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THEORETICAL AND EXPERIMENTAL RELATIONSHIP BETWEEN
TRIANGLE SINGULARITIES, PEIERLS MECHANISM,
AND RESONANCE POLES ⁺)

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The values $s_{\pm}$ correspond precisely to the location of the possible Landau singularities of the triangle graph Fig. 1c ⁹⁾, if we treat X as a real particle. Since 1c is a contraction of 1a, the structure of 1a must be considered within that of 1c. Thus the crucial question is : for which Riemann sheets are $s_{\pm}$ actually singularities, and which of these are near the physical region.

Define ¹⁰⁾

$$\begin{aligned} x &= (s - X^2 - V^2)/(2XV) \geq 1 && \text{above the } XV \text{ threshold} \\ y &= (Y^2 - X^2 - Z^2)/(2XZ) < -1 \\ z &= (\sigma - Z^2 - V^2)/(2ZV) \geq 1 \end{aligned} \quad (2)$$

Then $s_{\pm}$ are the two roots of

$$S: x^2 + y^2 + z^2 + 2xyz - 1 = 0 \quad (3)$$

i.e.,

$$x = -yz - (y^2 - 1)^{\frac{1}{2}}(z^2 - 1)^{\frac{1}{2}} \quad (1')$$

(Eq. (1') is easier to use than (1)). The curve S is shown in Figs. 2a and 2b ignoring $\text{Im } X$; and in Fig. 2c for finite $\text{Im } X < 0$. Only the part ab of S gives a singularity close to the physical region ¹⁰⁾. If and only if

$$(Z + V)^2 \leq \text{Re } \sigma \lesssim \text{Re } \sigma_B \quad (4)$$

with

There is great interest, both theoretical ^{1),2),3),4),5)} and experimental ^{6),7)}, in the study of three body energy peaks, involving unstable particles. Our aim is to clarify the distinction and similarity between the generalized Peierls mechanism ⁴⁾ and that due to triangle singularities; and to discuss the conditions under which these can be expected to be practically important, extending and re-emphasizing Goebel's work ⁵⁾.

Consider the production process and mechanism shown in Fig. 1a, where X represents an unstable particle with strong decay $X \rightarrow Y+Z$. If $V \equiv Y$, then $Z+V$ will re-form X, i.e., $\sigma = X^2$ will be enhanced, and we have $X+Y \rightarrow X+Y$ the original Peierls mechanism. If another resonance W exists, $W \rightarrow Z+V$, then $\sigma = W^2$ will be enhanced (if $s > (W+Y)^2$), and we have $X+V \rightarrow W+Y$. This we call the generalized Peierls mechanism, first suggested by Nauenberg and Pais ⁴⁾ for $K^* + \pi \rightarrow K + \rho$, and $K + \rho \rightarrow K^* + \pi$.

If, in Fig. 1a, one asks that all particles including X be on their mass shells, and that θ , the X to Y scattering angle in the s centre of mass system be real ($\cos\theta \leq 1$) this requires $s_+ \leq s \leq s_-$ with

$$2 Z^2 s_{\pm} = (X^2 + V^2)(\sigma + Y^2) - (X^2 + \sigma - Z^2)(Y^2 + V^2 - Z^2) \\ \mp \left\{ [\sigma - (Z+V)^2][\sigma - (Z-V)^2][Z^2 - (X+Y)^2][Z^2 - (X-Y)^2] \right\}^{\frac{1}{2}} \quad (1)$$

(note $\mp$) (cf., Ref. ⁴⁾). Here $s_{\pm}$ are given by $\cos^2\theta = +1$, i.e., X, Y, Z and hence also the internal V are all collinear. The s range $s_+ \leq s \leq s_-$ enters also on considering the "beam" experiment $X+V$ (Fig. 1b). Detailed calculations ^{1),2),3)} of this for the Peierls case $X+Y \rightarrow X+Y$ indicate that for the beam experiment a large peak in cross-section against s should occur near but somewhat above s_+ ⁸⁾, nothing significant occurring near s_- . (The details depend somewhat on the spins involved.)

$$3 = |y| \Rightarrow \sigma = \sigma_B = V^2 + Z^2 + V(X^2 + Z^2 - Y^2)/X \quad (5)$$

s_+ is on the relevant branch. In Fig. 2c we show a short path going through the physical cut, and through the woolly cusp ¹¹⁾ cut $\sqrt{s} = (X+V)^2$ and reaching s_+ on a singular sheet.

The branch ab has an elegant physical interpretation ¹²⁾. $s_{\pm}$ correspond to all particles on their mass shells, and all momenta collinear, in the over-all centre of mass frame, and hence also in the X rest frame. In this X frame, we demand that the directions of the intermediate V and Z be parallel (as opposed to antiparallel). This requires for a given s that σ be the lesser of the two σ roots of the equation $S = 0$. Further we demand that the speed of Z be greater than or equal to that of V ; this requires $x \leq -y$, i.e., $s \leq s_a$, where $s_a = (X^2 + V^2) + (X^2 + Z^2 - Y^2) V/Z$. The two conditions together restrict us to the branch ab . And this corresponds to the physical condition that, after the X resonance has existed for a time determined by $\text{Im } X$, the break up particle Z is heading in the correct direction and with sufficient speed to catch up and interact with the freely moving V ¹³⁾ (the condition is Lorentz invariant). The restriction on the lifetime of X for a large effect is now clear. If the lifetime is too short, X will not have moved very far away from the production vertex before break-up, so that owing to Compton wave-length spreading, the $\cos\theta = +1$ condition need not be very accurately satisfied. On the other hand, for $\Gamma \rightarrow 0$, the X will never break up.

We now turn to the practicalities.

A) The Peierls mechanism: here σ becomes essentially X^2 , and $V = Y$. It is then easy to see that $\text{Re } \sigma > \text{Re } \sigma_B$ so that no effect should occur ¹⁴⁾ (cf., Goebel ⁵⁾, and also Hwa ¹⁵⁾).

B) The generalized Peierls mechanism : here $\sigma \rightarrow W^2$, so that one must test whether $\text{Re } \sigma > \text{Re } \sigma_B$ ¹⁶⁾. This requires W to be near its threshold $V+Z$, and X to be rather high above its threshold. Further X should be fairly narrow, while W should be as wide as possible so that the VZ interaction is sufficiently strong. A possible example is

in the notation $\rightarrow X+V \rightarrow Y+W$. s_+ is about 2.25 GeV total centre of mass energy, considerably above the $K\Xi^*$ threshold though rather close to the competing $K^*\Xi$. Therefore the width of the K^* is likely to make this effect rather too small to be observable at present ¹⁷⁾.

C) The triangle mechanism : in this case, both σ and s are under experimental control, so that $\sigma < \sigma_B$ can always be arranged. One can then either i) with suitably fixed $\sqrt{s}$, observe the $\sqrt{\sigma}$ spectrum, or ii) for experimentally selected $\sqrt{\sigma}$, observe the number of events against $\sqrt{s}$. Chang and Tuan ¹⁸⁾ have strongly urged the experimental investigation of the triangle singularity in the process $K^-p \rightarrow \Xi^* (1533) + K \rightarrow \Xi + (K\pi) \rightarrow \Xi + K + \pi$. They suggest looking at the $K\pi$ spectrum for fixed beam energy, for a small range of beam energies. The analysis should also be performed the other way ¹⁹⁾.

While good examples of generalized Peierls and triangle singularities seem hard to come by in elementary particle physics ²⁰⁾, there seems more scope in low energy nuclear physics. Some calculations ²¹⁾ have been made and 10% effects are quite frequent, and probably observable by newly developing techniques.

Finally, we cannot refrain from commenting on the de facto success of the Peierls mechanism successfully predicting higher resonances. The possibility ²²⁾ of the wrong sheet singularity s_+ inducing a true resonance pole near s_+ seems very plausible in the light of the recent finding by Aitchison ²³⁾ of such

a pole in the Lee model. Such true poles should be easily distinguishable from Peierls singularities, by having definite quantum numbers, and by being seen in all different connected channels, including those in which the "final" resonant rescattering X does not occur^{24),25)}.

It has been shown^{3),22)} that the baryon systems are more likely to be real resonance poles. For these systems⁷⁾ Table 1²⁶⁾ seems to confirm the hypothesis of Goebel⁵⁾ that, the stronger the coupling, the nearer the pole to $\sqrt{s}_+$.

It is a pleasure to acknowledge stimulating discussions with S.F. Tuan, I.J.R. Aitchison, and others on this subject. I am grateful to Professor L. Van Hove for his hospitality at CERN ; and not least to the Graduate School of the University of Maryland for a Faculty Research Award for the summer of 1964, during which time this work was carried out.

T A B L E 1

Coupling Strength versus Position of
Experimental "Resonance" for Baryon Systems

System	$\text{Re } \sqrt{s}_+$	$\text{Re } \sqrt{s}_-$	Expt	$g^2/(4\pi) \quad ^*)$
$(N^* N \pi)$	1490 MeV	1605 MeV	1515 MeV	$14.5 \pm 2.5 \text{ GeV}^{-2}$
$(Y_1^* \Lambda \pi)$	1620	1700	1660 ± 10	8.2 ± 1.5
$(\Xi^* \Xi \pi)$	1729	1760	$\begin{cases} 1770 \pm 30 \\ 1810 \pm 10 \end{cases}$	3.4 ± 0.9

$$*) = \frac{6\Gamma M^2}{\Delta(M+m)^2 - \mu^2} \sqrt{k^3}$$

cf., S. Madelstam et al., Ann. Phys. (N.Y.),
18, 198 (1962).

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- 2) S.F. Tuan, Phys. Rev. 125, 1761 (1962).
M. Nauenberg and A. Pais, Phys. Rev. Letters 8, 82 (1962).
R.J. Oakes, Phys. Rev. Letters 12, 134 (1964).
Also G.B. Yodh and the present author (unpublished).
- 3) S.F. Tuan, Physics Letters 11, 248 (1964).
- 4) M. Nauenberg and A. Pais, Report to the 1964 Dubna Conference.
- 5) C. Goebel, University of Wisconsin Report, (1962), (unpublished) ;
Phys. Rev. Letters 13, 143 (1964).
- 6) Meson systems : see references cited in Ref. 4).
- 7) Baryon systems : πN^* , $T = \frac{1}{2}$, M. Olsson and G.B. Yodh (to be published).
 πN^* , $T = \frac{5}{2}$, G. Goldhaber et al., submitted to the 1964 Dubna Conference.
 Y_{13}^{**} (1660), as $\pi + Y_{13}^*$ (1382), L.W. Alvarez et al., Phys. Rev.
Letters 10, 184 (1963).
 Ξ^{**} (1770), as $\pi + \Xi^*$ (1533), P. Bellière et al., Proceedings of
the 1963 Sienna Conference, p. 173.
Also G.A. Smith et al., Phys. Rev. Letters 13, 61 (1964), who find
a mass 1810 ± 10 .
- 8) This may be relevant to the Ξ^{**} (Ref. 7), $\sqrt{s}_+$ in this case being
1.72 GeV ; see also Table 1.
- 9) For the Landau conditions are $q_i^2 = m_i^2$, i.e., all particles on their mass
shells ; and $\sum \alpha_i q_i = 0$, which in this case reduces to all q_i
parallel. s and σ then fix the α_i .

- 10) One must first deal with the unstable internal particle X , first considered properly by I.J.R. Aitchison and C. Kacser, Phys. Rev. 133, B 1239 (1964). It turns out that, after going through the three particle cut starting at $s = (V+Y+Z)^2$, the analytic properties on this sheet are found to exactly agree with those deduced by treating X as a true particle of definite mass. (This is the meaning of Goebel's footnote 4 in Ref. ⁵.) The latter properties are well known, cf., G. Barton and C. Kacser, Nuovo Cimento 21, 593 (and also 988) (1961).
G. Bonnevey, I.J.R. Aitchison and J.S. Dowker, Nuovo Cimento 21, 1001 (1961). And
M. Fowler, F.V. Landshoff and R.W. Lardner, Nuovo Cimento 17, 956 (1960).
- 11) M. Nauenberg and A. Pais, Phys. Rev. 126, 360 (1962).
- 12) First given, but not sufficiently emphasized, by J.B. Bronzan, Phys. Rev. 134, B 687 (1964).
- 13) Recently, R.E. Norton, UCLA preprint (1964), has suggested that each singularity near the physical sheet for all graphs will be in one-to-one correspondence with each real physically realizable intermediate process.
- 14) With $\sigma = X^2$, $s_+(\sigma)$ moves down through the $(X+V)^2$ cut. This does not affect the conclusion, which is independent of the continuation path for σ . In particular near b : $[\sqrt{s} - (X+V)] \propto (\sigma - \sigma_b)^2$ so that it is immaterial on which side σ passes σ_b .
- 15) R.C. Hwa, Phys. Rev. 130, 1580 (1963).
- 16) This condition is not satisfied for either $K + \varrho \rightarrow K^* + \pi$, or $K^* + \pi \rightarrow K + \varrho$. We note that the condition is not invariant under the interchange of intermediate and final states.
- 17) Another candidate, unfortunately having strangeness ± 2 , is $\Xi + \varrho \rightarrow \Xi^* (1533) + \pi$. Again the widths are not very satisfactory, but $\sqrt{s}_+$ is reasonably far from the $\Xi \varrho$ woolly cusp at 2.08 GeV, with $\sqrt{s}_+ = 2.17$ GeV. Observation of the woolly cusp phenomenon in itself would be very interesting. Another woolly cusp candidate is $\pi N \rightarrow \varrho N \rightarrow \pi N^*$. $\sqrt{s}_+$ is 1.74 GeV, rather close to the woolly cusp at 1.69 GeV., cf., Aitchison, Ref. ²⁰).

- 18) Y.F. Chang and S.F. Tuan, Purdue University preprint (1964).
- 19) We are ignoring the possible dependence of the XV production process on s. In general this variation should be small over the energy range involved.
- 20) We should mention the very interesting work of V.V. Anisovich and L.G. Dakhno, Physics Letters 10, 221 (1964), which is able to fit both the ABC data and the $\pi^- p \rightarrow n \pi^- \pi^+$ data of J. Kirz, J. Schwartz and R.D. Tripp, Phys. Rev. 130, 2481 (1963), using interference effects between a background and a triangle singularity amplitude, using one free complex constant. The fit seems good (I.J.R. Aitchison, private communication finds similar if slightly different best fits) but such results are not convincing proof of the existence of triangle singularities. The problem of widths is illustrated well by I.J.R. Aitchison, Phys. Rev. 133, B 1257 (1964) ; also Saclay preprint.
- 21) I.J.R. Aitchison and C. Kacser, to be presented at A.P.S. Topical Conference on Correlations of Particles in Nuclear Reactions (Gatlinburg, October 1964).
- 22) First suggested by Hwa (Ref. ¹⁵⁾), and studied in detail by I.P. Gyuk and S.F. Tuan, Nuovo Cimento 32, 227 (1964). See also Ref. ³⁾.
- 23) I.J.R. Aitchison, Saclay preprint (1964).
- 24) This has been particularly emphasized by S.F. Tuan (Ref. ³⁾, also ²²⁾).
- 25) The $\rho \pi$ system (A_1) does show one very interesting feature in that $\pi^+ p \rightarrow (\rho^0 \pi^+) p$ is seen, but $\pi^+ p \rightarrow (\rho^+ \pi^0) p$ is not seen, D.D. Carmony et al., to be presented at the 1964 Dubna Conference ; also Aachen-Berlin-CERN collaboration (private communication). This seems to suggest emission of a ρ^0 by the p, and then $\rho^0 \pi^+$ scattering via the simple (Peierls) π exchange. We remark that s_+

(cont.)

12.

25) corresponding to the 2π at threshold from the p is also located at the Peierls value s_+ (for equal masses). The significance of this is obscure, but cf., Ref. 5). Experimentally the momentum transfer to the p is found to be as small as is allowed, i.e., (A_1) production is very peripheral.

26) I am very grateful to S.F. Tuan for bringing this to my notice.

FIGURE CAPTIONS

Figure 1.

- a) A general production process involving the mechanism.
- b) The beam experiment, in which the X is a "true" particle.
- c) The basic triangle graph.

Figure 2.

- a) The curve of possible singularities S in the real (x, z) plane.
- b) The same in the real (s, σ) plane.
- c) Location of $s_{\pm}$ in the complex s plane as σ moves from threshold $\sigma = (Z+V)^2$ to infinity along the real axis. The dashed curve shows a path for which s_{+} is singular near the physical region.

FIG.1