(1964).

²Final appellation should follow the convention of G. F. Chew, M. Gell-Mann, and A. H. Rosenfeld, *Sci. Am.* **210**, 74 (1964). Since this convention depends upon the quantum numbers of the meson, the christening cannot take place until the former are definitely established.

³L. Bertanza et al., *Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962* (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 285.

⁴Channels (1) and (3) contain Σ^0 as well as Λ^0 events in unknown proportion. The existence of Σ^0 contamination does not have a significant effect on the interpretation of the data. See reference 9.

⁵The pion contamination in the 2.3-BeV/c K^- beam is $7 \pm 3\%$. For details see J. Leitner, G. C. Moneti, and N. P. Samios, *Nucl. Instr. Methods* **20**, 42 (1963).

⁶From such studies we find that the $\pi^- + p \rightarrow \Lambda^0(\Sigma^0) + K^0 + n\pi^0$ contamination is negligible.

⁷From a study of the $M^2(\pi^+ + \pi^- + \pi^0)$ spectrum from the reaction $K^- + p \rightarrow \Lambda + \pi^- + \pi^+ + \pi^0$, we determine the number of ω^0 's and η^0 's decaying via the charged ($\pi^+ + \pi^- + \pi^0$) mode (see reference 3 for details). Similarly, the charged $K^+ + K^-$ mode of ϕ^0 decay is directly observed from $K^- + p \rightarrow \Lambda + K^+ + K^-$ (see reference 8 for details). From this information and the known charged/neutral branching ratio of the η^0 , ω^0 , and ϕ^0 (see reference 8), we predict that 30 ± 10 , 20 ± 5 , and 10 ± 3 events should appear in the η^0 , ω^0 , and ϕ^0 peaks of Fig. 1, respectively. Three of the five normalization points correspond to these numbers. The other two correspond to the onset of $\Lambda\pi^0\pi^0$ phase space and the end point of the neutral spectrum.

⁸G. Puppi, *Ann. Rev. Nucl. Sci.* **13**, 287 (1963); P. L. Connolly et al., *Phys. Rev. Letters* **10**, 371 (1963).

⁹We emphasize that possible Σ^0 contamination cannot appreciably affect these conclusions. Firstly, the $\Sigma^0 + n\pi^0$ contribution is included in the "smooth-curve" background. Secondly, as mentioned in the text, the 960-MeV peak cannot be due to a special Σ^0 production mechanism because the missing γ ray would give rise to an X^0 width eight times the observed one. For the same reason, $\Sigma^0 + X^0$ production would be much broader than the experimental peak.

¹⁰To further investigate the effect of such intermediate resonance production on multipion mass distributions, we studied the (80-event) $K^- + p \rightarrow \Lambda + 4\pi$ channel, which we know to contain a strong $Y_1^*(1385)$. The $M^2(4\pi)$ distribution from this reaction fits phase space very well, indicating that resonance effects do not markedly distort the $M^2(4\pi)$ spectrum.

¹¹There is some indication that three of five pions are emitted as an η^0 , but we defer such discussion to a subsequent Letter.

¹²From a study of kinematically fitted $\pi^- + p \rightarrow \Lambda^0 + K^0 + \pi^+ + \pi^-$ events (where both the Λ^0 and K^0 decay visibly), we estimate that only 6 ± 3 events of this type are contained in the 415-event sample of channel (3).

¹³For reasons similar to those given in reference 1, possible Σ^0 contamination does not affect the conclusions drawn here.

¹⁴Identical behavior is exhibited by the events in the "960" peaks in channels (1) and (2). These data are not exhibited only because of space limitations.

¹⁵This estimate omits possible additional X^0 events which may decay electromagnetically, i.e., $X^0 \rightarrow \pi^+ + \pi^- + \gamma$.

¹⁶The Gaussian ideogram of the experimental resolution function has been obtained by multiplying the error of each individual event by $\sqrt{2}$ and summing over the events in this "960" peak region.

WEAK CURRENTS AND BROKEN UNITARY SYMMETRY

Reinhard Oehme*

Enrico Fermi Institute for Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois[†]
and CERN, Geneva, Switzerland

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It is the purpose of this note to compute the relative strength of the strangeness-changing and -nonchanging currents which enter into the weak interaction. In this report we consider only mesonic matrix elements of these currents.

We assume that the weak current for the interaction with leptons consists of vector and axial-vector terms which correspond to the following components of a unitary octet¹:

$$V_\alpha = \cos\theta_V [\hat{V}_\alpha^{(1)} + i\hat{V}_\alpha^{(2)}] + \sin\theta_V [\hat{V}_\alpha^{(4)} + i\hat{V}_\alpha^{(5)}],$$

and

$$A_\alpha = \cos\theta_A [\hat{A}_\alpha^{(1)} + i\hat{A}_\alpha^{(2)}] + \sin\theta_A [\hat{A}_\alpha^{(4)} + i\hat{A}_\alpha^{(5)}]. \quad (1)$$

Here the caret denotes the appropriately normalized currents. The weak interactions corresponding to Eq. (1) have been considered in detail by Cabibbo,² who finds that the experimental data for leptonic decays are consistent with an angle θ of about 0.26 radian, which is approximately the same for vector and axial-vector currents. In the following we want to give a theoretical explanation for this remarkable regularity. Our argument is based upon the equal-time commutation relations of the current densities, which have been discussed by Gell-Mann.³ More specifically, we need the commutators of $V_4^{(i)}(x)$ and $A_4^{(i)}(x)$ with the divergence $\partial_\beta A_\beta^{(j)}(x)$, as well

as some simple dispersion-theoretical considerations.

Essentially, we want to propose that the same SU(3)-breaking term in the strong Hamiltonian, which is causing the mass splitting within the meson octet, is also responsible for the orientation of the weak current in unitary space.

We write the strong Hamiltonian density in the form

$$H = H_0 + S^{(0)} + \gamma S^{(8)}, \quad (2)$$

where $\gamma S^{(8)}(x)$ is the symmetry-breaking term which transforms like the $I=0$ member of the unitary octet, and $S^{(0)}$ is the symmetric mass term. By $S^{(j)}(x)$ and $p^{(j)}(x)$ we denote the scalar and pseudoscalar densities, which, for example in a quark model, are proportional to $\bar{q}\lambda_j q$ and $-i\bar{q}\lambda_j \gamma_5 q$, respectively.⁴

In order to obtain a relation involving the angles θ_V and θ_A we consider first matrix elements of the form

$$\langle \pi^0 | [V_\alpha^{(i)}(x), \partial_\beta A_\beta^{(j)}(x')] | 0 \rangle. \quad (3)$$

Using the Hamiltonian (2) in the equation

$$\partial_\alpha A_\alpha^{(i)}(x) = [H(x), \int d^3y A_4^{(i)}(y)]_{x_0=y_0},$$

we find

$$\partial_\alpha (A_\alpha^{(1)} + iA_\alpha^{(2)}) = -[(\frac{2}{3})^{1/2} + \gamma(\frac{1}{3})^{1/2}](P^{(1)} + iP^{(2)}),$$

and

$$\partial_\alpha (A_\alpha^{(4)} + iA_\alpha^{(5)}) = -[(\frac{2}{3})^{1/2} - \gamma(\frac{1}{12})^{1/2}](P^{(4)} + iP^{(5)}).$$

With the equal-time commutation relations between the fourth components of the currents and the pseudoscalar densities, we obtain then the relations⁵

$$\begin{aligned} & \{V_4^{(1)}(x) - iV_4^{(2)}(x), \partial_\beta [A_\beta^{(1)}(x') + iA_\beta^{(2)}(x')]\}_{x_0=x'_0} \\ &= 2i\delta(\vec{x} - \vec{x}')[(\frac{2}{3})^{1/2} + \gamma(\frac{1}{3})^{1/2}]P^{(3)}(x), \\ & \{V_4^{(4)}(x) - iV_4^{(5)}(x), \partial_\beta [A_\beta^{(4)}(x') + iA_\beta^{(5)}(x')]\}_{x_0=x'_0} \\ &= 2i\delta(\vec{x} - \vec{x}')[(\frac{2}{3})^{1/2} - \gamma(\frac{1}{12})^{1/2}]\frac{1}{2}\{P^{(3)}(x) \\ & \quad + \sqrt{3}P^{(8)}(x)\}. \end{aligned} \quad (4)$$

We are interested in the matrix elements (3) corresponding to the relations (4). In the commutator we insert a complete set of intermediate states and retain only the single-particle states, namely a π^+ meson in the formula involving

$V^{(1)} - iV^{(2)}$ and a K^+ meson in the one with $V^{(4)} - iV^{(5)}$. Then there occur certain matrix elements which we write in the form

$$\begin{aligned} \langle \pi^0 | V_\alpha^{(1)} - iV_\alpha^{(2)} | \pi^+ \rangle &= (p + \pi)_\alpha \sqrt{2} \cos \theta_V F_\pi [-(p - \pi)^2], \\ \langle \pi^0 | V_\alpha^{(4)} - iV_\alpha^{(5)} | K^+ \rangle &= \{(p + \pi)_\alpha + (p - \pi)_\alpha \xi [-(p - \pi)^2]\} \\ & \quad \times \frac{1}{\sqrt{2}} \sin \theta_V F_K [-(p - \pi)^2], \end{aligned} \quad (5)$$

where p is the momentum of the π^+ meson or the K^+ meson, respectively. In addition we need

$$\begin{aligned} \langle \pi^+ | A_\alpha | 0 \rangle &= -ip_\alpha \cos \theta_A B_\pi (m_\pi^2), \\ \langle K^+ | A_\alpha | 0 \rangle &= -ip_\alpha \sin \theta_A B_K (m_K^2). \end{aligned} \quad (6)$$

Note that we have defined the form factors F and B in terms of the normalized currents. The conserved vector-current theory⁶ predicts $F_\pi(0) = 1$.

Let us now use the definitions (5) and (6) in the decomposition of the matrix elements of Eq. (4). Noting that the strong matrix element $\langle \pi | P^{(8)} | 0 \rangle$ vanishes because of isospin conservation, we can take the ratio and find

$$\tan \theta_V \tan \theta_A \frac{B_K(m_K^2) \bar{F}_K(-q_K^2)}{B_\pi(m_\pi^2) F_\pi(-q_\pi^2)} \approx \frac{m_\pi^2}{m_K^2} \frac{1 - \gamma/2\sqrt{2}}{1 + \gamma/\sqrt{2}}. \quad (7)$$

Furthermore, our approximation leads to the result⁷

$$\bar{F}_K(-q_K^2) \approx \text{const}, \quad (8)$$

where

$$\begin{aligned} & \bar{F}_K(-q_K^2) \\ &= F_K(-q_K^2) \left\{ \frac{m_K + E_\pi}{2m_K} + \frac{m_K - E_\pi}{2m_K} \xi(-q_K^2) \right\}, \end{aligned}$$

and $-q_K^2 = m_K^2 + m_\pi^2 - 2m_K E_\pi$. The quantity E_π is the energy of the π^0 meson, and in the matrix element for K_{e3} decay the argument $-q_K^2$ of F_K varies between $(m_K - m_\pi)^2$ and 0. In an approximation where one neglects the energy variation of F_K and ξ over the π^0 spectrum, our relation (8) indicates $\xi \approx +1$. We also obtain the formula⁸ $F_\pi(-q^2)(m_\pi + E_\pi)/2m_\pi \approx \text{const}$, where $-q_\pi^2 = 2m_\pi \times (m_\pi - E_\pi)$, but we are interested in F_π only for $E_\pi \approx m_\pi$, which corresponds to the decay $\pi^+ \rightarrow \pi^0 + e^+ + \nu$.

So far, we have considered only the commutation relations involving the vector current. An additional relation for $\tan \theta_A$ can be obtained

from the appropriate combinations of matrix elements like

$$\langle 0 | [A_{\alpha}^{(i)}(x), \partial_{\beta} A_{\beta}^{(j)}(x')] | 0 \rangle.$$

The corresponding calculation has been done by Gell-Mann³; it leads to the result

$$\tan^2 \theta \frac{B_K^2(m_K^2)}{A B_{\pi}^2(m_{\pi}^2)} \approx \frac{m_{\pi}^2}{m_K^2} \frac{1 - \gamma/2\sqrt{2}}{1 + \gamma/\sqrt{2}} \frac{1 - \rho/2\sqrt{2}}{1 + \rho/\sqrt{2}}, \quad (9)$$

where

$$\rho \equiv \langle 0 | S^{(8)} | 0 \rangle / \langle 0 | S^{(0)} | 0 \rangle.$$

In view of the success of the "mass formulas," which are obtained by treating the symmetry-violating interaction as a small perturbation,^{3,9} it seems plausible to assume that γ is a small parameter. We may also expect then that $\rho \ll 1$. If we suppose, in addition, that "renormalization effects" can be ignored, we may set the form factors in Eqs. (7) and (9) equal to one, and obtain as a zeroth approximation

$$\tan \theta_V \approx \tan \theta_A \approx m_{\pi}/m_K,$$

which corresponds to about 0.28 radian. As more complete formulas, we obtain from our approximate calculation

$$\tan \theta_V \approx \frac{m_{\pi} F_{\pi}}{m_K \bar{F}_K} \left(\frac{1 - \gamma/2\sqrt{2}}{1 + \gamma/\sqrt{2}} \right)^{1/2},$$

$$\tan \theta_A \approx \frac{m_{\pi} B_{\pi}}{m_K B_K} \left(\frac{1 - \gamma/2\sqrt{2}}{1 + \gamma/\sqrt{2}} \right)^{1/2} \left(\frac{1 - \rho/2\sqrt{2}}{1 + \rho/\sqrt{2}} \right)^{1/2}. \quad (10)$$

Empirically, the quantity $\tan \theta_V F_K / F_{\pi}$ has been determined² from the ratio of the rates for the decays $K^+ \rightarrow e^+ + \nu + \pi^0$ and $\pi^+ \rightarrow e^+ + \nu + \pi^0$, assuming that the variation of the form factor F_K over the pion spectrum is negligible. The parameter ξ can be obtained from the shape of the spectrum for $K_{\mu 3}$ decay and from measurements of the longitudinal polarization of the μ meson.¹⁰ Our formula (10), together with the empirical value of about 0.26 radian for $\tan \theta_V F_K / F_{\pi}$, indicates a value of ξ around +1. This may well be in agreement with experiments.¹⁰

The ratio of the decay rates for $K^+ \rightarrow \mu^+ + \nu'$ and $\pi^+ \rightarrow \mu^+ + \nu'$ determines $\tan \theta_A B_K / B_{\pi}$. The value of about 0.26 radian² is quite compatible with Eq. (10), provided the corrections to the

mass ratio are sufficiently small.

We see that the approximate theoretical expressions (10) give us some indication that the rotated orientation of the weak current in unitary space may be due to the same symmetry-breaking interaction which is also responsible for the mass splitting in the meson octet.

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*Guggenheim Fellow.

†Permanent address.

¹M. Gell-Mann and M. Lévy, *Nuovo Cimento* **16**, 705 (1960).

²N. Cabibbo, *Phys. Rev. Letters* **10**, 531 (1963).

³M. Gell-Mann, *Phys. Rev.* **125**, 1067 (1962).

⁴M. Gell-Mann, *Phys. Letters* **8**, 214 (1964); G. Zweig, (to be published).

⁵Although abstracted from field theory, these commutation relations for the densities may be too special. For example, there could be additional terms on the right-hand sides of Eq. (4), which involve derivatives of δ functions in such a way that they disappear after integration over $\vec{x}$ or $\vec{x}'$. We assume here that possible additional terms are absent or negligible.

⁶R. P. Feynman and M. Gell-Mann, *Phys. Rev.* **109**, 193 (1958); S. S. Gerstein and J. B. Zeldovich, *Zh. Eksperim. i Teor. Fiz.* **29**, 698 (1955) [translation: *Soviet Phys. - JETP* **2**, 567 (1956)].

⁷Our approximate equations imply also that the form factor $\bar{F}_K$ should be independent of $k_0 = [m_K^2 + \vec{k}^2]^{1/2}$. Here we consider $\bar{F}_K$ only for $k_0 = m_K$. Correspondingly, we choose $k_0 = m_{\pi}$ in the relation involving F_{π} .

⁸Relations like this are a reminder that the one-particle approximation to Eq. (4) may be rather crude, at least for values of E_{π} which deviate sizably from m_{π} . Note, in this connection, that it is the requirement of a conserved vector current which influences the form of the first formula in Eq. (5).

⁹S. Okubo, *Progr. Theoret. Phys. (Kyoto)* **28**, 64 (1962); S. L. Glashow and A. H. Rosenfeld, *Phys. Rev. Letters* **10**, 192 (1963).

¹⁰See, for example, J. L. Brown, J. A. Kadyk, G. H. Trilling, R. T. Van der Walle, B. P. Roe, and D. Sinclair, *Phys. Rev. Letters* **8**, 450 (1963); V. A. Smirnit-ski and A. O. Weissenberg, *Phys. Rev. Letters* **12**, 233 (1964); L. Okun, *Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962* (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 845. These papers contain further references.