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CONNECTION BETWEEN PRODUCTION MECHANISM AND DECAY

OF UNSTABLE PARTICLES : $KN \rightarrow K^*N$

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We wish to report some new results concerning the analysis of collisions in which unstable particles are produced. One important result is a direct relation between the spin space density matrix of the unstable particle, which can be deduced from its decay distribution, and the helicity amplitudes in the t channel. In the language of the peripheral model, this relation allows one to draw immediate inferences concerning the exchanged systems, and permits the generalization of the one meson exchange model. As an example of this type of analysis we consider the reaction $KN \rightarrow K^* N$ at moderately high energies, where we find that a Regge pole exchange model is compatible with the data on both the decay correlations and the production angular distributions.

The recent CERN experiment ¹⁾ (K^+ incident at a momentum of 3 GeV/c in a hydrogen bubble chamber, giving the 3 body final state $K^0 \pi^+ p$) yields information on the decay angular distribution of K^* and also on the differential cross-section for production. For our purposes it is convenient to summarize the data as follows :

1. Decay distribution of K^* . In the rest frame O_{K^*} of K^* , choose the z axis along the direction of the incident K meson as seen in O_{K^*} , and let m be the eigenvalue of the z component of the spin of the K^* referred to this axis. The spin space density matrix of K^* , $\rho_{mm'}$, can be almost completely determined from the decay distribution,

$$W(\theta, \phi) = N \left\{ \rho_{11} \sin^2 \theta + \rho_{00} \cos^2 \theta - \rho_{1,-1} \sin^2 \theta \cos 2\phi - \sqrt{2} \operatorname{Re} \rho_{10} \sin 2\theta \cos \phi \right\} \quad (1)$$

where N is a normalization constant and the angles (θ, ϕ) are defined by the z axis and the choice of the x - z plane as the plane of production ²⁾.

The data presently available at 3 GeV/c give $\rho_{11} \simeq 0.5$, $\rho_{00} \simeq 0$, $\rho_{1,-1} \simeq 0.4$, with $\operatorname{Re} \rho_{10}$ not yet determined. On the basis of these data

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we make the simplifying assumption that the $m = 0$ states of K^* are not populated in the production process. If one wishes to interpret the absence of $m = 0$ states within the framework of the one meson exchange model (OMEM), the quantum numbers of the exchanged object must be $J = 1^-$ (ω , ρ , or ϕ exchange) ¹⁾.

2. Angular dependence of production cross-section. The striking qualitative feature at 3 GeV/c is that the production cross-section falls off with increasing momentum transfer ($\sqrt{-t}$) much faster than can be understood on the basis of the OMEM, since, as we have just mentioned, this model requires the exchanged particle to be a vector meson. In fact, the effective propagator implied by the data corresponds to a mass near the threshold of the two-pion continuum. Put another way, if one insists on invoking ω - or ρ - or ϕ - exchange, one must introduce a form factor with a very strong t dependence into the formula for the cross-section. For example, the form factor $e^{-\lambda t}$ gives a fit if $\lambda \simeq 2 \text{ GeV}^{-2}$ ³⁾.

The necessity of very strong form factors in the OMEM puts its validity in doubt, in spite of the agreement with the decay angular distribution if vector meson exchange is assumed. In order to clarify the implications of the decay data for the production mechanism and to see to what extent $J = 1^-$ exchange is necessary, we have made a general analysis of the reaction amplitude for the process $ab \rightarrow cd$ and the density matrix $\rho_{mm'}$ for one of the final particles in terms of helicity amplitudes in the t channel, using the recent work on crossing relations of Trueman and Wick ⁴⁾ and Muzinich ⁵⁾. Two rigorous statements can be made on the basis of this analysis :

1. The density matrix $\rho_{mm'}$, as defined above, can be written as

$$\rho_{mm'} = N' \sum_{\bar{\lambda}\lambda} F_{\bar{\lambda}\lambda}^m(s,t) F_{\bar{\lambda}\lambda}^{m'}(s,t)^* \quad (2)$$

where N^* is a normalization constant, $F_{\lambda\lambda}^m(s,t)$ is the analytic continuation of the centre of mass amplitude in the t channel ($N\bar{N} \rightarrow \bar{K}K^*$), m , λ and $\bar{\lambda}$ being, respectively, the helicities of K^* , N and $\bar{N}$ ⁶⁾. The data on K^* decay therefore appear to imply that in the t channel the only states of K^* that interact are those having helicity ± 1 .

2. A partial wave analysis in the t channel shows that, for each angular momentum J , states with parity $(-1)^J$ can only produce K^* with helicity ± 1 , while states with parity $(-1)^{J+1}$ populate all helicities of K^* ($J = 0^-$ is an exception, giving only helicity 0).

One consequence of these two statements is that any superposition of exchanged systems of different angular momenta J , provided they have parities $(-1)^J$, has the possibility of yielding a decay distribution in agreement with the data. Turning now to the differential production cross-section we first remark that it seems improbable that any finite set of perturbation diagrams can give the necessary drastic momentum transfer dependence, while retaining agreement with the decay data. A mechanism qualitatively different from that of the OMEM must therefore be at work here. One simple possibility is that of Regge pole exchange. That Regge pole amplitudes decrease rapidly with increasing $-t$ for large s is well known. The concern that the energies considered here are not sufficiently large to allow the dominance of one, or at most a few, Regge poles is obviated when one notes that all the high lying Regge trajectories that can contribute ($P, P', \rho, \omega, \phi$) have the correct spin parity $(J, (-1)^J)$ to agree with the decay data. The trajectories of π and η lie considerably lower, and are presumably less and less important as the energy increases. The decay data at 1.45 GeV/c ⁷⁾ and at 1.96 GeV/c ⁸⁾ which are consistent with a OMEM with relative amounts of vector to pseudoscalar meson exchanges of roughly 1.5 and 2.5, respectively, are in accord with this idea. At 3 GeV/c the pseudo-scalar contribution has apparently become very small.

The well-known exponential dependence on momentum transfer follows from the Regge pole model provided that the P_α

$$\left(\text{or } -\frac{\tan \pi \alpha}{\pi} Q_{-\alpha-1} \right)$$

can be approximated by their leading terms which go as $(\cos \Theta_t)^\alpha$. Are the energies under consideration large enough to permit this approximation? The answer is yes. At 3 GeV/c $-\cos \Theta_t$ is in the range 2.5 - 3.3 for the important values of momentum transfer. Inspection of the relevant formulae shows that the error involved in approximating the Legendre functions by their leading terms is roughly 10 percent for $|\cos \Theta_t| = 2$ and less than 5 percent for $|\cos \Theta_t| = 3$ for all values of α of interest⁹⁾. The Regge formulae are too well known to need to be exhibited here¹⁰⁾. The essential point is that each Regge pole term can be written as a slowly varying (in t) function times an exponential factor $e^{\lambda t}$, where

$$\lambda \simeq \alpha'(0) \ln \left(\frac{s-u}{2M^2} \right),$$

$\alpha'(0)$ being the slope of the trajectory at $t = 0$ and M being a mass parameter that can be thought of as a reciprocal interaction radius. At 3 GeV/c for $KN \rightarrow K^* N$, $(s-u) \simeq 10 \text{ GeV}^2$. If M is taken as the pion (nucleon) mass the logarithm has the value 5.5 (2.3). The slopes are not at all well determined, but a popular estimate is $\alpha'(0) \simeq 0.4 - 0.7 \text{ GeV}^{-2}$. Consequently we expect λ to be of the order of 1 to 4, with the empirical value of approximately 2 being quite reasonable. It is perhaps worth emphasizing that an exponential decrease, consistent with experiment, does not depend on the assumption that one Regge pole dominates. We saw, in fact, that all the high-lying trajectories were acceptable contributors. Only when data on different charge states become available will one be able to assess the magnitudes of the contributions from the trajectories of different isotopic spin. One would say, however, that at

high enough energies the exchange should show mainly $T = 0$ character as the vacuum trajectory eventually dominates.

The Regge pole exchange model is undoubtedly a gross oversimplification of the production mechanism. Nevertheless, it can be viewed as an attempt, however crude, to escape from the OMEM and to include large families of states in the crossed channel, states which are plausibly important and which happen to have the spin-parity combinations required by the decay data. The type of generalization needed was made readily apparent through the direct relation (2) connecting the decay density matrix $\rho_{mm'}$ and the t channel helicity amplitudes.

A more general and detailed paper discussing production reactions of this type, with derivations of the various formulae and applications to other channels of the KN system, will be published elsewhere.

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FOOTNOTES AND REFERENCES

- 1) M. Ferro-Luzzi, R. George, Y. Goldschmidt-Clermont, V.P. Henri,
B. Jongejans, D. Leith, G. Lynch, F. Muller and J.M. Perreau,
Sienna Int. Conference on Elem. Particles (to be published).
- 2) Parity conservation in both production and decay and the condition $\text{Tr } \rho = 1$
limit the number of independent elements of the density matrix to three, two
real (ρ_{11} and $\rho_{1,-1}$) and one complex (ρ_{10}), with $\rho_{00} = 1 - 2\rho_{11}$.
- 3) J.D. Jackson and H. Pilkuhn (to be published).
- 4) T.L. Trueman and G.C. Wick, Annals of Physics (in press).
- 5) I.J. Muzinich, University of Washington preprint (1963).
- 6) Eq. (2) has an obvious generalization for a reaction in which all four
particles have spin. Its simplicity, whereby the t channel helicity of
 K^* becomes the magnetic quantum number along the z axis defined above
and no crossing matrices appear, is no accident. The crossing relations
of Trueman and Wick ⁴⁾ and Muzinich ⁵⁾ express an s channel amplitude in
terms of the t channel amplitudes by means of an orthogonal transformation
involving Wigner rotation matrices for each particle. The orthogonality of
the transformation means that when spin sums are performed the rotation
matrices for those particles whose spins are summed disappear from the end
result, as in Eq. (2). The absence in Eq. (2) of two rotation matrices for
the K^* can be understood by consideration of Fig. 4b of Trueman and Wick ⁴⁾.
The rotation needed to transform the s channel helicity density matrix of
 K^* into $\rho_{mm'}$ is just the inverse of the rotation that occurs in the
crossing relation. Muzinich's theorem (ii) ⁵⁾ on the transverse polarization
of one of the outgoing particles follows from Eq. (2) via $\langle s_y \rangle = \text{Tr}(\rho s_y)$.

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- 7) G.B. Chadwick et al., Physics Letters 6, 309 (1963).
- 8) S. Goldhaber, Proceedings of the Athens, Ohio, Conference on Recently Discovered Resonant Particles, April 26-27, 1963, p. 92.
- 9) Even at $\alpha = \frac{1}{2}$, where the usual expansion appears to diverge, a different expansion (from the complete elliptic integral of the second kind) gives corrections of only a few percent for $|\cos \theta_t| \geq 2$.
- 10) See, for example, S.C. Frautschi, M. Gell-Mann and F. Zachariasen, Phys.Rev. 126, 2204 (1962).