should therefore expect this to be the natural choice of quantization

axis ^{*}). Let $|m\rangle$ then be the spin states of d in this representation where $s_d \cdot p_b$ is diagonal. The amplitude required for the construction of the density matrix relative to this axis is then

$$\begin{aligned} & \langle m \lambda_c | G(\underline{p}_d = 0; \underline{s}, t) | \lambda_l \lambda_a \rangle \\ &= \sum_{\lambda_d} \langle m | \lambda_d \rangle \langle \lambda_d \lambda_c | G(\underline{p}_d = 0; \underline{s}, t) | \lambda_l \lambda_a \rangle \end{aligned} \quad (8)$$

The rotational matrix element $\langle m | \lambda_d \rangle$ corresponds to a rotation in O_d from the helicity direction $-\underline{p}_c$ to the quantization axis $\underline{p}_b$. From Fig. 1 the angle of rotation can be seen to be ψ_d . In fact, the rotation needed is exactly the inverse of the rotation involved in the crossing relation (4); thus $\langle m | \lambda_d \rangle = \langle \lambda_d | m \rangle = d_{\lambda_d m}^{s_d}(-\psi_d)$. This means that when (7) is inserted into (8) the rotation matrices for d are eliminated and we have

*) Although there is an increasing tendency for experimentalists to give decay correlations relative to this axis, there is some confusion about whether its "naturalness" has something to do with small momentum transfer and the Adair axis (incident direction in the centre of mass). While for small momentum transfers it is true that the Adair direction and the direction of $\underline{p}_b$ as seen in O_d are closely parallel, the emphasis should be placed on this axis being the momentum transfer axis in O_d , independent of the magnitude of momentum transfer.

$$\begin{aligned}
\langle m; \lambda_c | G(p_d=0; s, t) | \lambda_b \lambda_a \rangle &= \sum_{\lambda'_a \bar{\lambda}_b \bar{\lambda}_c} \langle m \bar{\lambda}_b | F(s, t) | \bar{\lambda}_c \lambda'_a \rangle \\
&\times d_{\lambda'_a \lambda_a}^{s_a}(\psi_a + \beta_a) d_{\bar{\lambda}_b \lambda_b}^{s_b}(\psi_b + \beta_b) d_{\bar{\lambda}_c \lambda_c}^{s_c}(\psi_c) .
\end{aligned}
\tag{9}$$

This formula shows that the helicity of d in the t channel centre of mass frame O_t equals the spin component along the direction of $\vec{p}_b$ in the rest frame O_d . Moreover, just as for the differential cross-section $\overline{\sigma}_d$ (5'), the rest frame density matrix of the reaction product d in terms of t channel amplitudes follows immediately from (9) :

$$\langle m | \rho_d | m' \rangle = N \sum_{\bar{\lambda}_b \bar{\lambda}_c \lambda_a} \langle m \bar{\lambda}_b | F(s, t) | \bar{\lambda}_c \lambda_a \rangle \langle m' \bar{\lambda}_b | F(s, t) | \bar{\lambda}_c \lambda_a \rangle^*
\tag{10}$$

where the normalization constant N is fixed by the condition $\text{tr } \rho_d = 1$. A determination of the density matrix ρ_d therefore yields rather direct information concerning the continuations of the helicity amplitudes in the t channel.

Reflection invariance imposes a certain condition on ρ_d , which, among other things, ensures that $\text{tr } s_d \rho_d$ is perpendicular to the production plane. This parity condition can be expressed as the following symmetry requirement (JW-44)

$$\langle -\lambda_d - \bar{\lambda}_b | F(s, t) | -\bar{\lambda}_c - \lambda_a \rangle = \eta_\pi (-1)^{\lambda_d - \bar{\lambda}_b - \bar{\lambda}_c + \lambda_a} \langle \lambda_d \bar{\lambda}_b | F(s, t) | \bar{\lambda}_c \lambda_a \rangle
\tag{11}$$

10.

where

$$\eta_x = \frac{\eta_d \eta_{\bar{e}}}{\eta_a \eta_{\bar{c}}} (-1)^{\Delta_d + \Delta_b - \Delta_c - \Delta_a} \quad (12)$$

η_i being the intrinsic parity of the indicated particle. Therefore

$$\langle m | \rho_d | m' \rangle = (-1)^{m-m'} \langle -m | \rho_d | -m' \rangle . \quad (13)$$

One can then readily verify that $\text{tr } s_x \rho_d$ and $\text{tr } s_z \rho_d$ vanish.

3. ANGULAR DISTRIBUTION OF DECAY

As we have already mentioned, it is the decay of the reaction product which makes ρ_d accessible to experimental study. Let us therefore evaluate the angular distribution of the decay $d \rightarrow \alpha + \beta$, where α and β have spin s_α and s_β , respectively ^{*)}. If $\underline{p}$ and $-\underline{p}$ are the momenta of α and β in O_d , the decay amplitudes are given by

$$A_m(\underline{p} \lambda_\alpha \lambda_\beta) = \langle \underline{p} \lambda_\alpha \lambda_\beta | U | s_d m \rangle, \quad (14)$$

where U is an operator which is invariant under rotations and reflections, and $\lambda_\alpha, \lambda_\beta$ are the helicities of the decay products. The angular dependence of (14) can be obtained directly from (JW-24) :

$$\begin{aligned} A_m(\underline{p} \lambda_\alpha \lambda_\beta) &= \langle \underline{p} \lambda_\alpha \lambda_\beta | s_d m \lambda_\alpha \lambda_\beta \rangle \langle s_d m \lambda_\alpha \lambda_\beta | U | s_d m \rangle \\ &= \left[(2s_d + 1) / 4\pi \right]^{-\frac{1}{2}} d_{m\Lambda}^{s_d}(\theta) e^{i\phi(m-\Lambda)} M_d(\lambda_\alpha \lambda_\beta) \end{aligned} \quad (15)$$

where $\Lambda = \lambda_\alpha - \lambda_\beta$, and (θ, ϕ) are the polar angles of $\underline{p}$ (see Fig. 2). The quantities $M_d(\lambda_\alpha \lambda_\beta)$ are not independent because of reflection invariance ; thus application of (JW-41) gives

^{*)} We consider explicitly only parity conserving two-body decays. For the three-body decay of the Ω the direction of the normal to the decay plane plays the role of the relative momentum in two-body decays. In the decay (via weak interactions) of spin $\frac{1}{2}$ baryons (Λ, Σ, Ξ) the only relevant quantity determined from the decay distribution is the transverse polarization, $\text{tr}(s_y \rho_d) = i \langle \frac{1}{2} | \rho_d | -\frac{1}{2} \rangle$. A discussion of decays in weak interactions, using helicity states, has been given by Jacob ¹⁶⁾.

$$M_d(\lambda_\alpha \lambda_\beta) = \eta_\alpha \eta_\beta i_d (-1)^{\Delta_d - \Delta_\alpha - \Delta_\beta} M_d(-\lambda_\alpha - \lambda_\beta) . \quad (16)$$

Hence if α and β are both spinless, or if $s_\alpha = \frac{1}{2}$ and $s_\beta = 0$, there is only one constant M_d characterizing the decay. With the help of (15) we can now write down the probability for decay into the direction (θ, ϕ) :

$$\begin{aligned} W(\theta, \phi) &= N' \sum_{\lambda_\alpha \lambda_\beta} \langle p_{\lambda_\alpha \lambda_\beta} | U \rho_d U^\dagger | p_{\lambda_\alpha \lambda_\beta} \rangle \\ &= N'' \sum_{\substack{\lambda_\alpha \lambda_\beta \\ m m'}} |M_d(\lambda_\alpha \lambda_\beta)|^2 e^{i(m-m')\phi} \\ &\quad \times d_{m\lambda}^{\Delta_d}(\theta) d_{m'\lambda}^{\Delta_d}(\theta) \langle m | \rho_d | m' \rangle . \end{aligned} \quad (17)$$

The invariance of the decay interaction under reflection, as expressed by (16), imposes the symmetry

$$W(\theta, \phi) = W(\pi - \theta, \pi + \phi) \quad (18a)$$

while the reflection invariance of the production mechanism, which requires ρ_d to satisfy (13), implies the further condition

$$W(\theta, \phi) = W(\theta, -\phi) \quad (18b)$$

This last symmetry, plus the reality of $W(\theta, \phi)$, means that in Eq. (17) the product $e^{i(m-m')\phi} \langle m | \rho_d | m' \rangle$ can be replaced by $\cos[(m-m')\phi] \times \text{Re} \langle m | \rho_d | m' \rangle$.

We shall discuss two examples of (17) : $s_d = 1, s_\alpha = s_\beta = 0$ (e.g., K^*), and $s_d = 3/2, s_\alpha = \frac{1}{2}, s_\beta = 0$ (e.g., N^*). We call these distributions W_{K^*} and W_{N^*} , although the formulae actually apply to any decay process with these angular momentum quantum numbers, whatever the intrinsic parities of the various particles. Let us first parametrize the density matrix ρ_{K^*} , with $s_d = 1$. The condition $\rho = \rho^\dagger$ and $\text{tr } \rho = 1$, together with the symmetry property (13), require ρ_{K^*} to have the form

$$\rho_{K^*} = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1,-1} \\ \rho_{10}^* & 1-2\rho_{11} & -\rho_{10}^* \\ \rho_{1,-1} & -\rho_{10} & \rho_{11} \end{pmatrix} \quad (19)$$

where ρ_{11} and $\rho_{1,-1}$ are real; furthermore, the eigenvalues of (19) must lie between zero and one. In terms of the parameters appearing in (19), the decay distribution is

$$W_{K^*}(\theta, \phi) = C \left\{ \rho_{11} \sin^2 \theta + (1-2\rho_{11}) \cos^2 \theta - \rho_{1,-1} \sin^2 \theta \cos 2\phi - \sqrt{2} \text{Re} \rho_{10} \sin 2\theta \cos \phi \right\}. \quad (20)$$

We note that this angular distribution determines almost completely the density matrix (19); only $\text{Im } \rho_{10}$ remains unknown.

The $s_d = 3/2$ density matrix is

$$\rho_{N^*} = \begin{pmatrix} \rho_{33} & \rho_{31} & \rho_{3,-1} & i\rho_{3,-3} \\ \rho_{31}^* & \frac{1}{2} - \rho_{33} & i\rho_{1,-1} & \rho_{3,-1}^* \\ \rho_{3,-1}^* & -i\rho_{1,-1} & \frac{1}{2} - \rho_{33} & -\rho_{31}^* \\ -i\rho_{3,-3} & \rho_{3,-1} & -\rho_{31} & \rho_{33} \end{pmatrix} \quad (21)$$

where ρ_{33} , $\rho_{3,-3}$ and $\rho_{1,-1}$ are all real. The corresponding decay distribution is

$$W(\theta, \phi) = C \left\{ \rho_{33} \sin^2 \theta + \left(\frac{1}{2} - \rho_{33} \right) \left(\frac{1}{3} + \cos^2 \theta \right) - \frac{2}{\sqrt{3}} \operatorname{Re} \rho_{3,-1} \sin^2 \theta \cos 2\phi - \frac{2}{\sqrt{3}} \operatorname{Re} \rho_{31} \sin 2\theta \cos \phi \right\}. \quad (22)$$

The parameters $\rho_{3,-3}$ and $\rho_{1,-1}$, which correspond to purely imaginary elements of the density matrix, do not enter the distribution [see below Eq. (18b)], but do contribute to other observables, such as spin correlations.

In passing we remark that the two decay distributions, (20) and (22), have the same general structure in the decay angles. This is a consequence of general theorems on the maximum complexity of angular distributions in parity-conserving interactions. The connection of these decay distributions with other angular correlations, such as the Treiman-Yang distribution, is discussed elsewhere¹⁷⁾.

4. THE REACTION $KN \rightarrow K^*N$ -
PARTIAL WAVE ANALYSIS IN THE t CHANNEL

Recent experiments at CERN ⁵⁾ on the process $K^+p \rightarrow K^*p$ at an incident laboratory momentum of 3.0 GeV/c indicate a decay distribution of the form of (20), with $\rho_{11} \simeq 0.50$, $\rho_{1,-1} \simeq 0.38$, $\text{Re } \rho_{10} \simeq 0$, where these values are averages over production angles. From (19) and (10) it can be seen that this implies that the dominant amplitudes in the t channel are those for which the K^* has helicity ± 1 . As mentioned in the introduction, one knows from the analysis in terms of one meson exchange diagrams that the absence of helicity zero for the K^* [or equivalently the absence of the $\cos^2 \theta$ term in (20)] requires vector meson exchange ($J = 1^-$), rather than pion exchange.

The question now arises as to whether or not this experimental information on the density matrix of K^* and the t channel helicity amplitudes implies uniquely that the dominant state in the t channel has $J = 1^-$. In order to answer this question it is necessary to make a partial-wave analysis of the helicity amplitudes for the t channel reaction ^{*)},

$$N + \bar{N} \rightarrow \bar{K} + K^* . \quad (23)$$

Instead of the somewhat cumbersome general notation of the preceding sections, we shall designate the twelve helicity amplitudes for (23) by $F_{\bar{\lambda}\lambda}^{\mu}(s,t)$, where μ is the helicity of K^* , and $\bar{\lambda}$, λ are the helicities of $\bar{N}$ and N , respectively. Because of (11), there are actually only six independent amplitudes for this process. The partial wave decomposition is ¹⁴⁾

$$F_{\bar{\lambda}\lambda}^{\mu}(s,t) = \sqrt{\frac{t}{q q'}} \sum_J (J + \frac{1}{2}) \langle \mu | T^J(t) | \bar{\lambda} \lambda \rangle d_{\bar{\lambda}-\lambda, \mu}^J(\theta_t) \quad (24)$$

) Note that our notation for the s channel reaction (1) and the t channel reaction (3) is $a = N$, $b = K$, $c = N$, $d = K^$.

where θ_t is the centre of mass scattering angle in the t channel, and q, q' are the magnitudes of the initial and final centre of mass 3 momenta, respectively. Equation (24) differs from (JW-31) only in our different choice of normalization for $F_{\lambda\lambda}^M$ and the specialization of the unessential azimuthal dependence to $\phi_t = 0$.

It is important to realize that, for a given angular momentum J , the T matrix element in (24) usually has contributions from states of both even and odd parity. In order to give some insight into this aspect we first consider briefly an analysis in terms of familiar non-relativistic states of definite spin, orbital and total angular momentum, and parity. For $J = 1$ the possible spin orbitals for the initial and final states of (23), together with the non-vanishing amplitudes, are displayed below in a self-explanatory scheme (the post-superscript being the parity) :

$\bar{N}N$	$^1P_1^+$	$^3P_1^+$	$^3D_1^-$	$^3S_1^-$	
$\bar{K}K^*$					
$^3S_1^+$	$A_1^{(1)}$	$A_2^{(1)}$	-	-	(25)
$^3D_1^+$	$A_3^{(1)}$	$A_4^{(1)}$	-	-	
$^3P_1^-$	-	-	$B_1^{(1)}$	$B_2^{(1)}$	

The four amplitudes A_i connect states of spin one and even parity, while the two amplitudes B_i connect states with spin one and odd parity. For arbitrary J analogous considerations show that for each $J > 0$ there are always four amplitudes, $A_1^{(J)} \dots A_4^{(J)}$, for states with parity $(-1)^{J+1}$, and two amplitudes $B_1^{(J)}$ and $B_2^{(J)}$, for states with parity $(-1)^J$. For $J = 0$ only one state (with odd parity) occurs, and only one amplitude.

The determination of the contributions of different parity states to the helicity amplitude $\langle \mu | T^J(t) | \bar{\lambda} \lambda \rangle$ can be accomplished in a similar way. One must first examine the behaviour under reflection of the $\bar{N}N$ state $|JM; \bar{\lambda} \lambda \rangle$ and the $\bar{K}K^*$ state $|JM; \mu \rangle$ that enter into the construction of $\langle \mu | T^J(t) | \bar{\lambda} \lambda \rangle$. From (JW-41) the effect of the parity operator is found to be

$$\begin{aligned} P |JM; \bar{\lambda} \lambda \rangle &= (-1)^J |JM; -\bar{\lambda} -\lambda \rangle \\ P |JM; \mu \rangle &= (-1)^{J+1} |JM; -\mu \rangle. \end{aligned} \quad (26)$$

By taking linear combinations of helicity states we can construct states of definite parity which are also characterized by the value of $n = |\bar{\lambda} - \lambda| = 0, 1$ for $\bar{N}N$ and $k = |\mu| = 0, 1$ for $\bar{K}K^*$. Displaying only the parity and n or k for compactness, we have the states with parity $(-1)^J$:

$$\bar{N}N: \quad |(-1)^J; n \rangle = \frac{1}{\sqrt{2}} \left[|JM; \bar{\lambda} \lambda \rangle + |JM; -\bar{\lambda} -\lambda \rangle \right] \quad (27)^*$$

$$\bar{K}K^*: \quad |(-1)^J; k \rangle = \frac{1}{\sqrt{2}} \left[|JM; 1 \rangle - |JM; -1 \rangle \right]. \quad (28)$$

Note that there are two $\bar{N}N$ states ($n = 0, 1$), but only one $\bar{K}K^*$ state ($k = 1$). For the opposite parity, $(-1)^{J+1}$, the states are

$$\bar{N}N: \quad |(-1)^{J+1}; n \rangle = \frac{1}{\sqrt{2}} \left[|JM; \bar{\lambda} \lambda \rangle - |JM; -\bar{\lambda} -\lambda \rangle \right] \quad (29)^*$$

*) In (27) and (29) there is a duplication of states if $\bar{\lambda}$ and λ take on all possible values. Our convention is to take $(\bar{\lambda}, \lambda) = (+ +)$ for $n = 0$ and $(\bar{\lambda}, \lambda) = (+ -)$ for $n = 1$.

$$\begin{aligned} \bar{K}K^*: \quad |(-1)^{J+1}; 1\rangle &= \frac{1}{\sqrt{2}} \left[|JM; 1\rangle + |JM; -1\rangle \right] \\ |(-1)^{J+1}; 0\rangle &= |JM; 0\rangle. \end{aligned} \quad (30)$$

One can now verify that for parity $(-1)^J$ there are two independent amplitudes for each $J > 0$ [coming from matrix elements of the transition operator between the one $\bar{K}K^*$ state (28) and the two $\bar{N}N$ states (27)], while for parity $(-1)^{J+1}$ there are four such amplitudes for each J .

Before exhibiting the different parity contributions to the reaction amplitude $\langle \mu | T^J | \bar{\lambda} \lambda \rangle$, we note from (28) and (30) that t channel states of angular momentum J and parity $(-1)^J$ only populate K^* states with helicities ± 1 , whereas the states with parity $(-1)^{J+1}$ populate all helicity states of K^* . This means that we can give a general answer to the question posed at the beginning of this section : does the experimental inference at 3 GeV/c, that the important t channel amplitudes have K^* helicity ± 1 , imply uniquely that vector meson exchange ($J = 1^-$) dominates ? The answer is no. All one can conclude from the absence of helicity zero for K^* is that the dominant exchanges are systems with angular momentum J and parity $(-1)^J$, henceforth called natural parity *).

A simple calculation, using the states (27) - (30), leads to expressions for the amplitudes $\langle \mu | T^J | \bar{\lambda} \lambda \rangle$ in terms of contributions from the two possible parities :

*) A rather obvious remark may be in order here. Our use of such phrases as "exchange of systems with angular momentum..." should not be taken to mean that we are speaking only about one particle exchange diagrams. The complexity and detailed structure of the intermediate states in the t channel are irrelevant for the preceding discussion ; only the total angular momentum and parity of the intermediate state has a bearing on the argument.

$$\begin{aligned}
\langle 1|T^J|++\rangle &= u_J + U_J \\
\langle 1|T^J|--\rangle &= u_J - U_J \\
\langle 1|T^J|+-\rangle &= v_J + V_J \\
\langle 1|T^J|-+\rangle &= v_J - V_J \\
\langle 0|T^J|++\rangle &= W_J \\
\langle 0|T^J|+-\rangle &= X_J
\end{aligned} \tag{31}$$

where u_J, v_J are the two amplitudes for the natural parity states and U_J, V_J, W_J, X_J are the four amplitudes for the unnatural parity combinations. In terms of the states (27) and (28) u_J and v_J are given by $u_J = \frac{1}{2} \langle (-1)^J ; 1|T^J|(-1)^J ; 0 \rangle$ and $v_J = \frac{1}{2} \langle (-1)^J ; 1|T^J|(-1)^J ; 1 \rangle$; similar definitions hold for the other four amplitudes. Helicity combinations other than those given in (31) can be obtained from them by symmetry [see Eq. (11) or (JW-43)].

If only the natural parity states contribute to the t channel amplitudes, there are only two independent amplitudes for each J and only three different, non-vanishing, helicity amplitudes $F_{\lambda\lambda}^{\mu}$. Explicitly we have

$$F_{++}^1 = F_{--}^1 = \sqrt{\frac{x}{q q'}} \sum_{J=1}^{\infty} \frac{(J+\frac{1}{2})}{\sqrt{J(J+1)}} u_J(x) \sqrt{1-x_*^2} P_J'(x_*) \tag{32}$$

$$F_{+-}^1 = \sqrt{\frac{x}{q q'}} \sum_{J=1}^{\infty} (J+\frac{1}{2}) v_J(x) \left[P_J(x_*) - \frac{(x_*-1)}{J(J+1)} P_J'(x_*) \right] \tag{33}$$

$$F_{-+}^1 = -\sqrt{\frac{x}{q q'}} \sum_{J=1}^{\infty} (J+\frac{1}{2}) v_J(x) \left[P_J(x_*) - \frac{(x_*+1)}{J(J+1)} P_J'(x_*) \right] \tag{34}$$

where we have exhibited the rotation matrices in terms of Legendre polynomials (and their derivatives) of argument $x_t = \cos \theta_t$. The two amplitudes F_{++}^0 and F_{--}^0 which contribute to zero helicity for K^* vanish identically, as expected.

While the remarks made in this section have been phrased in terms of the reaction $KN \rightarrow K^*N$ it is evident that the formalism applies equally well to any reaction involving particles of the same intrinsic spins (for example, $\pi N \rightarrow \rho N$, $\bar{K}N \rightarrow \phi \Lambda$, $\pi N \rightarrow K^* \Sigma$). In fact, the statements concerning the implications of the absence of helicity zero of K^* for the type of exchanged systems hold in reactions with more particles in the final state, provided we consider the crossed channel as $a + \bar{c} + \text{others} \rightarrow \bar{b} + d$.

5. THE REACTION $KN \rightarrow K^*N$ - MOMENTUM TRANSFER DISTRIBUTION

Data are available in the process $KN \rightarrow K^*N$ for the angular distribution of production of K^* , as well as for the decay correlations. In contrast to the analysis of the decay distributions, more specific assumptions must be made about the production mechanism before a meaningful confrontation can occur between theory and experiment. The one meson exchange models ⁷⁾⁻⁹⁾ are one such attempt. The striking qualitative feature of the data at 3 GeV/c is that the production cross-section falls off with increasing momentum transfer much faster than can be understood on the basis of a one meson exchange model [remember that within the framework of this model the exchanged particle must be a vector meson (ω, ρ, ϕ)] The effective propagator implied by the data corresponds to a mass near the threshold of the two-pion continuum. It is obvious that no finite set of diagrams can produce such a behaviour, nor is it reasonable to postulate the existence of as yet undiscovered resonances or bound states of the two-pion system.

A qualitatively different type of mechanism must therefore be invoked. One simple possibility which has suggested itself to us is that of Regge pole exchange. That Regge pole amplitudes in the t channel decrease rapidly for increasing momentum transfer at high energies is well known from elastic scattering, and some application to inelastic processes has been made ¹⁸⁾. One immediately wonders, however, whether the energies considered here (~ 3 GeV) are sufficiently large to allow the dominance of one, or at most a few, Regge poles. A necessary, but not sufficient, criterion is $\cos^2 \theta_t \gg 1$ in the physical domain. Figure 3 shows $-\cos \theta_t$ as a function of centre of mass production angle θ_s for the reaction $K^+p \rightarrow K^*p$ at incident momenta of 3 GeV/c and 6 GeV/c. At 3 GeV/c $-\cos \theta_t$ is larger than 2 for essentially the entire angular interval of interest to us. The familiar asymptotic form is actually surprisingly accurate for such values of $\cos \theta_t$ (see below). A much more telling point is that the whole notion of Regge pole exchange is probably very naive, a criticism in which

we fully concur. The main virtues of the Regge pole model are that, with a reasonable choice of parameters, it can be made to agree with the rather puzzling data on a variety of inelastic channels in KN collisions, and that it enables one to make a number of definite predictions which one may hope to test with future experiments at higher energies. The model is also compatible with the data at lower energies, where there is little difference between the Regge pole and the one meson exchange models ¹⁹⁾.

The most obvious Regge trajectories which can contribute are those containing the $Y = 0$ vector mesons, ω , ρ , and ϕ . The so-called vacuum trajectory (P) will also enter, though its $J = 0$ member does not, since the $J = 0$ $K^*\bar{K}$ state has odd parity. All of these trajectories contain only states of parity $(-1)^J$, and they will therefore produce a K^* density matrix (19) of the form :

$$\rho_{K^*} = \begin{pmatrix} \frac{1}{2} & 0 & \rho_{1,-1} \\ 0 & 0 & 0 \\ \rho_{1,-1} & 0 & \frac{1}{2} \end{pmatrix} \quad (35)$$

essentially consistent with present observations at 3 GeV/c. A priori, the π and η trajectories should also contribute, causing departures from (35). But in the naive "theory", these lie well below those just mentioned, and so provide the "explanation" of the smallness of the elements of ρ_{K^*} with m or m' equal to zero. In fact, this model would predict that the difference between (35) and the experimentally observed ρ_{K^*} [as inferred from W_{K^*} (20)] should decrease with increasing energy. If the various trajectories have roughly equal slopes, the deviations from (35) should vanish with energy roughly as $(1/s) \propto_1(0) - \propto_2(0)$, where $\propto_1(0)$ is representative of the natural parity trajectories and $\propto_2(0)$ of the π and η trajectories [see Eq. (40) below].

No real test of this prediction ^{*)} can be made as yet. But the data at 1.45 GeV/c ³⁾ and 1.95 GeV/c ⁴⁾, which are consistent with appreciable pseudoscalar, as well as vector, meson exchange, and the data at 3 GeV/c ⁵⁾, which indicate very little pseudoscalar meson contribution, are qualitatively consistent with this effect.

Let us now examine the conjecture of Regge pole exchange in more detail. From the preceding section we know that keeping only the high lying Regge trajectories, all with natural parity, reduces the general t channel problem to three amplitudes, (32), (33), and (34). The familiar incantation ²⁰⁾, employing the Watson-Sommerfeld formula, leads to the replacement of each angular momentum sum in these formulae by a sum of Regge pole terms. The standard Regge pole form ^{20), 21)} is modified only by the presence in (32) - (34) of the linear combinations of Legendre polynomials and their derivatives, rather than a single Legendre polynomial. It is straightforward to show that the sum of Regge pole terms can be written as ^{**)}

$$F^1 \simeq - \sum_n \frac{\sqrt{\pi} \Gamma(\alpha_n + \frac{3}{2})}{\Gamma(\alpha_n + 1)} S_n \frac{\beta_n(*)}{\sin \pi \alpha_n} \xi(\alpha_n, x_*) (2x_*)^{\alpha_n} \quad (36)$$

^{*)} It is somewhat unfair to claim this prediction, or at least its qualitative aspects, for the Regge pole model. The exchange of a particle of spin J will give rise asymptotically to an amplitude with a factor s^J , as is well known ²⁰⁾. Thus, in the one meson exchange model, vector meson exchange will become relatively more important as the energy increases.

^{**)} In writing (36) we have ostensibly followed the recipe of Read, Orear and Bethe ²²⁾ of using P_α (and its derivatives) for positive α and the Mandelstam form, $\frac{-\tan \pi \alpha}{\pi} Q_{-\alpha-1}$, for negative α , but we have kept only the leading term in the expansions. Inspection of the relevant formulae shows that the error introduced thereby is roughly 10 percent for $|\cos \theta_t| = 2$ and less than 5 percent for $|\cos \theta_t| = 3$ for all values of α of interest. Even for $\alpha = \frac{1}{2}$, where the usual expansion appears to diverge, a different expansion (from the complete elliptic integral of the second kind) gives corrections of only a few percent for $|\cos \theta_t| \geq 2$.

where $\alpha_n(t)$, $\beta_n(t)$, S_n are the trajectory, residue, and signature factor, respectively, for the n th Regge pole $\int S_n = \frac{1}{2} (1 \pm e^{-i\pi\alpha_n})$. There are two residues for each trajectory, one corresponding to the $u_J(t)$ in (32) and the other to the $v_J(t)$ in (33) and (34). The factor $\xi(\alpha_n, x_t)$ contains the slight modifications mentioned above :

$$\begin{aligned} F_{\pm\pm}^1 : \quad \xi &= -i \sqrt{\frac{\alpha_n}{\alpha_n+1}} \\ F_{+-}^1 : \quad \xi &= \frac{1}{\alpha_n+1} \left(\alpha_n + \frac{1}{x_t} \right) \\ F_{-+}^1 : \quad \xi &= \frac{-1}{\alpha_n+1} \left(\alpha_n - \frac{1}{x_t} \right) \end{aligned} \quad (37)$$

For very large x_t the ξ 's become independent of x_t and can be combined with the other factors in (36). But at the energies considered here, where $|x_t| \sim 2-3$, it would be necessary to keep the forms (37) if one were to attempt a detailed comparison with experiment.

The residues $\beta_n(t)$ contain all of the information about the dynamics of the specific reaction involved. It has been shown that :

- (a) the residues can be factored ²³⁾,
- (b) they vanish appropriately to avoid the appearance of "ghosts" ^{21), 24)},
- (c) a threshold behaviour can be extracted from them, leaving slowly varying functions of t .

The factorization property means that an effective coupling constant for a generalized vertex of a Regge pole and two external particles can be defined and, in principle, evaluated by comparison of different reactions in which some of the particles are the same. This possibility is perhaps afforded by the data on the

various channels of KN reactions, but such an analysis is premature. The exclusion of "ghosts" at $\alpha = 0$, or the stronger requirement that a certain angular momentum and parity state be absent, implies a specific dependence of $\beta_n(t)$ on α . The residue of the vacuum trajectory corresponding to v_J in (33) and (34), for example, must not only vanish as α in order to eliminate a "ghost" at $\alpha = 0$ ²¹⁾, but must have an additional factor $\sqrt{\alpha}$ in order to eliminate the $J = 0^+$ state which is forbidden at the KK^* vertex.

The explicit exhibition of the threshold behaviour of $\beta_n(t)$ is important, even for qualitative considerations. In analogy with the centrifugal barrier penetrability for states of definite angular momentum, one writes

$$\beta_n(t) = \left(\frac{g g'}{M^2} \right)^{\alpha_n} b_n(t) \quad (38)$$

where $b_n(t)$ is assumed to be slowly varying in t . The mass M plays the role of an inverse radius of interaction (often denoted by $\sqrt{s_0}$). When (38) is inserted into (36) we see that there will be a factor $(2g g' x_t / M^2)$, raised to the power α_n . In terms of the Mandelstam variables this factor is

$$\frac{2g g' x_t}{M^2} = \frac{1}{2M^2} \left[s - u + \frac{(m_a^2 - m_c^2)(m_b^2 - m_d^2)}{t} \right].$$

For the reaction $KN \rightarrow K^*N$, the last term vanishes. With the threshold behaviour extracted from the residues, (36) can be written

$$F^{-1} \simeq \sum_n \gamma_n(t) \xi \left(\frac{s-u}{2M^2} \right)^{\alpha_n(t)} \quad (39)$$

where $\gamma_n(t)$ is a supposedly slowly varying function of t that incorporates $b_n(t)$ and all the other factors in (36).

The amplitude (39) has the well-known exponential dependence on momentum transfer. With a linear approximation to $\alpha_n(t)$ near $t = 0$, one has

$$F^1 \simeq \sum_n \left[\gamma_n(t) \xi \left(\frac{s-u}{2M^2} \right)^{\alpha_n(0)} \right] \exp \left(t \alpha'_n(0) \ln \left(\frac{s-u}{2M^2} \right) \right). \quad (40)$$

For the $KN \rightarrow K^*N$ reaction at 3 GeV/c, $(s-u) \simeq 10 \text{ GeV}^2$. If M is taken as the pion mass, the logarithm is approximately 5.5, while with the nucleon mass it is 2.3. The slopes $\alpha'_n(0)$ are not known with any accuracy, but $\alpha'_n(0) \simeq 0.4 - 0.7 \text{ GeV}^{-2}$ is a popular estimate. Thus the momentum transfer dependence in (40) is roughly $e^{\lambda t}$, where λ is expected to be in the range, 1 - 4, at 3 GeV/c. Fits to the data, using the one meson exchange model with ad hoc exponential form factors in t , yield $\lambda \sim 2^9$). The relative values of $\rho_{1,1}$ and $\rho_{1,-1}$ inferred from the decay data can be fitted by appropriate choice of the various residues $\beta_n(t)$ *).

Without arguing about details we see that the Regge pole model involving the highest trajectories (P, ρ, ω, ϕ) provides a reasonable explanation of the very rapid decrease of the production cross-section with increasing momentum transfer, while at the same time preserving the density matrix structure (35). It is undoubtedly true that the simple Regge pole exchange is a gross oversimplification, especially at these energies. Nevertheless, it can be viewed as an attempt, however crude, to include a large class of states in the crossed channel, states which are plausibly important and which happen to have the natural spin-parity combination required by the decay data. If the model has even approximate validity, certain predictions can be made for energies above 3 GeV :

*) This is in complete correspondence with the one meson exchange model where, for vector meson exchange, the relative amounts of Dirac and Pauli coupling of the vector meson to the nucleons can be adjusted to fit the decay data ⁹⁾.

- (1) the density matrix (35) will approximate more closely the observed density matrix ;
- (2) the isospin properties of the reaction will increasingly be those characteristic of a $T = 0$ exchange (as the vacuum trajectory dominates) ;
- (3) the production cross-section will continue to show an exponential decrease with increasing momentum transfer (with the famous logarithmic shrinkage, if one takes the model seriously in all its aspects).

As mentioned in the introduction, there are other reactions that have the same peculiarity of being overly peripheral in comparison with the mass of the possible exchanged particles in the one-meson approximation. Obviously the Regge pole exchange model can remove this difficulty. The problem of which trajectories dominate and consequently what decay correlations are expected must be considered for each reaction separately since specific selection rules may operate. In the reaction $\pi N \rightarrow \rho N$, for example, G conjugation invariance requires that the exchanged objects have $G = -1$, and so limits the Regge trajectories to those for the ω (and ϕ^*) and π . The question of whether, and at what energy, the ω trajectory dominates over the π trajectory depends on the relative magnitude of the residues. It is fortuitous that for the present reaction, $KN \rightarrow K^*N$, a large number of the right type of trajectories can contribute ; the relative sizes of specific residues are less important.

In closing this section we remark that alternative models which can explain these data undoubtedly can be invented. It is possible that the narrowness of the forward peak of production cross-sections can be correlated with diffraction scattering via the optical model ²⁵⁾, rather than via Regge poles. The specific structure (35) of the K^* density matrix may be more difficult to obtain ; it is a virtue of the Regge pole exchange model that this follows without additional assumptions.

*) The absence of the $\rho\pi$ decay mode of the ϕ indicates a small effective coupling for the ϕ in this reaction.

6. THE REACTIONS $KN \rightarrow KN^*$ AND $KN \rightarrow K^*N^*$

The three-body final states ($K^+p \rightarrow K^0 \pi^+ p$) are divided roughly equally (at 3 GeV/c) between K^*p and K^0N^* . In addition there are many four-body final states ($K^+ \pi^- \pi^+ p$), somewhat over half of which can be interpreted as a K^* and an accompanying N^* . Thus, besides the K^*N reaction, already discussed, there are two other quasi-two-body processes, KN^* and K^*N^* . There also exist data on KN^* ⁴⁾ and K^*N^* ⁶⁾ at 1.95 GeV/c. The production angular distributions for KN^* and K^*N^* at 3 GeV/c are qualitatively similar to that of K^*N , being confined to momentum transfers appreciably less than 1 GeV/c ⁵⁾ and indicating a peripheral mechanism. The same applies to the data at 1.95 GeV/c.

Data on the decay of the resonances give information on the density matrix of each resonance and the production mechanism through the angular distributions (20) and (22). We first consider the decay correlations for K^* produced in $KN \rightarrow K^*N^*$. At 1.95 GeV/c the data ⁶⁾ are compatible with almost pure $\cos^2 \theta$ in (20), corresponding to more than 80 percent pion exchange in the one meson exchange model. At 3 GeV/c there is a mixture of $\sin^2 \theta$ and $\cos^2 \theta$, but the dominant term is $\cos^2 \theta$, again indicating mainly pion exchange. The differential production cross-section expected from pion exchange increases with increasing momentum transfer, in spite of the pion propagator, because of the large spins of the products and the resulting numerator factors in the cross-section. A rapidly decreasing form factor is again necessary to obtain a fit to the data ^{6),9)}.

In terms of the crossed channel ($K\bar{K}^* \rightarrow \bar{N}N^*$) we see that the production of N^* necessitates $T = 1$ for the intermediate states. In the one meson exchange model only the π and ρ are eligible. The decay data thus indicate that ρ exchange is relatively unimportant at energies of a few GeV. Now if one were to invoke a Regge pole exchange mechanism in order to explain the very peripheral nature of the production, one would be embarrassed by the absence of the ρ

trajectory. We have no explanation of our own to offer for this absence. We can only appeal to the so-called Low quantum number ²⁶⁾ which would make the $K \rho K^*$ vertex much weaker than the $K \pi K^*$ vertex.

In the reaction $KN \rightarrow KN^*$ the t channel intermediate states must also have $T = 1$. But pion exchange is forbidden by parity conservation at the $K\bar{K}$ vertex; only ρ exchange occurs in the one meson exchange model. Stodolsky and Sakurai ²⁷⁾ and Jouvett ²⁸⁾ have discussed the ρ exchange model for this and related reactions using the analogy of magnetic dipole photoproduction for the ρNN^* vertex. One result is that the decay distribution of the isobar should be of the form ²⁷⁾, $(1 + 3 \cos^2 \theta')$ where θ' is measured from the normal to the production plane.

Because of the interest in decay distributions relative to the normal we consider briefly the transformation of (22) from the axes shown in Fig. 2 to axes with the normal $\underline{n}$ as the z axis. The new spherical angles will be denoted (θ', ϕ') . The decay distribution (22) becomes

$$W_{N^*}(\theta', \phi') = A + \sin^2 \theta' [B + C \sin(2\phi' + \delta)] \quad (41)$$

where the parameters A, B, C are functions of the coefficients in (22) and the phase angle δ depends on these coefficients as well as the choice of x axis. The density matrix ρ'_{N^*} , referred to the normal, can be obtained from (21) by a suitable rotation. Its general structure is

$$\rho'_{N^*} = \begin{pmatrix} \rho'_{33} & 0 & \rho'_{3,-1} & 0 \\ 0 & \rho'_{11} & 0 & \rho'_{1,-3} \\ \rho'^{*}_{3,-1} & 0 & \rho'_{-1,-1} & 0 \\ 0 & \rho'^{*}_{1,-3} & 0 & \rho'_{-3,-3} \end{pmatrix} \quad (42)$$

where the elements are linear combinations of the elements of (21). The absence of elements with $(m - m')$ odd is a reflection of the fact that the orbital angular momenta in the initial and final states each have only components along the normal with $(\ell - m_\ell)$ even. The magnetic dipole absorption model for exchange corresponds to having only the two central elements present in (42).

The N^* decay data at $3 \text{ GeV}/c$ ⁵⁾ are in rough, if not perfect, agreement with the Stodolsky-Sakurai model, but just as for K^*N , the production angular distribution indicates that the mass of the exchanged vector meson is much less than the ρ mass. Apart from the obvious remark that the behaviour of the production differential cross-section can be explained by a Regge-like mechanism, we can add little since the decay distribution in this reaction is apparently completely dependent on the details of the couplings, even though only one trajectory is present^{*)}.

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*) The reason for the absence of an elegant result like the statement concerning the exchange of states with natural parity in the $KN \rightarrow K^*N$ reaction can be found in the fact that the relevant spins are half integral rather than integral, and the connections (28) and (30) between helicities and parity are absent.

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FIGURE CAPTIONS

- Figure 1 - Velocity space diagram which serves to define the angles ψ appearing in (4). The lines emanating from a point P represent the velocities of the various particles as seen by an observer using a frame corresponding to the point P. Thus O_s represents the s channel centre of mass, and an observer at O_s observes particles b and d moving in the directions $O_s O_b$ and $O_s O_d$, respectively, the angle between these being the production angle θ_s . Similarly, an observer in the rest frame O_d sees the centre of mass and particle b moving in the directions $O_d O_s$ and $O_d O_b$, respectively. The actual values of the angles cannot be obtained from the diagram by trigonometry, however, except in the non-relativistic limit. For further details, see Wick¹⁵⁾.
- Figure 2 - Definition of the decay angles (θ, ϕ) in the rest frame O_d of particle d. $\vec{p}$ is the momentum of one of the decay products of d, $\hat{n}$ is perpendicular to the production plane, and $\vec{p}_b$ is the momentum of b as seen in O_d .
- Figure 3 - Cosine of centre of mass angle θ_t in the t channel for the s channel reaction $KN \rightarrow K^*N$ as a function of the square of the momentum transfer $(-t)$ for incident K meson laboratory momenta of 3 GeV/c and 6 GeV/c.

FIG.1

FIG. 2

FIG.3