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Influence of Dust-Charge Fluctuation on Coherent Structures in Magnetized, Inhomogeneous Dusty Plasmas

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In dusty plasmas, the fluctuation of charge on dust grains may have a significant influence on collective phenomena in the plasma. We study this dust charging effect on the electrostatic drift flow in dusty plasmas paying particular attention to the formation of quasi-stationary coherent structures. Two regimes of different time scales, the ion-drift regime and the dust-drift regime of much lower frequency, are considered by deriving for each regime a pair of coupled equations describing the temporal change of the stream function of drift flow and the dust-charge fluctuation. For both regimes, the current to dust grains must be small enough to yield a quasi-stationary flow. In the case of ion-drift flow, the small current is consistent with the low level excitation of the charge fluctuation, giving no appreciable influence on the coherent structure. In dust-drift flow, the charge fluctuation with a finite amplitude may behave in phase with the potential, acting to reduce the increase of the propagation speed of coherent vortices with the dust density reported earlier without taking account of the dust-charging effect.

KEYWORDS: coherent structure, drift vortex, dust-charge fluctuation, dusty plasma

§1. Introduction

There has been much attention in the formation of coherent structure in magnetized, inhomogeneous plasmas as well as atmospheric or oceanic flows on rotating planets.¹⁾ The subject was recently studied by Chen and Yu²⁾ for a dusty plasma,³⁾ a three-component plasma system containing appreciable amount of negatively charged heavy dust grains in addition to the usual plasma components, ions and electrons. For the dusty plasma in a cylindrical vessel, they found quasi-stationary vortex-like structure in two regimes of different time scales, the ion-drift and dust-drift regimes, and concluded that the presence of the dust causes an increase in the rotation speed of the structure for both cases. However, their treatment was based on the assumption that the variation of the dust charge with respect to the local electrostatic potential was rapidly damped and could

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be ignored. The assumption may not be justified, since the electrostatic potential, playing the role of a stream function for the electrostatic drift flows under consideration, is in fact closely related to the dust-charge fluctuation.⁴⁻⁷⁾ Indeed, the damping rate for the potential is of the same order of magnitude as that for the charge fluctuation in the case of ion-drift flow, so that the coherent structure would be also damped when the charge fluctuation had been damped as assumed. In the present paper, we derive a coupled set of equations for the potential and the charge fluctuation for each of the two regimes considered in ref. 2), and investigate the influence of the charge fluctuation on the flow dynamics which is described by the electrostatic potential.

In §2, we give the basic assumptions to treat the subject, including a simplified model for the dust grains, and derive the dust-charging equation. In §§3 and 4, the ion-fluid equations and the dust-fluid equations are used, respectively, to derive the equation for the time variation of potential vorticity for each fluid taking account of the effects of ion-dust collisions. Then the equation is coupled with the charging equation using the short wavelength drift-mode ordering, and the influence of dust-charge fluctuation on quasi-stationary solutions is examined. Concluding remarks are given in §5.

§2. Basic Assumptions and Charging Equation

Suppose a dusty plasma with the homogeneous and constant temperatures T_j and inhomogeneous densities n_{j0} ($j = e, i, d$) is embedded in a uniform magnetic field B pointing to the z -direction, and consider quasi-two-dimensional motions in the (x, y) plane. Here and hereafter, we denote by the suffices $j = e, i$ and d the quantities related to electrons, ions and dust grains, respectively. We also decompose a quantity Q into the equilibrium value Q_0 and the perturbed one $\tilde{Q}$. In equilibrium, the charge neutrality condition reads

$$Z_i n_{i0} = n_{e0} + Z_{d0} n_{d0}, \quad (1)$$

where Z_i is the charge number of plasma ions, and the charge of dust grains is expressed as $-Z_d e$. The quasi-neutrality is assumed for the perturbed state, too, since only low-frequency processes will be considered. Denoting by $\tilde{n}_j$ the perturbed densities, we have

$$Z_i \tilde{n}_i = \tilde{n}_e + \tilde{Z}_e n_{d0} + Z_{d0} \tilde{n}_d. \quad (2)$$

For simplicity, we treat the dust grains as spherical particles of definite average size and mass, neglecting their distributions, and put $T_d = 0$. They are considered as a continuous fluid like electrons and ions by taking their radius much smaller than the inter-grain distance and plasma Debye length. In particular, since Z_d has large values, we treat $\tilde{Z}_d$ as a field variable, a continuous function of $\mathbf{r} = (x, y)$ and t .

First of all, we derive the equation for $\tilde{Z}_d$ on the assumption that the change of the grain charge

is caused solely by the currents of electrons and ions to the grain.^{5,7)} We have

$$-e \left(\frac{\partial}{\partial t} + \mathbf{v}_d \cdot \nabla \right) Z_d = I_e + I_i, \quad (3)$$

where $\mathbf{v}_d$ is the velocity of dust fluid, and the currents I_j ($j = e, i$) are calculated by assuming appropriate sticking cross sections and velocity distributions of electrons and ions impinging on the grain. For Maxwellian distributions far from the grain, they are given by⁵⁾

$$I_e = -\pi a^2 e \left(\frac{8T_e}{\pi m_e} \right)^{1/2} n_e \exp \left(-\frac{Z_d e^2}{a T_e} \right), \quad I_i = \pi a^2 Z_i e \left(\frac{8T_i}{\pi m_i} \right)^{1/2} n_i \left(1 + \frac{Z_d Z_i e^2}{a T_i} \right), \quad (4)$$

where a is the grain radius, and the velocity of grain has been neglected compared with the thermal velocities of electrons and ions. In equilibrium, $I_{e0} + I_{i0} = 0$, yielding

$$f_e e^{-z_0} = \left(\frac{m_e T_i}{m_i T_e} \right)^{1/2} \left(1 + \frac{z_0}{\theta_i} \right); \quad z_0 \equiv \frac{Z_{d0} e^2}{a T_e}, \quad \theta_i \equiv \frac{T_i}{Z_i T_e}, \quad (5)$$

where z_0 is the magnitude of grain potential relative to T_e , and

$$f_e = \frac{n_{e0}}{Z_i n_{i0}}, \quad f_d = \frac{Z_{d0} n_{d0}}{Z_i n_{i0}} = 1 - f_e \quad (6)$$

are the fractions of electrons and dust grains in the equilibrium negative charge. Equation(5) determines the relation between the dust parameters f_d and z_0 when the plasma parameters $Z_i, m_e/m_i$ and T_i/T_e are given (cf. Fig. 1). Equation(3) holds for $\tilde{Z}_d$ and $\tilde{I}_e + \tilde{I}_i$ with

$$\tilde{I}_e = -I_{i0} \left(\frac{\tilde{n}_e}{n_{e0}} - z_0 \frac{\tilde{Z}_d}{Z_{d0}} \right), \quad \tilde{I}_i = I_{i0} \left(\frac{\tilde{n}_i}{n_{i0}} + \frac{z_0}{z_0 + \theta_i} \frac{\tilde{Z}_d}{Z_{d0}} \right), \quad (7)$$

where the expressions for $\tilde{n}_j/n_{j0}$ ($j = e, i$) depend on the wave mode to be considered.

§3. Ion-Drift Mode

In the first place, we consider the ion-drift mode for which the characteristic frequency ω satisfies the condition $\omega_{cd} \ll \omega \ll \omega_{ci}$, where ω_{cj} is the gyro-frequency of j -th species, and the thermal velocities v_{Tj} of particles and the wave phase velocity v_{ph} along the magnetic field satisfy $v_{Td} \ll v_{Ti} \lesssim v_{ph} \ll v_{Te}$. The grains may be regarded as at rest, so that $\mathbf{v}_d = 0$ and $\tilde{n}_d = 0$. On the other hand, the ions are described by the fluid equations of continuity and momentum balance supplemented by the collision terms with the grains:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = -\nu_{id} n_i + \nu_{id0} n_{i0}, \quad (8)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \nabla \right) \mathbf{v}_i = -\frac{Z_i e}{m_i} \nabla \psi - \frac{T_i}{m_i n_i} \nabla n_i + \omega_{ci} (\mathbf{v}_i \times \hat{z}) - \nu_{id} \mathbf{v}_i, \quad (9)$$

where $\mathbf{v}_i$ is the velocity of ion fluid, ψ the electrostatic potential, and ν_{id} the ion-dust collision frequency. In (8), we have let the right-hand side vanish at equilibrium. It is assumed that all the ions as well as electrons are absorbed by the heavy grains at collisions, and the same frequency ν_{id} is used in (8) and (9). With this assumption, ν_{id} is related to the ion current I_i by

$$\nu_{id} = \left(\frac{n_d}{Z_i e n_i} \right) I_i, \quad (10)$$

and expressed also as $\nu_{id} = \nu_{id0} + \tilde{\nu}_{id}$, where $\tilde{\nu}_{id} = \nu_{id1} \tilde{Z}_d$ and $\nu_{id1} \equiv (\partial \nu_{id} / \partial Z_d)_0$. Now we write $\psi = \psi_0 + \tilde{\psi}$ and $n_i = n_{i0} + \tilde{n}_i$ on the right-hand side of (9), and subtract the equilibrium term $-(Z_i e / m_i) \nabla [\psi_0 + (T_i / Z_i e) \ln n_{i0}] = 0$. Thus we get

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \nabla \right) \mathbf{v}_i = -\frac{Z_i e}{m_i} \nabla \tilde{\phi} + \omega_{ci} (\mathbf{v}_i \times \hat{z}) - \nu_{id} \mathbf{v}_i, \quad (11)$$

where we have defined the function $\tilde{\phi}$ by

$$\tilde{\phi}(\mathbf{r}, t) = \tilde{\psi}(\mathbf{r}, t) + \frac{T_i}{Z_i e} \ln \left(1 + \frac{\tilde{n}_i}{n_{i0}} \right). \quad (12)$$

Taking the curl of (11) and using (8), we derive the equation for the potential vorticity¹⁾ $(\omega_{ci} + \Omega) / n_i$ for the drift flow, where $\Omega = \hat{z} \cdot (\nabla \times \mathbf{v}_i)$ is the z-component of vorticity:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \nabla \right) \ln \left(\frac{\omega_{ci} + \Omega}{n_i} \right) = \nu_{id} \left(\frac{\tilde{n}_i}{n_i} - \frac{\Omega}{\omega_{ci} + \Omega} \right) + \tilde{\nu}_{id} \frac{n_{i0}}{n_i} - \frac{(\nabla \nu_{id} \times \mathbf{v}_i) \cdot \hat{z}}{\omega_{ci} + \Omega}. \quad (13)$$

Finally, the electrons obey the Boltzmann distribution

$$n_e(\mathbf{r}, t) = n_{e0}(\mathbf{r}) \exp \left[\frac{e \tilde{\psi}(\mathbf{r}, t)}{T_e} \right], \quad (14)$$

where $\psi_0(\mathbf{r})$ has been absorbed in $n_{e0}(\mathbf{r})$.

Equation(13) is to be coupled with the perturbed one of (3), and a set of equations for $\tilde{\phi}$ and $\tilde{Z}_d$ will be derived by expressing $\tilde{n}_j / n_{j0}$ ($j = e, i$) in terms of these variables. To facilitate the following analysis, we here introduce the dimensionless quantities by

$$\begin{aligned} \omega_{ci} t &\mapsto t, & (x, y) / \rho_i &\mapsto (x, y), \\ \phi &= e \tilde{\phi} / T_e (1 + f_e \theta_i), & \zeta &= \tilde{Z}_d / Z_{d0} (1 + f_e \theta_i), \\ \nu &= \nu_{id0} / \omega_{ci}, & \nu_1 &= \nu_{id1} / \omega_{ci}, \end{aligned} \quad (15)$$

where $\rho_i^2 = (Z_i T_e + f_e T_i) / m_i \omega_{ci}^2$; we use the same symbols (t, x, y) for the dimensionless independent variables to save the notations. Further, we assume the following ordering which is appropriate for the short wavelength drift mode under consideration:^{1,8)}

$$\nabla \sim \theta_i \sim O(1); \quad \partial / \partial t \sim \phi \sim \tilde{n}_j / n_{j0} \sim \nabla \ln n_{j0} \sim \zeta \sim \nu \sim \nu_1 \sim O(\epsilon), \quad (16)$$

where ϵ is a small ordering parameter. Then the velocity $\mathbf{v}_i$ and Ω in (13) may be approximated by $\mathbf{v}^0 = \hat{z} \times \nabla \phi$ and $\Omega = \nabla^2 \phi$, and we obtain, using (2), (6), (12) and (14),

$$\tilde{n}_e/n_{e0} = \phi - f_e \theta_i \zeta, \quad \tilde{n}_i/\tilde{n}_{i0} = f_e \phi + f_d \zeta. \quad (17)$$

Retaining terms of $O(\epsilon^2)$ and omitting higher order ones, we can now bring (3) and (13) into the following set for the stream function ϕ and dust-charge fluctuation ζ :

$$\frac{\partial \zeta}{\partial t} + \eta \zeta = \nu \phi \quad (18)$$

$$\frac{\partial \tilde{q}}{\partial t} + [\phi, \tilde{q} - \ln n_{i0}] = -\nu \tilde{q} + \nu_1 \zeta \quad (19)$$

where, in (18),

$$\tilde{q} = \nabla^2 \phi - f_e \phi - f_d \zeta, \quad (20)$$

and $q \equiv \tilde{q} - \ln n_{i0}$ is the potential vorticity in the present ordering (16). Other symbols are given by

$$\eta = I \left\{ z_0 \left(1 + \frac{1}{z_0 + \theta_i} \right) + f_d \frac{1 + \theta_i}{1 + f_e \theta_i} \right\}, \quad I = \frac{I_{i0}}{e Z_{d0} \omega_{ci}}, \quad \nu = f_d I, \quad \nu_1 = \nu z_0 \frac{1 + f_e \theta_i}{z_0 + \theta_i}, \quad (21)$$

and $[A, B] = \hat{z} \cdot (\nabla A \times \nabla B)$. We note that the stream function ϕ is related to the potential $\tilde{\psi}$ and the charge fluctuation ζ by

$$\phi = e \tilde{\psi} / T_e + f_d \theta_i \zeta, \quad (22)$$

which follows from (12), (15) and (17).

In the absence of dust grains, $f_d = 0$ ($f_e = 1$), $\nu = 0$ and $\zeta = 0$, (18) is trivial while (19) reduces to the Charney-Hasegawa-Mima (CHM) equation^{9,10)} for nonlinear drift flow in a pure plasma; a slight generalization has been made to include the finite ion temperature, being necessary to account for the ion current to grains, in the unit of length, ρ_i . It expresses the conservation of potential vorticity along the flow described by the stream function ϕ , and allows as its stationary solution a coherent structure such as the wellknown dipole vortex ("modon") first derived by Larichev and Reznik¹¹⁾ for the equivalent case of geostrophic flow. When the grains are present, however, the set (18)-(20) does not allow a stationary solution and ϕ and/or ζ will decay if given initial finite values. Indeed, an explicit analysis¹²⁾ of (18)-(20) for a cylindrically bounded dusty plasma²⁾ shows that the amplitude of the function ϕ representing a rotating vortex-like structure decays due to the ion-dust collisions, while the phase of the rotation lags as a result of coupling with the dust-charge fluctuation ζ which, starting from initial zero value, also soon decays after arriving at a certain level.

A long-lived structure will be possible only when all the coefficients η , ν and ν_1 in (18) and (19) have small values, higher than $O(\epsilon)$ as assumed in (16). Then ζ will also remain smaller than $O(\epsilon)$ as inferred from (18) if it is small initially, and (19) reads $\partial q/\partial t + [\phi, q] = 0$ with $q = \nabla^2 \phi - f_e \phi - \ln n_{i0}$. This equation is just the same as that derived in ref. 2) for the ion-drift flow in a dusty plasma, but obtained here on somewhat different ground. As shown in ref. 2), for the plasma contained in a cylinder of radius R with the density profile $n_{i0} \sim \exp(-r^2/r_0^2)$, it gives the stationary solution $\phi = \phi_m J_m(kr) \cos m(\theta - \omega t)$ in the polar coordinates (r, θ) , where m is an integer, ϕ_m a constant amplitude, J_m the Bessel function of order m , $k = x_{ms}/R$ with x_{ms} the s -th zeros of $J_m(x)$, and $\omega = 2/(f_e + k^2)r_0^2$ the angular velocity; the solutions are just the linear eigenmodes in cylindrical geometry. Thus the rotation speed increases as the dust density increases. For the modon moving along the y -axis with the velocity u in a dusty plasma with $n_{i0} \sim \exp(-\kappa x)$, where κ is a constant, u is subject to a condition for the solution to decay exponentially at infinity. It is given by $u > \kappa/f_e$ or $u < 0$, thus a greater positive speed than the ordinary modon with $f_e = 1$.

For f_d not very small, the magnitude of damping coefficients η , ν and ν_1 may be represented by the parameter I , expressing the equilibrium current, as can be seen from (21). Denoting by $l_d = (3/4\pi n_{d0})^{1/3}$ the mean distance between dust grains of radius a , we can estimate I as follows from (4), (6) and (21):

$$I \simeq \frac{3}{4f_d} \left(\frac{a}{l_d} \right)^2 \frac{\rho_i}{l_d} \left(1 + \frac{z_0}{\theta_i} \right), \quad (23)$$

where now $\rho_i \equiv (8T_i/\pi m_i)^{1/2}/\omega_{ci}$. The crucial factors are $(a/l_d)^2$ and (ρ_i/l_d) , where the former should be very small while the latter depends on actual conditions of interest. The condition for quasi-stationary structures may be expressed as $I \leq O(\epsilon^2)$.

§4. Dust-Drift Mode

Next, we consider the dust-drift mode having the characteristic frequency much smaller than ω_{cd} and satisfying the condition $v_{Td} \ll v_{ph} \ll v_{Ti}$. In this case, the ions also obey the Boltzmann distribution like electrons

$$n_i(\mathbf{r}, t) = n_{i0}(\mathbf{r}) \exp(-Z_i e \tilde{\psi}/T_i). \quad (24)$$

To derive the equation for the potential vorticity of dust flow, we use the fluid equations assuming cold grains:

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{v}_d) = 0, \quad (25)$$

$$\frac{\partial}{\partial t} (m_d n_d \mathbf{v}_d) + \nabla \cdot (m_d n_d \mathbf{v}_d \mathbf{v}_d) = -Z_d e n_d (-\nabla \tilde{\phi} + \mathbf{v}_d \times \mathbf{B}), \quad (26)$$

where, in (26), we take account of the change of the grain mass m_d due to the sticking of ions.¹³⁾ It

is derived from the conservation of total mass density which reads neglecting electron contribution

$$\frac{\partial}{\partial t}(m_d n_d + m_i n_i) + \nabla \cdot (m_d n_d \mathbf{v}_d + m_i n_i \mathbf{v}_i) = 0. \quad (27)$$

Making use of (8), (10) and (25), we have $dm_d/dt = (m_i/Z_i e) \tilde{I}_i$, and may rewrite (26) as follows:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_d \cdot \nabla \right) \mathbf{v}_d = \frac{Z_d e}{m_d} [\nabla \tilde{\psi} - B(\mathbf{v}_d \times \hat{z})] - \frac{m_i}{m_d} \left(\frac{\tilde{I}_i}{Z_i e} \right) \mathbf{v}_d. \quad (28)$$

Hereafter, we treat the mass m_d on the right-hand side of (28) as constant, but decompose the charge Z_d into Z_{d0} and $\tilde{Z}_d$ and put $\omega_{cd} = Z_{d0} e / m_d$.

As in the case of ion-drift flow, we can derive the equation for the potential vorticity $(\omega_{cd} - \Omega_d)/n_d$ from (25) and (28), where $\Omega_d = \hat{z} \cdot (\nabla \times \mathbf{v}_d)$:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_d \cdot \nabla \right) \ln \left(\frac{\omega_{cd} - \Omega_d}{n_d} \right) = \frac{\omega_{cd}}{\omega_{cd} - \Omega_d} \left\{ \hat{z} \cdot (\nabla \times \tilde{I} \mathbf{v}_d) - \nabla \cdot \left(\frac{\mathbf{v}_d^{(1)} \tilde{Z}_d}{Z_{d0}} \right) \right\}, \quad (29)$$

where $\tilde{I} \equiv \tilde{I}_i / Z_{d0} e \omega_{ci}$, and $\mathbf{v}_d^{(1)} = \mathbf{v}_d - \mathbf{v}^0$ with $\mathbf{v}^0 = (\hat{z} \times \nabla \tilde{\psi}) / B$. According to the ordering (32) below, $\mathbf{v}_d^{(1)}$ is of higher order in ϵ and need not be given here explicitly. To introduce dimensionless variables, we define the effective temperature T_{eff} by²⁾

$$T_{eff} = f_d \left/ \left(\frac{Z_i}{T_i} + \frac{f_e}{T_e} \right) \right. = f_d T_i / Z_i (1 + f_e \theta_i), \quad (30)$$

and use $\rho_d = c_d / \omega_{cd}$, with $c_d = (Z_{d0} T_{eff} / m_d)^{1/2}$ the dust-sound velocity, as the unit of space coordinates. We put

$$\begin{aligned} \omega_{cd} t &\mapsto t, & (x, y) / \rho_d &\mapsto (x, y) \\ \phi_d &= e \tilde{\psi} / T_{eff}, & \zeta_d &= \tilde{Z}_d / Z_{d0} \end{aligned} \quad (31)$$

and assume the following ordering:

$$\nabla \sim \theta_i \sim O(1); \quad \partial / \partial t \sim \phi_d \sim \zeta_d \sim \tilde{n}_j / n_{j0} \sim \nabla \ln n_{j0} \sim O(\epsilon). \quad (32)$$

Then we have from (2), (14) and (24),

$$\tilde{n}_e / n_{e0} = [f_d \theta_i / (1 + f_e \theta_i)] \phi_d, \quad \tilde{n}_i / n_{i0} = -[f_d / (1 + f_e \theta_i)] \phi_d, \quad \tilde{n}_d / n_{d0} = -(\phi_d + \zeta_d), \quad (33)$$

and from (4) and (7)

$$\tilde{I} = \left(-\frac{f_d}{1 + f_e \theta_i} \phi_d + \frac{z_0}{z_0 + \theta_i} \zeta_d \right) I, \quad (34)$$

where I has been defined in (21).

With the ordering (32), we find that the dominant terms on the left-hand side of (29) are $O(\epsilon^2)$ as before, while both the two terms on the right-hand side may be ignored, being $O(\epsilon^3)$ or higher, provided that the parameter $\tilde{I}$ is at most $O(\epsilon^2)$, $\tilde{I} \lesssim O(\epsilon^2)$. It is equivalent to $I \lesssim O(\epsilon)$ from (34) for $\phi_d \sim \zeta_d \sim O(\epsilon)$. Under this condition, the velocity $\mathbf{v}_d$ may be replaced by $\mathbf{v}^0 = \hat{z} \times \nabla \phi_d$ and (29) becomes

$$\frac{\partial q_d}{\partial t} + [\phi_d, q_d] = 0, \quad (35)$$

with

$$q_d = \nabla^2 \phi_d - \phi_d - \zeta_d + \ln n_{d0}, \quad (36)$$

which is the potential vorticity in the present case with reversed sign. On the other hand, the charging equation (3) may be written in the form:

$$\frac{\partial \zeta_d}{\partial t} + [\phi_d, \zeta_d] = g (\nu_d \phi_d - \eta_d \zeta_d), \quad (37)$$

where, $g = \omega_{ci}/\omega_{cd} = (m_d Z_i)/(m_i Z_{d0})$ and

$$\nu_d = f_d \frac{1 + \theta_i}{1 + f_e \theta_i} I, \quad \eta_d = z_0 \left(1 + \frac{1}{\theta_i + z_0} \right) I. \quad (38)$$

Equations (35)-(37) are the coupled set for ϕ_d and ζ_d valid when the condition $I \lesssim O(\epsilon)$ is satisfied. (Note that it has been used only for deriving (35), but not for (37).) This condition for I is weaker than the corresponding one, $I \lesssim O(\epsilon^2)$, for the quasi-stationary ion-drift flow treated in the preceding section, while the charging equation (37) is much different from the corresponding one (18) in that a very large factor g is multiplied on the right-hand side. It is then possible that $gI \gtrsim O(\epsilon)$ even if $I \lesssim O(\epsilon^2)$. In that case, ζ_d will quickly approach $(\nu_d/\eta_d)\phi_d$ within the time-scale of interest, independent of initial conditions. It means that the charge on dust grains and the plasma potential behave in phase for very slow process and weak charging current. Putting $\zeta_d = (\nu_d/\eta_d)\phi_d$ in (36), we have the conservation of potential vorticity (35) with

$$q_d = \nabla^2 \phi_d - \lambda \phi_d + \ln n_{d0} \quad (39)$$

where $\lambda = 1 + (\nu_d/\eta_d)$. A remarkable difference from the ion-drift case is that the coefficient λ of ϕ_d is now greater than unity and *increases* as f_e decreases. In the case of cylindrical plasma with the radius R and $n_{d0} \sim \exp(-r^2/r_0^2)$, the vortex-like solutions are given in a similar form as the ion-drift case, and $\omega = -2/(\lambda + k^2)r_0^2$. Since the normalization length ρ_d also depends on f_d , we give the angular velocity of rotation in the dimensional form as follows:

$$\frac{\omega}{\omega_{cd}} = -\frac{2\rho_{d0}^2}{r_0^2} F(f_d), \quad \rho_{d0}^2 = \frac{T_i Z_{d0}}{Z_i m_d \omega_{cd}^2} \quad (40)$$

where $F(f_d) = 1/[\Lambda(f_d) + k_{ms}^2]$, and we have put $k_{ms} = \rho_{d0} x_{ms}/R$ and

$$\Lambda(f_d) = \frac{1 + f_e \theta_i}{f_d} \lambda. \quad (41)$$

The case $\zeta_d \equiv 0$, treated in ref. 2), corresponds to put $\lambda = 1$. In the case of the modon in plane geometry with $n_{d0} \sim \exp(-\kappa x)$, the condition for the propagation velocity u is $u > 0$ or $u/c_{d0} < -(\kappa\rho_{d0})\Lambda^{-1}(f_d)$, where $c_{d0} = \rho_{d0}\omega_{cd}$.

If the dust-charge fluctuation is ignored as in ref. 2), the presence of dust grains manifests itself only through the effective temperature T_{eff} in the normalization length, and the speed of the quasi-stationary vortex (ω or negative u) increases monotonically with f_d , the relative dust density. When the charge fluctuation is properly taken into account and the condition $gI \gtrsim O(\epsilon)$ holds, there may arise such a situation that the charge fluctuation behaves in phase with the electrostatic potential. Then the "mass" term, ϕ_d , in the potential vorticity is effectively increased with the result that the increase of speed of the vortex is reduced. The two counter effects are expressed in the expression of $\Lambda(f_d)$ given by (41). As an example, the inverse of the function $\Lambda(f_d)$ is shown in Fig. 1, together with the parameter z_0 , for a hydrogen plasma with $T_i = T_e$; the range of f_d is limited to yield $z_0(f_d) \gtrsim 0$. It can be seen that the increase of the vortex speed is much reduced for f_d greater than about 0.6. The qualitative behavior of Λ^{-1} is not much different from the illustrated one for other reasonable choice of the parameters θ_i and m_i/m_e .

§5. Concluding Remarks

In this paper, we considered the low-frequency drift waves in a magnetized, inhomogeneous dusty plasma in two regimes of different time scales, the ion-drift and dust-drift regimes, in each of which the ion fluid and the dust fluid govern the flow dynamics, respectively. The finite ion temperature was included in the flow dynamics of ions to take account of the ion current to grains. A set of coupled equations for the stream function of drift flow and the fluctuation of dust charge was derived for each regime on the basis of the short wavelength ordering for the drift flow.

The coupled equations were considered for each regime separately to see the influence of the dust-charge fluctuation on the flow dynamics, in particular on the formation of possible quasi-stationary coherent structures. First of all, it would be obvious that the ion-dust collision frequency, or, equivalently, the equilibrium ion current to dust grains, must be sufficiently small in order to obtain a stationary solution. In the ion-drift regime, this condition was found to be consistent with the low-level excitation of the charge fluctuation, and the modified CHM equation obtained was formally the same as derived in ref. 2) ignoring the charging process from the beginning.

In the dust-drift regime, it was found that the very slow time scale of variation could make the dust charge fluctuate in phase with the plasma potential. This brings the effective increase of the "mass" term in the potential vorticity, and acts to reduce the speed of stationary vortices from the speed without the charge fluctuation, especially for greater values of the relative dust density.

In conclusion, we used in this paper a simplified model for the dust grains: spherical particles of definite size and mass, without distributions. The model has been used in many previous works,⁴⁻⁷⁾ and it is expected that the results obtained in the present paper will be valid approximately. It is

because the explicit magnitudes of the collisional process, depending on the radius a and mass of grains, do not appear in the final results as long as we neglect the collisional effects as small, and the parameter $z_0 = Z_{d0}e^2/aT_e$ in the expression of λ does not depend on a , Z_{d0} being proportional to the capacitance a . For more consistent treatment, however, the distribution of grain charge and size^{14,15)} should be carefully incorporated in the theory.

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- 1) For a recent review article, see J. Nycander: *Chaos* **4** (1994) 253.
 - 2) Y. Chen and M. Y. Yu: *Phys. Letter A* **185** (1994) 475.
 - 3) U. de Angelis: *Phys. Scripta* **45** (1992) 465.
 - 4) J. X. Ma and M. Y. Yu: *Phys. Plasma* **1** (1994) 3520.
 - 5) M. R. Jana, A. Sen and P. K. Kaw: *Phys. Rev.* **E48** (1993) 3930.
 - 6) M. R. Jana, A. Sen and P. K. Kaw: *Phys. Scripta* **51** (1995) 385.
 - 7) V. N. Tsytovich and O. Havnes: *Comments Plasma Phys. Control. Fusion* **5** (1993) 267.
 - 8) K. Orito, M. Sato and H. Irie: *J. Phys. Soc. Jpn.* **64** (1995) 267.
 - 9) J. G. Charney: *Geophys. Public. Kosjones Nors. Videnshap. Akad. Oslo* **17** (1948) 3.
 - 10) A. Hasegawa and K. Mima: *Phys. Fluids* **21** (1978) 87.
 - 11) V. D. Larichev and G. M. Reznik: *Dkl. Akad. Nauk SSSR* **231** (1976) 1077.
 - 12) N. Koide and M. Sato: *Relaxation of Drift Vortices in Dusty Plasmas*, pre-print, NUP-A-97-7 (1997), unpublished.
 - 13) B. P. Pandey and C. B. Dwivedi: *PRAMANA—journal of physics*, **45** (1995) 255.
 - 14) T. K. Aslaksen and O. Havnes: *J. Plasma Phys.* **51** (1994) 271.
 - 15) A. G. Sitenko, A. G. Zagorodny, Yu. I. Chutov, P. Schram and V. N. Tsytovich: *Plasma Phys. Control. Fusion* **38** (1996) A105.

Fig. 1. The parameter $z_0 = Z_{d0}e^2/aT_e$ and inverse of the function $\Lambda(f_d)$, defined by eq. (41), as functions of the relative dust-grain density f_d . Hydrogen plasma with $T_e = T_i$ ($\theta_i = 1$) is assumed. The curve marked $\lambda = 1$ corresponds to $1/\Lambda(f_d)$ ignoring the dust-charge fluctuation.