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ON THE ITERATION PROCEDURE IN THE PERATIZATION
OF WEAK LEPTONIC INTERACTIONS

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A B S T R A C T

The field theory of weak interactions as formulated by
Feinberg and Pais leads for the sum of uncrossed ladder graphs in
weak leptonic scattering to the solution of two singular integral
equations. A slight modification, which seems to be in the spirit
of the whole approach, makes the full solution of one of them pos-
sible. In perturbation expansions in the coupling constant g^2 ,
the limiting process for the invariant cut-off is tested together
with the Feinberg-Pais iteration procedure of solving the integral
equation. Their result is reproduced up to second order in an
unambiguous way.

1. INTRODUCTION

The discoveries of many new resonances, some of which have spin larger than $1/2$, has encouraged recent investigations of the question whether field theories which are unrenormalizable in the sense of perturbation theory, may give finite answers if appropriate limiting processes are invented for the whole solution. Supported by similar effects in many-body problems Lee and Yang¹⁾ assume that, e.g., in the electrodynamics of the charged vector boson such a limit exists and they arrive at interesting conclusions. Another very important result is the one obtained by Feinberg and Pais²⁾ that there is really a class of diagrams, namely the uncrossed ladder graphs in weak leptonic reactions with an intermediate vector boson, which give a finite contribution of the order of the Born approximation. They introduce a cut-off Λ and define the amplitude M by the limit

$$M(p) = \lim_{\Lambda \rightarrow \infty} M(p, \Lambda) \quad (1.1)$$

which should be performed as the very last step of calculation. (1.1) means that the essential singularity for $\Lambda = \infty$ is approached in a certain way.

It is, of course, not obvious that a solution really exists which has a finite limit for $\Lambda \rightarrow \infty$. Soluble field theoretical models can be constructed in which (1.1) for the sum of ladder graphs, leads to a divergent result³⁾. It is possible that only in a definite class of theories the prescription (1.1) gives something finite. This may even be the case for theories where the sum of ladder graphs diverges.

We concentrate here on the problem for the class of ladder graphs only as it has been considered by Feinberg and Pais. These authors establish an iteration scheme for calculating this amplitude. In this scheme one takes the part which would give the "most divergent diagrams" in usual perturbation

theory, and treats the rest as perturbation in the (small) coupling constant. Now the question arises if the infinite sum of the less divergent (or finite) parts may not add up as well to something finite, which is of the same order of magnitude, as the contribution of the "most divergent" one. The problem can be expressed also in another way : according to the fundamental assumption of regularization with the cut-off Λ , the limit (1.1) should be taken in principle after having done a calculation of the whole amplitude to all orders of any iteration scheme. The problem is then to establish the soundness of performing the limit (1.1) separately in every order of a given iteration scheme ⁴⁾.

In order to check this, we would need the whole solution of the problem of Feinberg and Pais ²⁾. We do not want to repeat their calculation but, referring entirely to the notation of FP II ⁵⁾, we simply write down the integral equation for the two body leptonic reaction, in the ladder approximation, for the amplitude $M_{\mu\nu}^{\pm}$ defined by

$$M^{\pm} = M^{\text{allowed}} \pm M^{\text{forbidden}} \quad (1.2)$$

$$M^{\pm} = M_{\mu\nu}^{\pm} \gamma_{\mu}^{(1)} (1 + \gamma_5^{(1)}) \gamma_{\nu}^{(2)} (1 + \gamma_5^{(2)}) \quad (1.3)$$

M^{allowed} contains the ladder graphs with an odd number of rungs, $M^{\text{forbidden}}$ contains the ladder graphs with an even number of rungs. The integral equations in the low energy limit (incoming four momenta equal to zero) are (c.f. FP II (2.17), (2.20))

$$\begin{aligned} T^{\pm}(p, \Lambda) = & - \frac{ig^2(4 + p^2/m^2)}{p^2 + m^2} \pm \\ & \pm \frac{4ig^2\Lambda^4}{(2\pi)^4} \int \frac{d^4p' (4 + (p'-p)^2/m^2) T(p', \Lambda)}{p'^2 (p'^2 + \Lambda^2)^2 [(p'-p)^2 + m^2]} \end{aligned} \quad (1.4)$$

$$\beta^{\pm}(p, \Lambda) = - \frac{ig^2}{m^2(p^2 + m^2)} \pm \frac{4ig^2\Lambda^4}{3(2\pi)^4 m^2 p^4} \int d^4 p' \frac{\{3[p^2 - (pp')]^2 + (pp')^2 - p^2 p'^2\} \beta^{\pm}(p', \Lambda)}{(p'^2 + \Lambda^2)^2 [(p - p')^2 + m^2]} \quad (1.5)$$

$T^{\pm}$ is the trace of $M_{\mu\nu}^{\pm}$

$$M_{\mu\nu}^{\pm} = \alpha^{\pm} g_{\mu\nu} + \beta^{\pm} p_{\mu} p_{\nu} \quad (1.6)$$

The subject of this paper is to find a closed solution of (1.4). This is indeed possible after a slight change in the integral equation which is entirely in the spirit of the problem, namely to use the fact that the high frequencies give the most important contributions. Performing then the limit (1.1) and making a perturbation expansion in the coupling constant afterwards, it is possible to check the equivalence with the result of FP II. Unfortunately, (1.5) is not so easy to attack (c.f. footnote ¹⁰) but a verification of the iteration procedure in (1.4) would certainly make one feel more confident in doing the iteration in (1.5) similarly. If $\beta^{\pm}$ in (1.6) turns out to be finite for $p \rightarrow 0$, (1.4) gives all information for the low energy limit anyhow.

In the second chapter, we give the solution of the modified equation ; chapter 3 contains the limit (1.1), whereas the fourth one is devoted to the proof that the two lowest orders of a perturbation expansion in g^2 coincide with the result of FP II in their iteration scheme, the famous

$$\begin{aligned} \alpha^{\pm}(0) &= \frac{T^{\pm}(0)}{4} = \frac{1}{4} \lim_{\Lambda \rightarrow \infty} T^{\pm}(0, \Lambda) = \\ &= - \frac{3}{4} \frac{ig^2}{m^2} \left(1 \mp \frac{3g^2}{4\pi^2} \right) \end{aligned} \quad (1.7)$$

if the full solution is developed in a series first and then the limit is performed which corresponds essentially to the way the iteration is done. But also a development of M after having assumed $\Lambda \rightarrow \infty$ gives the same result. This is by no means a trivial statement since the essential singularity at $\Lambda = \infty$ induces in general a certain ambiguity which, however, disappears just in the problem under consideration.

2. SOLUTION OF THE TRACE EQUATION

Because of the analytic properties of the propagators in (1.4), we are allowed to transform the integration over p'_0 into one over the imaginary axis of the complex p'_0 plane, provided T has similar properties. An inspection of our solution will make this clear. (1.4) becomes then

$$T = -\frac{ig^2(s+4m^2)}{m^2(s+m^2)} + \frac{\lambda}{3} \frac{\Lambda^4}{\pi^2 m^2} \int \frac{[d^4 p'] T(s', \Lambda)}{s' (s' + \Lambda^2)^2} +$$

$$+ \frac{\lambda \Lambda^4}{\pi^2} \int \frac{[d^4 p'] T(s', \Lambda)}{s' (s' + \Lambda^2)^2 [(p'-p)^2 + m^2]} \quad (2.1)$$

where

$$s = p^2$$

$$[d^4 p'] = dp'_1 dp'_2 dp'_3 dp'_4$$

with $-\infty \leq p_\mu \leq +\infty$ real, $p_4 = ip_0$, and

$$\lambda = \mp \frac{3g^2}{4\pi^2} \quad (2.2)$$

In the definition of λ we have to take the signs minus or plus for T^+ and T^- respectively. The first advantage of (2.1) is that it represents in this form no longer an hyperbolic but an (inhomogeneous) elliptic boundary problem in a Euclidean $R_4^{(6),7)}$.

Now we may ask for the consequences of putting $m = 0$ in the second integral. Clearly we expect no influence on the high frequency contributions which are significant for the solution ; however, we get an infra-red divergence

at low momenta. Hence m plays in this integral mainly the role of an infra-red cut-off. There should be no essential change if we cut the low frequencies differently, e.g., by replacing s' in the denominator which stems from the lepton propagators with mass zero by $s' + \mu^2$. We give the cut-off a different name in order to be able to show that the result in lowest order perturbation theory does not depend on μ . For the next order it is sufficient to take $m = \mu$.

In this philosophy the equation (2.1) is replaced by our "model equation" ⁸⁾

$$T = - \frac{ig^2(s+4m^2)}{m^2(s+m^2)} + \frac{\lambda \Lambda^4}{3\pi^2 m^2} \int \frac{[d^4 p'] T(s', \Lambda)}{s' (s' + \Lambda^2)^2} +$$

$$+ \frac{\lambda \Lambda^4}{\pi^2} \int \frac{[d^4 p'] T(s', \Lambda)}{(s' + \mu^2)(s' + \Lambda^2)^2 (p' - p)^2} \quad (2.3)$$

Performing the iteration of FP II we find the same result (1.7) for the two lowest orders in g^2 .

On the other hand (2.3) has the nice property of being equivalent to a boundary value problem of a second order differential equation of the hypergeometric type ^{7),9)}. One can verify easily that having performed the angular integrations (we write for the moment simply $T(s)$ instead of $T(s, \Lambda)$) in

$$T(s) = T_B(s) + T_C + \lambda \Lambda^4 \int_0^\infty \frac{s' ds' T(s')}{(s' + \mu^2)(s' + \Lambda^2)^2} \left[\frac{\theta(s' - s)}{s'} + \frac{\theta(s - s')}{s} \right] \quad (2.4)$$

$$T_B(s) = - \frac{ig^2(s+4m^2)}{m^2(s+m^2)}. \quad (2.5)$$

$$T_c = \frac{\lambda \Lambda^4}{3m^2} \int_0^\infty \frac{ds' T(s')}{(s' + \Lambda^2)^2} \quad (2.6)$$

T obeys the differential equation ¹⁰⁾

$$\left(\frac{d^2}{ds^2} + \frac{2}{s} \frac{d}{ds} \right) (T - T_B - T_c) = - \frac{\lambda \Lambda^4 T}{s(s + \mu^2)(s + \Lambda^2)^2} \quad (2.7)$$

Expressing the integrand of (2.4) through (2.7), the boundary conditions are obtained

$$\begin{aligned} \lim_{s \rightarrow 0} \left\{ s^2 \frac{d}{ds} (T - T_B - T_c) \right\} &= 0 \\ \lim_{s \rightarrow \infty} \{ T - T_B - T_c \} &= 0 \end{aligned} \quad (2.8)$$

(2.7) and (2.8) can be rewritten as

$$\left\{ \frac{d^2}{ds^2} + \frac{2}{s} \frac{d}{ds} + \frac{\lambda \Lambda^4}{s(s + \mu^2)(s + \Lambda^2)^2} \right\} T = g(s) \quad (2.9)$$

$$g(s) = \frac{6ig^2 m^2}{s(s + m^2)^3} \quad (2.10)$$

with the boundary conditions

$$\left[s^2 \frac{dT}{ds} \right]_{s=0} = 0 \quad (2.11)$$

$$[T]_{s=\infty} = - \frac{ig^2}{m^2} + \frac{\lambda}{3m^2} \Lambda^4 \int_0^\infty \frac{ds' T(s')}{(s' + \Lambda^2)^2} \quad (2.12)$$

We arrive at the solution of (2.9) by variation of constants in the homogeneous solution. The result is

$$T(s, \Lambda) = h(\Lambda) X_1(s, \Lambda) + \int_0^s g(s') s'^2 ds' [X_2(s', \Lambda) X_1(s, \Lambda) - X_1(s', \Lambda) X_2(s, \Lambda)] \quad (2.13)$$

$$h(\Lambda) = N(\Lambda) D^{-1}(\Lambda) \quad (2.14)$$

$$N(\Lambda) = -\frac{ig^2}{m^2} + \frac{\lambda \Lambda^4}{3m^2} \int_0^\infty \frac{ds''}{(s'' + \Lambda^2)^2} \int_0^{s''} g(s') s'^2 ds' \times \\ \times [X_2(s', \Lambda) X_1(s'', \Lambda) - X_1(s', \Lambda) X_2(s'', \Lambda)] - \\ - \int_0^\infty g(s') s'^2 ds' [X_2(s', \Lambda) X_1(\infty, \Lambda) - X_1(s', \Lambda) X_2(\infty, \Lambda)] \quad (2.15)$$

$$D(\Lambda) = 1 - \frac{\lambda \Lambda^4}{3m^2} \int_0^\infty \frac{ds'}{(s' + \Lambda^2)^2} X_1(s', \Lambda) \quad (2.16)$$

X_1 and X_2 are two linearly independent solutions of the homogeneous equation of (2.9). They have been normalized so that near $s = 0$

$$X_1(s, \Lambda) = 1 + O(s) \\ X_2(s, \Lambda) = s^{-1} + O(\ln s, 1) \quad (2.17)$$

and can be found by introducing

$$z = - \frac{\Lambda^2 - \mu^2}{\Lambda^2 + s} \frac{s}{\mu^2} \quad (2.18)$$

in the homogeneous equation :

$$X_1 = F\left(\frac{1-\kappa}{2}, \frac{1+\kappa}{2}; 2; z\right) \quad (2.19)$$

$$X_2 = - \frac{\Lambda^2 - \mu^2}{\mu^2 \Lambda^2} \Gamma\left(\frac{3+\kappa}{2}\right) \Gamma\left(\frac{3-\kappa}{2}\right) \times \\ \times (1-z) F\left(\frac{3-\kappa}{2}, \frac{3+\kappa}{2}; 2; 1-z\right) \quad (2.20)$$

F is the hypergeometric function in standard notation, κ is the abbreviation for

$$\kappa = \left[1 - 4\lambda \Lambda^2 (\Lambda^2 - \mu^2)^{-1}\right]^{1/2} \quad (2.21)$$

The analytic continuation of F in (2.20) into the point $z = 0$ can be found, e.g., in ¹¹⁾. From there we take also the analytic continuation of (2.19), (2.20) to large negative z

$$X_1 \xrightarrow{z \rightarrow \infty} \frac{\Gamma(\kappa)}{\Gamma\left(\frac{1+\kappa}{2}\right) \Gamma\left(\frac{3+\kappa}{2}\right)} (-z)^{-\frac{1-\kappa}{2}} \left[1 + O\left(\frac{1}{z}, \frac{1}{z^\kappa}\right)\right] \quad (2.22)$$

$$X_2 \xrightarrow{z \rightarrow \infty} \frac{(\Lambda^2 - \mu^2) \Gamma(\kappa) \Gamma\left(\frac{3-\kappa}{2}\right)}{\Lambda^2 \mu^2 \Gamma\left(\frac{1+\kappa}{2}\right)} e^{\pm i\frac{\pi}{2}(1-\kappa)} \times \\ \times (-z)^{-\frac{1-\kappa}{2}} \left[1 + O\left(\frac{1}{z}, \frac{1}{z^\kappa}\right)\right] \quad (2.23)$$

The logarithmic singularity in X_2 , which produces a cut starting from zero, appears multiplied by the regular solution as is always the case ¹¹⁾

$$(1-z) F\left(\frac{3-\kappa}{2}, \frac{3+\kappa}{2}; 2; 1-z\right) = G(z) + \frac{\log(z)}{\Gamma(\frac{1-\kappa}{2}) \Gamma(\frac{1+\kappa}{2})} X_1(z)$$

where $G(z)$ is analytic in the neighbourhood of $z = 0$. Hence a function X_2' , which contains $\log(-z)$ and is therefore regular for $z < 0$, differs from X_2 only by a constant multiple of X_1 . (2.13) - (2.16) are invariant against such a transformation of X_2 , hence the solution is independent of the definition of the cut in X_2 . The analytic continuation

$$\log(-z) = \log z \pm i\pi, \quad z^{\frac{1-\kappa}{2}} = e^{\pm \frac{i\pi}{2}(1-\kappa)} (-z)^{\frac{1-\kappa}{2}}$$

where either the two upper or the two lower signs are taken, enables us to define

$$X_2' = X_2 + \frac{\Lambda^2 - \mu^2}{\Lambda^2 \mu^2} i\pi \frac{\Gamma(\frac{3+\kappa}{2}) \Gamma(\frac{3-\kappa}{2})}{\Gamma(\frac{1-\kappa}{2}) \Gamma(\frac{1+\kappa}{2})} X_1 \quad (2.20')$$

From (2.20') we find (c.f. Eqs. (2.22), (2.23)) the asymptotic form

$$\begin{aligned} X_2' \xrightarrow{z \rightarrow \infty} & \frac{(\Lambda^2 - \mu^2) \Gamma(\kappa) \Gamma(\frac{3-\kappa}{2})}{\Lambda^2 \mu^2 \Gamma(\frac{1+\kappa}{2})} \sin \frac{\pi \kappa}{2} \times \\ & \times (-z)^{-\frac{1-\kappa}{2}} \left[1 + O\left(\frac{1}{z}, \frac{1}{z^k}\right) \right] \end{aligned} \quad (2.23')$$

3. REMOVAL OF THE CUT-OFF

If Λ tends to infinity we then notice that the singular point of regular type of the differential equation at $z = \infty$ or $s = -\Lambda^2$ shifts into $s = \infty$. Since K remains bounded the homogeneous solutions behave essentially as (2.22), (2.23) with z replaced by s . These are changes at big s so that for finite s in (2.13) the limit can be performed immediately everywhere except in $h(\Lambda)$. In the numerator of this quantity the second integral is of order $O(\gamma^{(K-1)/2})$. $-\frac{ig^2}{m^2}$ has to be neglected compared with the double integral which gives in the limit a leading contribution proportional to $\gamma^{(1+K)/2}$ where

$$\gamma = (\Lambda^2 - \mu^2) \mu^{-2} \quad (3.1)$$

The same is true for the integral in D so that with the remaining parts $\bar{N}$, $\bar{D}$ of N and D

$$h = \lim_{\Lambda \rightarrow \infty} h(\Lambda) = \lim_{\Lambda \rightarrow \infty} \bar{N}(\Lambda) \bar{D}^{-1}(\Lambda) \quad (3.2)$$

with

$$\bar{N}(\Lambda) = \frac{2ig^2 \lambda}{(1 - \frac{m^2}{\Lambda^2})^3} \int_0^{-\gamma} dz'' \int_0^{z''} \frac{z' dz' [X_2(z') X_1(z'') - X_1(z') X_2(z'')]}{\left[z' - \frac{m^2(\Lambda^2 - \mu^2)}{\mu^2(\Lambda^2 - m^2)} \right]^3} \quad (3.3)$$

$$\bar{D}(\Lambda) = \frac{\lambda \mu^2}{3m^2(1 - \frac{\mu^2}{\Lambda^2})} \int_0^{-\gamma} dz'' X_1(z'') \quad (3.4)$$

4. EXPANSION IN g^2

The perturbation expansion before performing the limit is calculated first. Up to first order in a quantity α

$$\alpha = \lambda (1+y) y^{-1} \quad (4.1)$$

(compare the definition of κ , (2.21)) the two solutions, normalized as (2.17) are

$$\chi_1 = 1 + \alpha [1 + (1-z) z^{-1} \ln(1-z)] \quad (4.2)$$

$$\chi_2 = -y(1+y)^{-1} \mu^{-2} [z^{-1}(1-z) + \alpha \ln z] \quad (4.3)$$

More convenient for our integration is a second solution χ_2' (corresponding to (2.23'))

$$\chi_2' = -y(1+y)^{-1} \mu^{-2} [z^{-1}(1-z) + \alpha \ln(-z)] \quad (4.3')$$

It should be noticed that in such an expansion the superposition of two cuts at infinity, coming from factors $z^{-(1 \pm \kappa)/2}$, does not appear any more. Only logarithmic cuts survive. Elementary integrations in (3.4) give

$$\bar{D} = \frac{\lambda \mu^2}{3m^2} [-y + \lambda(y \ln y - 2y) + O(\ln y, \ln^2 y)] \quad (4.4)$$

It can be seen from (4.4) that $\bar{D}$ diverges more rapidly in order λ than in the lowest order; however, together with $\bar{N}$ the term $y \ln y$ will drop out.

In terms of the perturbation expansions (4.2), (4.3) also the second integral in (2.15) gives a contribution of order $\ln y$. It can be neglected compared with the first one which has been retained in $\bar{N}$. With (4.2), (4.3) we get from (3.3)

$$\bar{N} = -\frac{ig^2 \lambda \mu^2}{m^2} \left[-y + \lambda (y \ln y - 2y - \frac{b \ln b}{b-1} y) + O(\ln y, \ln^2 y) \right] \quad (4.5)$$

$$b = m^2 \mu^{-2} \quad (4.6)$$

The final result for h which is identical with the low energy limit for the amplitude (c.f. Eq. (2.13)) is

$$\begin{aligned} T(0) = h &= \lim_{\Lambda \rightarrow \infty} h(\Lambda) = \lim_{y \rightarrow \infty} \bar{N}(y) D^{-1}(y) = \\ &= -\frac{3ig^2}{m^2} \left(1 + \lambda \frac{b \ln b}{b-1} \right) \end{aligned} \quad (4.7)$$

As it could be expected the lowest order is independent of m/μ (c.f. Eq. (4.6)). For $b = 1$ even the second order coincides with (1.7), the result of FP II.

We now develop h in a series in λ (or g^2) after having performed the limit (1.1). Since we can use in (3.8) for the two hypergeometric functions again the closed expressions (4.2), (4.3), we notice that the result is identical to (4.7).

5. CONCLUSIONS

Since we believe that our "model equation" (2.3) agrees at least qualitatively with the original trace equation (2.1), the last chapter shows that the iteration procedure, adopted by Feinberg and Pais (FP II) in the trace equation, really agrees with the exact solution. This is related to the fact that the sequence of the limits $\Lambda \rightarrow \infty$ and $g^2 \rightarrow 0$ can be exchanged. The low energy limit h of the amplitude T (cf. (3.2), (3.3), (3.4)) is of the form

$$h = \lim_{y \rightarrow \infty} \frac{O(1) + y^{\frac{1+\kappa}{2}} [a + O(y^{-1}, y^{-\kappa})]}{O(1) + y^{\frac{1+\kappa}{2}} [b + O(y^{-1}, y^{-\kappa})]} = \frac{a}{b} \quad (5.1)$$

The result is the same in a series expansion in λ with

$$y^{\frac{\kappa-1}{2}} = (1 - \ln y) + O(y^{-1} \ln y, y^{-1}) \quad (5.2)$$

As an example that the essential singularity in $\Lambda = \infty$ in general causes ambiguities consider (5.1) in the fictitious case $a = b = 0$

$$f = \lim_{y \rightarrow \infty} \frac{O(1) + y^{\frac{\kappa-1}{2}} [c_1 + O(y^{-1})] + y^{\frac{1-\kappa}{2}} [c_2 + O(y^{-1})]}{O(1) + y^{\frac{\kappa-1}{2}} [d_1 + O(y^{-1})] + y^{\frac{1-\kappa}{2}} [d_2 + O(y^{-1})]}$$

With the limit $y \rightarrow \infty$ first there are two results

$$f = \begin{cases} c_1/d_1 & \text{for } \kappa > 1 \\ c_2/d_2 & \text{for } \kappa < 1 \end{cases}$$

In the expansion (5.2), however, we arrive at

$$f = \lambda \frac{c_2 - c_1}{d_2 - d_1}$$

We thus come back to our conjecture of the introduction that peratization works only for certain classes of theories. Moreover, it may happen that the prescription (1.1) gives a finite result which, however, differs from the one in an iteration scheme. Fortunately, this is not the case for the leptonic reaction with an intermediate vector meson in the ladder approximation.

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