CERN LIBRARIES, GENEVA

CM-P00056859

A LOWER BOUND FOR ELASTIC CROSS-SECTIONS IN THE HIGH-ENERGY REGION

A.Martin

CERN, Geneva

It was noticed some time ago by Rarita ¹⁾ and Lévy ²⁾ that if only L partial waves contribute to a scattering process, the elastic cross-section cannot be arbitrarily small for a given total cross-section. More precisely :

$$\frac{\sigma_e}{(\sigma_t)^2} > \frac{k^2}{4\pi(L+1)^2} \quad (1)$$

where k is the c.m. momentum.

On the other hand, Froissart ³⁾ has shown that in the high-energy region, the dominant contribution to a scattering amplitude comes from orbital angular momenta $\ell < C k \log k$ if this amplitude satisfies Mandelstam representation. This was shown also to be true under less stringent assumptions by myself ⁴⁾. Combination of these two results makes us guess that :

$$\sigma_e > \text{Const} \times \frac{(\sigma_t)^2}{(\log s)^2} \quad (2)$$

where s is the square of the c.m. energy.

In this note, the only non-trivial point is to prove that the contributions from angular momenta $\ell > C k \log k$ to σ_t are really negligible provided C is large enough, and that inequality (2) holds even if the total cross-section is rapidly decreasing with energy.

Let us assume, for simplicity of the argument, the validity of Mandelstam representation, though this is certainly not necessary (analyticity and temperedness in an ellipse in the $\cos\theta$ plane, with foci at -1 and $+1$ and semi major axis $1 + \text{const} \times k^{-2}$ is certainly enough).

2.

We can write the imaginary part of the forward scattering amplitude

$$A(s,0) = \frac{\sqrt{s}}{k} \sum_{e=0}^L (2e+1) a_e + R_L \quad (3)$$

with

$$0 \leq a_e \leq 1$$

The rest R_L is given by :

$$R_L = (L+1) \int_{t_0}^{\infty} \frac{Q_L(1+\frac{t}{2k^2}) - Q_{L+1}(1+\frac{t}{2k^2})}{t} \rho_u(s,t) dt \\ + (-1)^L (L+1) \int_{t'_0 > t_0}^{\infty} \frac{Q_L(1+\frac{t}{2k^2}) + Q_{L+1}(1+\frac{t}{2k^2})}{t + 4k^2} \rho_t(s,t) dt \quad (4)$$

where L is larger than the number of subtractions. $\rho_u(s,t)$ and $\rho_t(s,u)$ are the spectral functions contributing to the absorptive part in channel s . Equal masses have been assumed.

We shall just consider the contribution of

$$X_L = (L+1) \int_{t_0}^{\infty} Q_L(1+\frac{t}{2k^2}) \frac{\rho_u(s,t)}{t} dt \quad (5)$$

A similar argument holds for the other contributions. $\rho_u(s,t)$ is assumed to be a tempered distribution. Here we shall over-simplify a little by taking it to be a function bounded by a polynomial in s and t . By making a sufficient number of integrations by parts, one can always work with functions anyway and since the derivatives of the Q_L 's have bounds of the same type as Q_L itself this makes no difference. Since t_0 is positive and s large, we can just take

$$\rho_u(s, t) < C_0 s^N t^N \quad (6)$$

On the other hand, from

$$Q_L(z) = \int_0^\infty \frac{dt}{(z + \cosh t \sqrt{z^2 - 1})^{L+1}}, \quad z > 1$$

one can easily prove the inequality

$$Q_L\left(1 + \frac{t}{2k^2}\right) < \frac{1}{2} \left[1 + \frac{\sqrt{t}}{k}\right]^{-L} \log \frac{4k^2 + t}{t}$$

Inserting this in X_L one finds

$$X_L < \text{const} \times \frac{L+1}{L-2N} \log \left(\frac{4k^2 + t_0}{t_0} \right) s^{2N} \left(1 + \frac{\sqrt{t_0}}{k}\right)^{-L+2N}$$

If we take

$$L = C_1 \sqrt{\frac{s}{t_0}} \log \left(\frac{s}{s_0} \right) \quad (7)$$

where $C_1 = N + \frac{M}{2} + \varepsilon$

and s_0 is some unknown scale factor, we see that X_L decreases faster than s^{-M} as s goes to infinity. So by taking C_1 large enough, we can manage to make R_L negligible, in the high-energy region, versus the first L terms in (3) provided that the absorptive part of the amplitude does not decrease faster than any negative power of s .

Since by the optical theorem

$$\sigma_t = \frac{4\pi}{k^2} \frac{k}{\sqrt{s}} A(s, 0)$$

we see that if at high energies σ_t is larger than s^{-M} , we have in the high-energy region

$$\sigma_e > \frac{(\sigma_t)^2 t_0}{16\pi \left(N + \frac{M}{2} + \epsilon\right) \left[\log\left(\frac{s}{s_0}\right)\right]^2} \quad (8)$$

Of course one would like to have some idea about s_0 . In fact, we have no information on this quantity which is related to C_0 appearing in Eq. (6). There is no reason to believe that it is of the order of t_0 ; it could be, as well, of the order of the square masses of the particles involved in the scattering. In the case of nucleon-nucleon scattering t_0 is $4\mu^2$, where μ is the pion mass, which is much less than the square of the nucleon mass; even when one takes the logarithm, it still makes a big difference. On the other hand, it is plausible that N be an integer larger than unity but not too big.

It may be interesting to compare formulae (2) or (7) with the predictions of a detailed model, for instance the Regge pole model. Then $A(s, t)$, where t is the momentum transfer, behaves like $f(t)s^{\alpha(t)}$. Then $\sigma_t \leq \text{const. } s^{\alpha(0)-1}$ and

$$\sigma_e \simeq \text{const} \times \frac{[s^{\alpha(0)-1}]^2}{\log s}$$

so that one has

$$\sigma_e = \text{const} \times (\sigma_t)^2 (\log s)^{-1} \quad (9)$$

which is of course consistent with (2) or (7).

To end these remarks we should say that in the case σ_t const, which appears to hold to a good approximation in NN scattering from 3 GeV to probably 10000 GeV⁵⁾, the elastic cross-section cannot decrease faster than $1/(\log^2 s)$, i.e., very slowly. This is encouraging for work on future high energy machines.

R E F E R E N C E S

- 1) W. Rarita and P. Schwed, Phys.Rev. 112, 271 (1958).
- 2) M. Lévy, Proceedings of the Padua-Venice Conference, 1957 (unpublished); and Phys.Rev. Letters 5, 380 (1960).
- 3) M. Froissart, Phys.Rev. 123, 1054 (1961).
- 4) A. Martin, Proceedings of the 1962 High Energy Physics at CERN, p. 566. A more detailed account will appear in the Physical Review.
- 5) G. Cocconi, private communication.