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A POSSIBLE INTERPRETATION OF ULTRA-HIGH ENERGY EXPERIMENTAL DATA

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A B S T R A C T

Arguments are given in favour of the conjecture that experimental results at cosmic ray energies do not indicate the existence of any strong correlation between secondaries. Most of the observed features of jets may be considered as consequences of two, empirically well established facts :

- i) transverse momenta of secondaries are small,
- ii) average inelasticity is small.

These two facts may be, on the other hand, easily understood on the basis of the multiperipheral model.

The origin of the two centre jets has been examined previously ¹⁾. In this work, the properties of the asymmetric jets are discussed. It is shown that, as in the case of the two-centre jets, the introduction of the notion of a fire-ball is not necessary to explain the data. A statistical model taking i) and ii) as basic conditions is constructed; it should help to discriminate between these effects which are only consequences of the modification of the phase space due to i) and ii) and those which may have a deeper dynamical meaning.

I. INTRODUCTION AND SUMMARY

I.1 The aim of this paper is to discuss certain problems connected with the interpretation of experimental data obtained while observing interactions of cosmic ray particles with matter. More explicitly, we shall be interested only in interactions which can be roughly considered as elementary ones, and for definiteness we shall limit ourselves to nucleon-nucleon collisions. Many of the results obtained can be, however, without difficulty generalized to the case of pion-nucleon collisions. This paper may be considered as a continuation of the work done by O. Czyzewski and the author ¹⁾ some time ago (this work will be hereafter referred to as (I)).

I.2 While facing the problem of multiple production, the simplest conjecture to make is probably that, due originally to Fermi ²⁾ : the matrix element responsible for the production process is essentially a constant and it is the dimension of the phase space volume corresponding to a given physical situation which determines the probability of the realization of this situation in the collision process.

Everybody knows that, at present, experimental data definitely disprove the above conjecture. Two important facts of a dynamical nature have been established which prove that the behaviour of the matrix element is of decisive importance for the properties of the production process. These facts are the following :

- i) transverse momenta of secondaries are small ($\langle k_T \rangle \approx 0.3 - 0.4 \text{ GeV}$).
- ii) in the c.m. system, to each of the two incident nucleons there corresponds a final baryon, which is (on the average) much more energetic than any other secondary.

We visualize this in Fig. 1. The "leading" secondary $A'(B')$ can be considered as a "continuation" of the incident particle $A(B)$ - it is, however, frequently in an excited state (for instance N^*). All secondaries other than the "leading" ones shall be called pionization secondaries, and the process of their production, the pionization process. The kinematical separation of "leading" secondaries allows an introduction of the notion of inelasticity which is the fraction of energy transferred to pionization. ii) can be, therefore, formulated also in the following way :

ii') inelasticity is on the average small, of the order of 20 - 30%.

The meaning of i) and ii) is well understood, and these facts are usually associated with peripherality of the interaction. Indeed, a self-consistent model of high-energy interactions, the so-called multiperipheral model proposed by Amati, Fubini and Stanghellini ³⁾, gives a very natural explanation to i) and ii), and this is independent of the detailed dynamics of this model. Within the framework of the multiperipheral model, i) and ii) are not considered as independent facts ; they both follow from the smallness (on the average) of momentum transfers carried by the exchanged virtual object. However, in this paper, for later convenience (and also because experimentally they correspond to different observations) we shall consider i) and ii) as two separate conditions.

I.3 It seems not to be commonly recognized that i) and ii) are quite strong conditions and have numerous consequences. Thus, a dynamical meaning independent of i) and ii) has been given to several effects found experimentally (at energies $E > 10^{11}$ eV), in spite of the possibility that those effects are only consequences of i) and ii). In our opinion, the existence of two-centre jets ⁴⁾⁻⁷⁾ as well as the properties of asymmetric jets ⁸⁾⁻¹⁰⁾, can be understood as following from i) and ii) without introducing the notion of a fire-ball, that is without assuming a very special kind of correlation between secondaries.

In (I) the problem of two-centre jets has been discussed. We shall give here further arguments in favour of the conjecture, that experimental results obtained with cosmic rays until now (which are, however, unfortunately very inaccurate), do not indicate the existence of any strong correlations between secondaries (apart from the occurrence in the final state of "ordinary" resonances like N^* , ρ , etc.) and many of the observed effects simply follow from i) and ii).

Section II is devoted to a brief recapitulation of the results obtained in (I). In Section III a statistical theory taking the conditions i) and ii) of this Section as basic conditions is sketched. In Section IV, the properties of asymmetric jets are discussed. Finally, in Section V, some general remarks concerning the relation of the results of this work to the multiperipheral model are presented.

II. A RECAPITULATION OF PREVIOUS RESULTS

II.1 Before going further, let us recall to the reader the main results of (I). A very simple model was proposed there ; it was assumed that transverse momenta of secondaries are small and that energies of pionization secondaries are bounded from above, this bound being chosen to get a correct average inelasticity. The second of these conditions is, strictly speaking, not a very realistic one. However, it allows one not to specify the energy transferred to pionization in each particular Monte-Carlo jet^{*)}, care being taken only that it should be smaller than the total available energy. This simplified considerably the calculations. Moreover, it was assumed that pionization secondaries are emitted independently from each other. Only effects, independent of the particular values of model parameters (e.g., inelasticity),

*) The Monte-Carlo method has been used to study the statistical properties of the model.

were discussed. It was shown that the existence and properties of two-centre jets follow in a natural way from the model. The discussion was limited to roughly symmetric jets with not too small multiplicity ; the properties of asymmetric jets will be examined in this work. To aid in the proper understanding of these results, let us make a few comments.

The $\log \tan \theta$ (θ is the laboratory angle of emission of a secondary) distribution one gets in the Fermi theory is, in fact, a superposition of $\log \tan \theta$ distributions, each corresponding to a particular transverse momentum. When all transverse momenta are allowed, one gets a roughly Gaussian $\log \tan \theta$ distribution. This is certainly in disagreement with experiment. When, however, as required by i), large transverse momenta are forbidden, one has to superpose only long rectangles ^{*}). The $\log \tan \theta$ distribution then gets a long, flat part. Now, if there is enough energy transferred to pionization to allow, kinematically, the formation of "fire-balls", they are indeed frequently produced. Due to the smallness of transverse momenta, secondaries are, so to say, pushed toward the ends of the $\log \tan \theta$ distribution. Notice that all that has been proved on the basis of experimental data, is the existence of a strong deviation of the experimental $\log \tan \theta$ distribution from the Gaussian shape in the direction of the two-centre model ¹¹⁾. In (I) it was shown that the magnitude and type of this experimentally observed deviation are the same as the magnitude and type of the deviation from the Gaussian shape of a quasi-rectangular $\log \tan \theta$ distribution ^{**)}. The fact that a simple model without correlations can produce all the two-centre effects led to the conclusion that the existence of such dynamical objects like fire-balls does not necessarily follow from the present experimental data.

^{*}) To every fixed, small transverse momentum, there corresponds a rectangular $\log \tan \theta$ distribution.

^{**)} A quasi-rectangular distribution is, of course, a much more "bimodal" one than the Gaussian distribution.

III. STATISTICAL THEORY WITH MODIFIED PHASE SPACE

III.1 Although the multiperipheral model explains qualitatively certain salient features of multiple production, for various reasons it is, at present, unable to provide any realistic numerical predictions to be compared with experiment. From the point of view of the analysis of experimental data it would be, however, useful to have a simple, roughly realistic model, of course less grounded than the field-theoretical models, but able to provide unambiguous numerical answers to some of experimenter's questions. It is a virtue of statistical models that they do not present very important calculational difficulties. We know, however, that the properties of the matrix element are crucial and cannot be ignored. Let us adopt, therefore, the following point of view : we shall assume certain very general features of the matrix element ^{*)}, and we shall use the corresponding assumptions as additional conditions in an otherwise statistical treatment. There is indeed quite an interesting question : what is the smallest number of constraints which need to be put on the Fermi theory to get a picture resembling the present experimental situation ?

We hope that the scheme of calculation we shall present can be helpful in detecting some unknown dynamical features of interactions, as the conventional statistical theory helped to pick out resonant states at lower energies, allowing one to distinguish dynamical effects from phase space effects.

III.2 Formulation of the kinematics

The following notation will be used :

$p^{(A)}, p^{(B)}$	-	4-momenta of incident particles,
$p^{(A')}, p^{(B')}$	-	4-momenta of final baryons,
q	-	total 4-momentum of pionization secondaries,
E	-	total energy of the collision in the lab. system,
m, μ	-	nucleon and pion masses.

*) Taking the general predictions of the multiperipheral model as a hint.

6.

We shall use the following two "inelasticity coefficients" defined in the c.m. system :

$$\kappa_1 = (p_0^{(A)} - p_0^{(A')}) / p_0^{(A)} \quad (1)$$

$$\kappa_2 = (p_0^{(B)} - p_0^{(B')}) / p_0^{(B)}$$

We have in the c.m. system ^{*)}

$$q_0 = p_0^{(A)} (\kappa_1 + \kappa_2) \quad (2)$$

$$q_L \approx p_0^{(A)} (\kappa_1 - \kappa_2)$$

since

$$p_L^{(A')} \gg p_T^{(A')}, \quad p_L^{(B')} \gg p_T^{(B')} \quad (3)$$

The total "effective mass" of pionization secondaries is therefore

$$M \approx \sqrt{2 m E \kappa_1 \kappa_2} \quad (4)$$

The momenta of individual, first generation ^{**) pionization secondaries will be denoted by $k^{(i)}$}

^{*)} Subscripts "L" and "T" will denote the longitudinal and transverse component of a 3-dimensional vector with respect to the direction of the particle (A).

^{**) An isobar produced in the collision will be called first generation secondary, while its decay products will be called second generation secondaries.}

$$\sum_i k^{(i)} = q \quad (5)$$

It is easy to verify that for E sufficiently high, we have in the lab. system :

$$q_0^{\text{lab}} \approx q_L^{\text{lab}} \approx k_1 E \quad (6)$$

The inelasticity in the c.m. system is $(k_1 + k_2)/2$; k_1 and k_2 play the role of the inelasticity in the laboratory and antilaboratory system, respectively. The Lorentz factor for the transformation relating the lab. system to the pionization rest frame is

$$\gamma_P = \frac{q_0^{\text{lab}}}{M} \approx \sqrt{\frac{E}{2m}} \cdot \sqrt{\frac{k_1}{k_2}} \quad (7)$$

Denoting by γ_c the corresponding Lorentz factor for the c.m. system, we get

$$\gamma_P \approx \gamma_c \sqrt{k_1/k_2} \quad (8)$$

III.3 Basic assumptions

Let us formulate now the basic assumptions of our statistical model. We shall assume that first generation secondaries are produced independently from each other. Furthermore, we shall assume that conditions i) and ii) of Section I are satisfied. The distributions of k 's and of transverse momenta of secondaries have to be assumed ad hoc - they must be taken from experiment or deduced from a more sophisticated theory. In our model these distributions represent the influence of the matrix element on the production process. The

probability to find n first generation pionization secondaries in a given energy momentum state is therefore (for the justification of this form see (I))

$$dp(n) = \text{const.} \delta_4(q - \sum_i k^i) \cdot \frac{a^n}{n!} \times \\ \times \prod_i f(k_T^i) \mathcal{V}(k_0^i) \delta(k_i^2 - \mu_i^2) d_4 k^i \quad (9)$$

$f(k_T)$ is here normalized as follows

$$\int f(k_T) k_T dk_T = 1, \quad (10)$$

a is a constant which will be determined later. (9) permits, in principle, the calculation of the distribution of any desired quantity. In order to use it effectively one has to give a simple, approximate method of calculation of phase space integrals of the type

$$\Omega_n(Q) = \int \delta_4(Q - \sum_i k^i) \times \\ \times \prod_i f(k_T^i) \mathcal{V}(k_0^i) \delta(k_i^2 - \mu_i^2) d_4 k^i \quad (11)$$

III.4 Calculation of phase space integrals

The method of calculation of the phase space integral $\Omega_n(Q)$ presented below is a straightforward generalization of the method proposed by Lurçat and Mazur¹²⁾. We shall not enter here into the details of the derivation, which can be found in Ref. ¹²⁾ (for the convenience of the reader, we shall keep the notation used by Lurçat and Mazur). Let us define

$$\Phi(\alpha_\nu) = \int e^{-\alpha_\nu Q^\nu} \Omega_n(Q) d_4 Q \quad (12)$$

Of course

$$\Phi(\alpha_\nu) = \prod_{i=1}^n \varphi_{(i)}(\alpha_\nu) \quad (13)$$

where

$$\varphi_{(i)}(\alpha_\nu) = \int e^{-\alpha_\nu k^\nu} f(k_T) \mathcal{V}(k_0) \delta(k_\nu^2 - \mu_{(i)}^2) d_4 k \quad (14)$$

Lurçat and Mazur introduce an auxiliary function

$$\Omega_n^{(\alpha)}(Q) = \frac{\Omega_n(Q) e^{-\alpha_\nu Q^\nu}}{\Phi(\alpha_\nu)} \quad (15)$$

which is positive definite and normalized to unity. By virtue of the relation

$$\Omega_n^{(\alpha)}(Q) = \int \delta_4(Q - \sum_i k^i) \times \\ \times \prod_i \left\{ \frac{e^{-\alpha_\nu k^\nu} f(k_T^i) \mathcal{V}(k_0^i) \delta(k_i^2 - \mu_i^2) d_4 k^i}{\varphi_{(i)}(\alpha_\nu)} \right\} \quad (16)$$

$\Omega_n^{(\alpha)}(Q)$ can be considered as a frequency function of a random variable Q which is a sum of n independent random variables $k^{(i)}$. This permits one to

approximate $\Omega_n^{(\alpha)}(Q)$ for large n , by Edgeworth series ¹³⁾. It is shown in Ref. ¹²⁾ that for a conveniently chosen 4-vector α_ν this approximation may be a fairly good one even for small n 's (for given n and Q one chooses α_ν so as to make the average of the frequency function $\Omega_n^{(\alpha)}(Q)$ as close as possible to Q).

In our case $\varphi(\alpha_\nu)$, due to invariance reasons, is a function of $\alpha = \sqrt{\alpha_\nu^2}$ and α_T

$$\varphi(\alpha, \alpha_T) = \pi \int dk_T k_T f(k_T) I_0(\alpha_T k_T) \times \\ \times K_0(\sqrt{(\alpha^2 + \alpha_T^2)(k_T^2 + \mu^2)}) \quad (17)$$

The cumulants of the frequency function $\Omega_n^{(\alpha)}$ can be found from the logarithmic derivatives of $\Phi(\alpha_\nu)$. Following Lurçat and Mazur we define

$$A_\nu = - \frac{\partial \log \Phi(\alpha_\nu)}{\partial \alpha_\nu} \equiv \int Q_\nu \Omega_n^{(\alpha)}(Q) d_4 Q \\ B_{\nu\mu} = \frac{\partial^2 \log \Phi(\alpha_\nu)}{\partial \alpha_\nu \partial \alpha_\mu} \equiv \int (Q_\nu - A_\nu)(Q_\mu - A_\mu) \Omega_n^{(\alpha)}(Q) d_4 Q \quad (18)$$

We have

$$A_\nu = - \sum_i \left\{ \frac{\partial \log \varphi_i}{\partial \alpha} \frac{\alpha_\nu}{\alpha} + \frac{\partial \log \varphi_i}{\partial \alpha_T} \varepsilon_\nu \right\} \\ B_{\nu\mu} = \sum_i \left\{ \frac{\partial^2 \log \varphi_i}{\partial \alpha^2} \frac{\alpha_\nu \alpha_\mu}{\alpha^2} + \frac{\partial^2 \log \varphi_i}{\partial \alpha \partial \alpha_T} \left(\frac{\alpha_\nu \varepsilon_\mu + \varepsilon_\nu \alpha_\mu}{\alpha} \right) \right. \\ \left. + \frac{\partial \log \varphi_i}{\partial \alpha} \left(\frac{\alpha^2 g_{\nu\mu} - \alpha_\nu \alpha_\mu}{\alpha^3} \right) + \frac{\partial^2 \log \varphi_i}{\partial \alpha_T^2} \varepsilon_\nu \varepsilon_\mu + \frac{\partial \log \varphi_i}{\partial \alpha_T} \varepsilon_{\nu\mu} \right\} \quad (19)$$

where

$$\epsilon_\nu = \frac{\partial \alpha_T}{\partial \alpha^\nu}, \quad \epsilon_{\nu\mu} = \frac{\partial^2 \alpha_T}{\partial \alpha^\nu \partial \alpha^\mu} \quad (20)$$

and $g_{\nu\mu}$ is the metric tensor. We shall now put

$$\alpha_\nu = \beta_\nu \quad (21)$$

where β_ν is specified by the following conditions

$$\begin{aligned} A_\nu &= Q_\nu & \nu &= 1, 0 \\ \frac{\partial \beta_T}{\partial \beta^\nu} Q^\nu &= 0 \\ \beta_T &\rightarrow 0 \end{aligned} \quad (22)$$

One can then verify that

$$\begin{aligned} A_\nu &= - \left\{ \sum_i \left[\frac{\partial \log \varphi_i}{\partial \alpha} \frac{\alpha_\nu}{\alpha} \right] \right\} \alpha_\nu = \beta_\nu \\ B_{\nu\mu} &= \left\{ \sum_i \left[\frac{\partial^2 \log \varphi_i}{\partial \alpha^2} \frac{\alpha_\nu \alpha_\mu}{\alpha^2} + \right. \right. \\ &\quad \left. \left. + \frac{\partial \log \varphi_i}{\partial \alpha} \left(\frac{\alpha^2 g_{\nu\mu} - \alpha_\nu \alpha_\mu}{\alpha^3} \right) + \frac{\partial^2 \log \varphi_i}{\partial \alpha_T^2} \epsilon_\nu \epsilon_\mu \right] \right\} \alpha_\nu = \beta_\nu \end{aligned} \quad (23)$$

By virtue of the Central Limit Theorem, we have

$$\begin{aligned} \Omega_n^{(\alpha)}(Q) &= \frac{(2\pi)^2}{(\text{Det } B)^{1/2}} \exp \left[-\frac{1}{2} (B^{-1})_{\nu\mu} (Q^\nu - A^\nu)(Q^\mu - A^\mu) \right] \times \\ &\quad \times \left\{ 1 + O\left(\frac{1}{\sqrt{n}}\right) \right\} \end{aligned} \quad (24)$$

The next term of the series can be calculated easily. It contains higher derivatives of $\log \Phi(\alpha_\nu)$.

Let us assume for simplicity all masses to be equal

$$\mu_i = \mu, \quad i = 1, \dots, n \quad (25)$$

(23) gives ($Q_1 \equiv Q_L$)

$$\left\{ - \frac{\partial \log \varphi}{\partial \alpha} \right\}_{\alpha=\beta} = \frac{W}{n}, \quad W = \sqrt{Q_0^2 - Q_L^2} \quad (26)$$

From (22) and (23) it follows that

$$\begin{aligned} [\text{Det } B]_{\alpha_\nu = \beta_\nu} &= \left\{ \frac{n^4}{\alpha^2} \left(\frac{\partial \log \varphi}{\partial \alpha} \right)^2 \frac{\partial^2 \log \varphi}{\partial \alpha^2} \times \right. \\ &\quad \left. \times \left[-\frac{1}{\alpha} \frac{\partial \log \varphi}{\partial \alpha} + \frac{\partial^2 \log \varphi}{\partial \alpha^2} \right] \right\}_{\alpha_\nu = \beta_\nu} \\ &= \frac{\beta Q_T^2}{W} \quad (27) \end{aligned}$$

We get finally

$$\begin{aligned} \Omega_n(W, Q_T) &= \frac{(2\pi)^2 \beta [\varphi(\beta)]^n}{W} \left[\frac{nh(\beta)}{2} \left(\frac{W}{\beta} + \right. \right. \\ &\quad \left. \left. + nh(\beta) + n\mu^2 - \frac{W^2}{n} \right) \right]^{-1/2} \exp\left(\beta W - \frac{\beta Q_T^2}{2W}\right) \{1 + \dots\} \quad (28) \end{aligned}$$

where

$$\varphi(\beta) = \pi \int dk_T k_T f(k_T) K_0(\beta \sqrt{k_T^2 + \mu^2})$$

$$h(\beta) = \frac{\pi}{\varphi} \int dk_T k_T^3 f(k_T) K_0(\beta \sqrt{k_T^2 + \mu^2}) \quad (29)$$

$\beta = \beta(n, W)$ is determined by the equation

$$\frac{\int dk_T k_T f(k_T) \sqrt{k_T^2 + \mu^2} K_1(\beta \sqrt{k_T^2 + \mu^2})}{\int dk_T k_T f(k_T) K_0(\beta \sqrt{k_T^2 + \mu^2})} = \frac{W}{n} \quad (30)$$

The multiplicity distribution for given M and q_T is

$$p(n) = \text{const} \frac{a^n}{n!} \Omega_n(M', q_T), \quad (31)$$

$$M' = \sqrt{q_0^2 - q_L^2} \approx M$$

The energy momentum distribution for one particle takes the form

$$dp = \text{const} \frac{f(k_T) dk_T^2 dk_L}{k_0} \left\{ \sum_n \frac{a^n}{n!} \Omega_n(M', q'_T) \right\}, \quad (32)$$

$$M' = \sqrt{(q_0 - k_0)^2 - (q_L - k_L)^2}$$

$$q'_T = (q - k)_T$$

For k_0 not too large in the pionization rest frame ($k_0 \ll M$) one gets

$$dp \approx \text{const} \frac{f(k_T) dk_T^2 dk_L}{k_0} \quad (33)$$

since energy conservation is not very important (compare (I)). For large k_0 the factor in brackets deforms and cuts the energy distribution.

III.5 The ultra-relativistic limit

Assume that

$$\frac{W}{n} \rightarrow \infty \quad (34)$$

From (30) we get

$$\beta \rightarrow 0 \quad (35)$$

One has up to the terms vanishing as $\beta \rightarrow 0$

$$\begin{aligned} K_0(\beta \sqrt{k_T^2 + \mu^2}) &\rightarrow -\log \frac{\beta}{2} \sqrt{k_T^2 + \mu^2} - \gamma \\ K_1(\beta \sqrt{k_T^2 + \mu^2}) &\rightarrow \frac{1}{\beta \sqrt{k_T^2 + \mu^2}} \end{aligned} \quad (36)$$

(γ is the Euler constant).

Equation (30) can now be written

$$\frac{1}{\beta \log \frac{2c}{\beta}} = \frac{W}{n} \quad (37)$$

or

$$\frac{1}{\beta} = \frac{W}{n} \log 2c \frac{1}{\beta} \quad (38)$$

where

$$c = \frac{1}{\mu} \exp \left[-\gamma - \frac{1}{2} \int dk_T k_T f(k_T) \log \left(1 + \frac{k_T^2}{\mu^2} \right) \right] \quad (39)$$

For $W/n \rightarrow \infty$ we have an asymptotic solution

$$\frac{1}{\beta} \approx \frac{W}{n} \log \frac{2cW}{n} \quad (40)$$

Inserting (40) in (31) we get a roughly Poisson-like multiplicity distribution with the approximate average

$$\langle n \rangle \approx \pi a \log 2cM \quad (41)$$

IV. PROPERTIES OF ASYMMETRIC JETS

IV.1 Some general qualitative arguments

We call asymmetric jets for which the pionization rest frame ($\vec{q} = 0$) does not coincide with the c.m. system. They were discovered experimentally by the Moscow group^{8),9)} at energies of the incident particles of the order of 10^{11} eV. The evidence for the existence of still more asymmetric events at energies above 10^{12} eV has been given in the work of Gierula and Miesowicz¹⁰⁾.

Asymmetric jets offer, in our opinion, a good illustration of the ideas expressed in this paper and we shall discuss their properties in more detail. We shall start with a very general qualitative discussion.

Assume that the average number of charged particles corresponding to a given momentum interval is given (in the lab. system) by

$$dn = f(k_T) dk_T \frac{dk_L}{k_L}, \quad k_L^{\min} \lesssim k_L \lesssim k_L^{\max} \quad (42)$$

This is (apart from the cut-off factor replaced here by the inequality for k_L) the high-energy limit of (32). (42) is a general result one gets in the Amati, Fubini and Stanghellini model (this result can be obtained starting from even more general hypotheses).

From (42) we get the average multiplicity

$$\langle n \rangle \approx \text{const} \log(k_L^{\max}/k_L^{\min}) \quad (43)$$

Since, roughly, $k_L^{\max}/k_L^{\min} \sim M$, this is essentially the same as (41). The distribution of the $\log \tan \theta$ variable corresponding to (42) is (for sufficiently large $k_L^{\max}$) well approximated by a rectangle with a length $\log(k_L^{\max}/k_L^{\min})$. The average dispersion of the $\log \tan \theta$ plot is thus roughly

$$\langle \sigma \rangle \approx \text{const} \log(k_L^{\max}/k_L^{\min}) \quad (44)$$

Therefore

$$\frac{\langle \sigma \rangle}{\langle n \rangle} \approx \text{const} \quad \text{for large } M \quad (45)$$

For small M (45) is no longer true. It will be seen later that, for small M , the angular distribution is roughly isotropic and the average energy per secondary M/n is ≈ 0.4 GeV. This leads to

$$\langle \sigma \rangle \approx 0.4 \quad \text{for small } M \quad (46)$$

The striking consequences of the modification of the phase space, following from assumptions i) and ii), become apparent only for sufficiently large M/n ^{*)}. For sufficiently small M , we have $M/\langle n \rangle < \langle k_T \rangle$; the condition i) is then evidently not very important and those predictions of our model which concern only pionization, practically coincide with the predictions of the conventional Fermi statistical theory, as applied to pionization. In order to give a more precise meaning to the expression "small M " we have to find the $M - \langle n \rangle$ correlation and see how $M/\langle n \rangle$ depends on M .

IV.2 A simple application of the statistical theory of Section III

We shall apply the results of Section III to the calculation of the dependence of the average multiplicity of charged pions on M_c , the total effective mass of charged pionization secondaries

$$M_c = \sqrt{\left(\sum_i k^{(i)'} \right)^2} \quad (47)$$

$(k^{(i)'})$ are the 4-momenta of charged second generation secondaries).

*) In order to observe the effects of "one dimensionality" imposed by i), it is sufficient to have a large M since the average n increases slowly. With increasing M the fraction of events for which M/n is small decreases relatively quickly.

In general, there is a certain amount of correlation between second generation secondaries, namely between those which come from the decay of the same mother isobar. This correlation has, however, not much importance if one considers only charged secondaries.

The result of the numerical calculation is shown in Fig. 2. The formula (31) was applied. We replaced M by M_0 and put for simplicity $q_T = 0$. (31) does not depend strongly on the exact form of $f(k_T)$ which appears only under the integrals. We assumed for $f(k_T)$ the form

$$f(k_T) = \frac{36}{m^2} e^{-\frac{6k_T}{m}} \quad (48)$$

which corresponds to the distribution of transverse momentum

$$dp = \frac{4k_T}{\langle k_T \rangle^2} e^{-\frac{2k_T}{\langle k_T \rangle}} dk_T \quad \text{with} \quad \langle k_T \rangle = \frac{m}{3} \quad (49)$$

suggested by accelerator experiments.

In order to get agreement with the data given by Guseva et al. we put

$$a = 8.7/\pi \quad (50)$$

By this assignment one fixes the average multiplicity for one value of the energy.

IV.3 Discussion of Guseva et al. and Gierula and Miesowicz results

As is seen from Fig. 2, the dependence of $\langle n \rangle$ on M is very well reproduced by our calculations. Taking the distribution of K from the Moscow results one gets a good fit to the multiplicity distribution. Starting from (32), one can obtain the momentum distribution and the angular distribution. It does not seem, however, to be worthwhile - for the energies discussed (above 10^{11} eV) - to make very exact numerical calculations. One should use rather rough input data and compare the results with very inaccurate experimental results. Even in the presence of a discrepancy, it would be difficult to make any definite conclusion about the significance of this discrepancy, due to large experimental errors. On the other hand, a qualitative or semi-quantitative discussion is, in our opinion, very instructive.

From Fig. 2, we see that for small M ($M \lesssim 5$ GeV, say) the ratio $M/\langle n \rangle$ changes very slowly and is close to $\langle k_T \rangle$. In this case the condition i) does not imply any important modification of the phase space and the predictions of our model are, in fact, the same as those of the Fermi theory. One can expect, thus, an isotropic angular distribution of pions in the pionization rest frame. With increasing M , the probability increases for a pionization secondary to have a large momentum. Since its transverse momentum is bounded, increase of M will be accompanied by an increase of anisotropy. According to (4) and (8), the magnitude of M is determined by the magnitude of the product $K_1 K_2$, while the ratio K_1/K_2 determines the anisotropy. At the energy of the Moscow group experiment ($E \approx 2 \cdot 10^{11}$ eV), in order to observe a distinct anisotropy in angular distribution, both K_1 and K_2 must be relatively large (of course $K_i < 1$). Thus, anisotropic jets should be almost symmetric. On the other hand, for an asymmetric jet, at least one of the K_i must be small. Therefore, M is small and the emission of pions in the pionization rest frame is isotropic.

All that is indeed observed in the Guseva et al. experiment. We do

not see, however, the necessity of introducing several dynamical mechanisms responsible for the dependence of the behaviour of the collision on the inelasticity parameters K_1 and K_2 as proposed in Ref. 8).

Consider now Gierula and Miesowicz results. They examined low multiplicity jets ($n_s < 10$) found in emulsion, mostly by tracing back electromagnetic cascades (due to neutral secondary pions). The energies of these events are thus $> 10^{12}$ eV. The scanning method was evidently very efficient in finding forward asymmetric jets, while backward asymmetric jets (and to some extent also symmetric jets, with very small inelasticity) were missed.

For most of examined jets, the anisotropy parameter σ has been found to be close to 0.4 (isotropy in the pionization rest frame) and the energy when estimated by the Castagnoli method frequently turned out to be anomalously large (for instance, $5.4 \cdot 10^5$ GeV, $3.2 \cdot 10^5$ GeV, $1.8 \cdot 10^5$ GeV *).

At energies $E > 10^{12}$ eV, M is on the average large. The abundance of strongly anisotropic jets is a manifestation of this fact. However, selecting small multiplicity events, one gets a sample characterized by small average M , due to the weak independence of $\langle n \rangle$ on M **). In order to get a large change of $\langle n \rangle$ one needs a large change of M . For a very high energy, in order to have small M , one must have $K_1 K_2 \ll 1$. Since $K_i < 1$, one frequently gets $K_1/K_2 \ll 1$ or $K_1/K_2 \gg 1$ (i.e., strong

*) Notice that the energy spectrum of the primary cosmic radiation drops quite rapidly and it is not probable to find a low multiplicity jet with energy $> 10^{14}$ eV.

**) It is not very probable to have a low multiplicity jet in the case when M is large. For instance, according to our calculations, the probability that the number of charged pions is < 10 is $\approx 10^{-3}$, for $M = 30$.

asymmetry, a large fraction of the energy lost by incident particles goes over to the motion of pionization as a whole). As was already noticed, the situation where $K_1/K_2 \gg 1$ has been especially favoured by the scanning conditions. This leads to very large γ_p according to (8). Since the Castagnoli method gives us γ_p rather than γ_c , anomalously large energies have to be observed when this method is applied. For fixed inelasticity in the lab. system K_1 and $K_1 > K_2$ (forward asymmetric jet) one gets from (41) and (8)

$$\frac{\langle n \rangle_{K_2=K_1} - \langle n \rangle_{K_2 \ll K_1}}{\langle n \rangle_{K_2=K_1}} \approx \frac{\log \gamma_p / \gamma_c}{\log \gamma_c} \quad E \rightarrow \infty \quad (51)$$

Due to the logarithmic dependence of $\langle n \rangle$ on M , one expects asymptotically, for relatively low multiplicity jets, very large and increasing deviations of the γ_p / γ_c ratio from unity. With decreasing energy, we enter the region of smaller average M . Here, $\langle n \rangle$ depends more strongly on M and the deviations of γ_p / γ_c from unity become on the average smaller. This is indeed observed. Among low multiplicity jets investigated by Gierula and Miesowicz, the energy, when estimated by the Castagnoli method, has been too large by a factor of $\approx 10 - 100$, while in the Moscow experiment, the corresponding factor has been $\approx 4 - 6$ for most asymmetric jets.

For very asymmetric jets, where M is small, the anisotropy should correspond to an isotropic emission with average energy per pion ≈ 0.4 GeV. Thus we expect $G \approx 0.4$, in agreement with experiment.

V. GENERAL REMARKS

V.1 The relation of the results of this paper to the multiperipheral model

It is worthwhile to make some comments on the connection between our results (especially the results of (I)) and the multiperipheral model.

Mathematically, the most important feature of the multiperipheral model is the factorization of the collision amplitude and the smallness of momentum transfers carried by the exchanged object. This leads ^{*)} to the result that the situation where invariant momentum transfers carried by the virtual particle are equal, is roughly the most probable one. This, in turn, means that the uniform $\log \tan \theta$ plot roughly corresponds to the most probable configuration of emission angles ^{**)}

*) Private communication from Professor S. Fubini.

**) The energy momentum conservation in the vertices of a multiperipheral graph reads (Δ 's are momentum transfers)

$$\Delta_i + k_i = \Delta_{i+1} \quad (a)$$

Introduce the hyperbolic angles η and ξ

$$\begin{aligned} k_0^i &= m \operatorname{ch} \eta^i, & \Delta_0^i &= \Delta \operatorname{sh} \xi^i \\ k_L^i &= m \operatorname{sh} \eta^i, & \Delta_L^i &= \Delta \operatorname{ch} \xi^i \end{aligned}$$

Neglecting, for simplicity, transverse components one gets from (a)

$$\eta_{i+1} - \eta_i = \xi_{i+1} - \xi_i = \operatorname{ch}^{-1} \left(1 + \frac{m^2}{2\Delta^2} \right) = \text{const}$$

In the lab. system where $k_L \gg m$, all η 's are very large. Therefore

$$\log \tan \theta^{(i+1)} - \log \tan \theta^{(i)} \approx \log k_L^i / k_L^{i+1} \approx \text{constant}.$$

In order to get frequent formation of two centres, however, very large deviations from the uniform $\log \tan \theta$ plot must be possible and probable. This is realized when the correlations between first generation secondaries, at least the correlations between those secondaries which are not too close to the leading ones, are weak. The amount of correlation between produced particles depends, in the multiperipheral model, on the detailed structure of the collision amplitude. From the point of view of this model, the results of (I) and of this paper (if they are correct) are equivalent to an empirical establishment of the following fact :

a multiperipheral mechanism is compatible with present experimental data if it allows large deviations from the most probable configuration.

This means that the maximum of the modulus of the collision amplitude, corresponding to a uniform $\log \tan \theta$ distribution, must be a very broad one. This is an empirical condition to be satisfied by the terms into which the collision amplitude factorizes.

V.2 The properties of jets, which follow from the conditions i) and ii) of Section I, together with the assumption of weak correlation between secondaries, clearly coincide with the experimental results on the two-centre jets as well as those on the so-called one fire-ball jets found by the Moscow group and by Gierula and Miesowicz. We do not claim that the correlations do not exist. We think, however, that within the present experimental accuracy, there is no empirical reason to believe they are strong.

Let us stress here, that what we propose is not a theory in the proper sense of the word. We have not proposed any model which should give a more or less complete description of multiple production. What we are doing is only a phenomenology free of intuitive pictures and based on clear-cut, mathematically simple, assumptions.

It is obvious that the phase space factor plays an incomparably more important role in multiple production than at low energies. Every limitation imposed on momenta of secondaries, every correlation between particles, induces a modification of the available phase space and has consequences which, at first sight, look like completely new dynamical effects. Thus all the consequences of known facts have to be determined and proved not to coincide with the observed effect before we conclude that this effect brings some new information on the dynamics of the interaction. A simple way of doing such a check is to construct a statistical model where the known facts are treated as constraints.

The number and quality of assumptions in the kind of investigation we propose is determined by the experimental situation. The more we shall learn about multiple production, the more "memory" it will be necessary to attribute to secondaries. In this paper (and in (I)), we assumed that pionization secondaries "remember" the direction of motion of the incident particle only ^{*)}. This is at present probably sufficient as far as ultra-high energies are concerned, but almost certainly will prove to be insufficient at accelerator energies where the measurements are much better.

*) Due to this assumption f 's depend on k_T only. Assuming that pionization secondaries "remember", for instance, the energy momentum of pionization, one gets a correlation between the transverse momentum and the energy of a secondary since, now, f 's may depend also on $k_y q^\nu / M$ ($= k_0$ on the pionization rest frame).

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FIGURE CAPTIONS

- Figure 1 The figure visualizes the kinematical separation of "leading" secondaries A' and B' from the rest of produced particles, called pionization secondaries.
- Figure 2 The dependence of the average multiplicity of charged pions on the total effective mass of charged pions, calculated using the statistical model of Section III. The results of the calculation are compared with the data obtained by the Moscow group.

FIG. 1

$\langle ns \rangle$ —average multiplicity of charged secondaries

Mc—effective mass of all charged pions (GeV)

FIG. 2