We emphasize that possible Σ^0 contamination cannot appreciably affect these conclusions. Firstly, the Σ^0 + π^0 contribution is included in the "smooth-cur-" background. Secondly, as mentioned in the text, the 960-MeV peak cannot be due to a special Σ^0 production mechanism because the missing γ ray would give rise to an Χ^0 width eight times the observed one. For the same reason, Σ^0 + Χ^0 production would be much broader than the experimental peak.

To further investigate the effect of such intermediate resonance production on multipion mass distributions, we studied the (80-event) Κ^- + p → Λ + Χ^0 channel, which we know to contain a strong Y^0(1385). The M^2(4π) distribution from this reaction fits phase space very well, indicating that resonance effects do not markedly distort the M^2(4π) spectrum.

1There is some indication that three of five pions are emitted as an η^0, but we defer such discussion to a subsequent Letter.

13From a study of kinematically fitted Π^- + p → Λ^+ + Κ^- + η^0 + π^- events (where both the Λ^+ and Κ^- decay visibly), we estimate that only 6 ± 3 events of this type are contained in the 415-event sample of channel (3).

14For reasons similar to those given in reference 1, possible Σ^0 contamination does not affect the conclusions drawn here.

15Identical behavior is exhibited by the events in the "960" peaks in channels (1) and (2). These data are not exhibited only because of space limitations.

16This estimate omits possible additional Χ^0 events which may decay electromagnetically, i.e., Χ^0 → φ^- + γ.

17The Gaussian ideogram of the experimental resolution function has been obtained by multiplying the error of each individual event by \sqrt{2} and summing over the events in this "960" peak region.

WEAK CURRENTS AND BROKEN UNITARY SYMMETRY

Reinhard Oehme*

Enrico Fermi Institute for Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois†
and CERN, Geneva, Switzerland
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It is the purpose of this note to compute the relative strength of the strangeness-changing and -nonchanging currents which enter into the weak interaction. In this report we consider only mesonic matrix elements of these currents.

We assume that the weak current for the interaction with leptons consists of vector and axial-vector terms which correspond to the following components of a unitary octet:

\[ V_\alpha = \cos \theta_\alpha \left[ \tilde{V}_\alpha \right] + \sin \theta_\alpha [\tilde{V}_\alpha \left( 4 \right) + i \tilde{V}_\alpha \left( 5 \right)], \]

\[ A_\alpha = \cos \theta_\alpha \left[ \tilde{A}_\alpha \right] + \sin \theta_\alpha \left[ \tilde{A}_\alpha \left( 4 \right) + i \tilde{A}_\alpha \left( 5 \right) \right], \tag{1} \]

Here the caret denotes the appropriately normalized currents. The weak interactions corresponding to Eq. (1) have been considered in detail by Cabibbo, who finds that the experimental data for leptonic decays are consistent with an angle θ of about 0.26 radian, which is approximately the same for vector and axial-vector currents.

In the following we want to give a theoretical explanation for this remarkable regularity. Our argument is based upon the equal-time commutation relations of the current densities, which have been discussed by Gell-Mann. More specifically, we need the commutators of \( V_4(x) \) and \( A_4(x) \) with the divergence \( A_\beta(0)(x) \), as well...
as some simple dispersion-theoretical considerations.

Essentially, we want to propose that the same SU(3)-breaking term in the strong Hamiltonian, which is causing the mass splitting within the meson octet, is also responsible for the orientation of the weak current in unitary space.

We write the strong Hamiltonian density in the form

$$H = H_0 + S^{(0)} + \gamma S^{(8)},$$

(2)

where $\gamma S^{(8)}(x)$ is the symmetry-breaking term which transforms like the $I=0$ member of the unitary octet, and $S^{(0)}$ is the symmetric mass term. By $S^{(0)}(x)$ and $p^{(0)}(x)$ we denote the scalar and pseudoscalar densities, which, for example in a quark model, are proportional to $\bar{u}u$ and $-i\bar{u}\gamma_5u$, respectively.\(^4\)

In order to obtain a relation involving the angles $\theta_V$ and $\theta_A$ we consider first matrix elements of the form

$$\langle q^0 | [V^{(i)}_\alpha(x), \bar{\partial} A^{(i)}_\beta(x')]|0\rangle.$$  

(3)

Using the Hamiltonian (2) in the equation

$$\partial x_{\alpha} A^{(i)}_{\alpha}(x) = [H(x), \int d^3 y A^{(i)}_{\alpha}(y)] x_0 = y_0,$$

we find

$$\partial x_{\alpha} A^{(1)}_{\alpha} + iA^{(2)}_{\alpha} = -[(\frac{2}{3})^{1/2} + \gamma(\frac{1}{3})^{1/2}][P^{(1)} + iP^{(2)}],$$

and

$$\partial x_{\alpha} A^{(3)}_{\alpha} + iA^{(4)}_{\alpha} = -[(\frac{2}{3})^{1/2} - \gamma(\frac{1}{3})^{1/2}][P^{(3)} + iP^{(4)}].$$

With the equal-time commutation relations between the fourth components of the currents and the pseudoscalar densities, we obtain then the relations\(^2\)

$$\{V^{(1)}_4(x) - iV^{(2)}_4(x), \bar{\partial} A^{(1)}_\beta(x') + iA^{(2)}_\beta(x')\} x_0 = x_0,$$

$$= 2i\delta(x - x')[(\frac{2}{3})^{1/2} + \gamma(\frac{1}{3})^{1/2}] P^{(1)},$$

$$\{V^{(3)}_4(x) - iV^{(4)}_4(x), \bar{\partial} A^{(3)}_\beta(x') + iA^{(4)}_\beta(x')\} x_0 = x_0,$$

$$= 2i\delta(x - x')[(\frac{2}{3})^{1/2} - \gamma(\frac{1}{3})^{1/2}] \{P^{(3)} + \sqrt{3}P^{(4)}\}.\ (4)$$

We are interested in the matrix elements (3) corresponding to the relations (4). In the commutator we insert a complete set of intermediate states and retain only the single-particle states, namely a $\pi^+$ meson in the formula involving $V^{(1)} - iV^{(2)}$ and a $K^+$ meson in the one with $V^{(4)} - iV^{(5)}$. Then there occur certain matrix elements which we write in the form

$$\langle \pi^+ | V^{(1)}_\alpha - iV^{(2)}_\alpha | \pi^- \rangle = \langle \pi^+ | 1/\sqrt{2} \cos\theta_V F \pi^- | (p + \pi) \rangle,$$

$$\langle K^+ | V^{(4)}_\alpha - iV^{(5)}_\alpha | K^- \rangle = \langle (p + \pi) | \pi^- \rangle \langle \pi^- | \epsilon \rangle\ [-(p - \pi)^2],$$

$$= \pi \sin\theta_V F, K^- | (p - \pi)^2],$$

(5)

where $p$ is the momentum of the $\pi^+$ meson or the $K^+$ meson, respectively. In addition we need

$$\langle \pi^+ A^1_{\alpha} | 0 \rangle = -i\bar{p}_\alpha \cos\theta_A A^{1}_{\pi} (m^2_\pi),$$

$$\langle K^+ A^{1}_{\alpha} | 0 \rangle = -i\bar{p}_\alpha \sin\theta_A A^{1}_{K} (m^2_K).\ (6)$$

Note that we have defined the form factors $F$ and $B$ in terms of the normalized currents. The conserved vector-current theory\(^8\) predicts $F_\pi(0) = 1$.

Let us now use the definitions (5) and (6) in the decomposition of the matrix elements of Eq. (4). Noting that the strong matrix element $\langle \pi^+ | P^{(1)} | 0 \rangle$ vanishes because of isospin conservation, we can take the ratio and find

$$B_K(m_K^2) F_K(-q^2) = m^2_\pi - \gamma/2\sqrt{2}\tan\theta_V \tan\theta_A B_\pi(m_\pi^2) F_\pi(-q^2) = m^2_\pi - \gamma/\sqrt{2}\tan\theta_V \tan\theta_A B_\pi(m_\pi^2).\ (7)$$

Furthermore, our approximation leads to the result\(^7\)

$$F_K(-q^2) \approx \text{const},\ (8)$$

where

$$F_K(-q^2) = F_K(-q^2) \left\{ \frac{m_K + E_\pi}{2m_K} - \frac{m_K - E_\pi}{2m_K} \xi(-q^2) \right\},$$

and $-q^2 = m_K^2 + m_\pi^2 - 2m_K E_\pi$. The quantity $E_\pi$ is the energy of the $\pi^0$ meson, and in the matrix element for $K_{e3}\pi$ decay the argument $-q^2$ of $F_K$ varies between $(m_K - m_\pi)^2$ and 0. In an approximation where one neglects the energy variation of $F_K$ and $\xi$ over the $\pi^0$ spectrum, our relation (8) indicates $\xi = +1$. We also obtain the formula\(^8\)

$$F_\pi(-q^2)(m_\pi + E_\pi)/(2m_\pi) \approx \text{const},$$

where $-q^2 = 2m_\pi E_\pi$, but we are interested in $F_\pi$ only for $E_\pi = m_\pi$, which corresponds to the decay $\pi^+ - \pi^0 + e^+ + \nu$.

So far, we have considered only the commutation relations involving the vector current. An additional relation for $\tan\theta_A$ can be obtained

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from the appropriate combinations of matrix elements like

\[ \langle 0 | A^{(i)}_\alpha (x), \delta A^{(j)}_\beta (x') | 0 \rangle. \]

The corresponding calculation has been done by Gell-Mann; it leads to the result

\[ \frac{B_K}{m_K^2} \cdot \frac{B_\pi}{m_\pi^2} \approx \frac{m_\pi^2}{m_K^2} \cdot \frac{1 - \gamma/2\sqrt{2}}{1 + \gamma/2\sqrt{2}} \cdot \frac{1 - \rho/2\sqrt{2}}{1 + \rho/2\sqrt{2}}, \]

(9)

where

\[ \rho = \langle 0 | S^{(0)} | 0 \rangle / \langle 0 | S^{(0)} | 0 \rangle. \]

In view of the success of the "mass formulas," which are obtained by treating the symmetry-violating interaction as a small perturbation, it seems plausible to assume that \( \gamma \) is a small parameter. We may also expect then that \( \rho \ll 1 \).

If we suppose, in addition, that "renormalization effects" can be ignored, we may set the form factors in Eqs. (7) and (9) equal to one, and obtain as a zeroth approximation

\[ \tan^2 \theta_V = \frac{m_\pi^2}{m_K^2}, \]

which corresponds to about 0.28 radian. As more complete formulas, we obtain from our approximate calculation

\[ \tan^2 \theta_V = \left( \frac{1 - \gamma/2\sqrt{2}}{1 + \gamma/2\sqrt{2}} \right)^{1/2}, \]

\[ \tan^2 \theta_A = \left( \frac{1 - \gamma/2\sqrt{2}}{1 + \gamma/2\sqrt{2}} \right)^{1/2} \left( \frac{1 - \rho/2\sqrt{2}}{1 + \rho/2\sqrt{2}} \right)^{1/2}. \]

(10)

Empirically, the quantity \( \tan^2 \theta_V F_K / m_\pi \) has been determined\(^2\) from the ratio of the rates for the decays \( K^+ \rightarrow \mu^+ + \nu \) and \( \pi^+ \rightarrow e^+ + \nu \) assuming that the variation of the form factor \( F_K \) over the pion spectrum is negligible. The parameter \( \xi \) can be obtained from the shape of the spectrum for \( K_{\mu3} \) decay and from measurements of the longitudinal polarization of the \( \mu \) meson.\(^{10}\)

Our formula (10), together with the empirical value of about 0.26 radian for \( \tan^2 \theta_V F_K / m_\pi \), indicates a value of \( \xi \) around 0. This may well be in agreement with experiments.\(^{10}\)

The ratio of the decay rates for \( K^+ \rightarrow \mu^+ + \nu \) and \( \pi^+ \rightarrow \mu^+ + \nu \) determines \( \tan^2 \theta_A B_K / B_\pi \). The value of about 0.28 radian\(^2\) is quite compatible with Eq. (10), provided the corrections to the mass ratio are sufficiently small.

We see that the approximate theoretical expressions (10) give us some indication that the rotated orientation of the weak current in unitary space may be due to the same symmetry-breaking interaction which is also responsible for the mass splitting in the meson octet.

Most of this work has been done during my stay at the Imperial College in London, and it is a pleasure to thank Professor Abdus Salam for his kind hospitality and for interesting discussions. I am also much indebted to Professor L. Van Hove for his hospitality at CERN.

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\(^{1}\) Guggenheim Fellow.

\(^{2}\) Permanent address.


\(^{5}\) M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

\(^{6}\) M. Gell-Mann, Phys. Letters 5, 214 (1964); G. Zweig, (to be published).

\(^{7}\) Although abstracted from field theory, these commutation relations for the densities may be too special. For example, there could be additional terms on the right-hand sides of Eq. (4), which involve derivatives of \( \delta \) functions in such a way that they disappear after integration over \( \vec{x} \) or \( \vec{r} \). We assume here that possible additional terms are absent or negligible.


\(^{9}\) Our approximate equations imply also that the form factor \( F_K \) should be independent of \( k_8 = m_K^2 + \vec{R}_8^2 \). Here we consider \( F_K \) only for \( k_8 = m_K \). Correspondingly, we choose \( k_8 = m_{\pi^+} \) in the relation involving \( F_\pi \).

\(^{10}\) Relations like this are a reminder that the one-particle approximation to Eq. (4) may be rather crude, at least for values of \( E_\pi \) which deviate sizably from \( m_\pi \). Note, in this connection, that it is the requirement of a conserved vector current which influences the form of the first formula in Eq. (5).
