CM-P00057010

ON THE COMMUTATION RELATIONS OF WEAK CURRENTS

Reinhard Oehme ^{*)} and Gino Segrè ^{**)}

CERN - Geneva

^{*)} Guggenheim Fellow, on leave from the University of Chicago.

^{**)} National Science Foundation Postdoctoral Fellow.

There are several ways in which one can introduce the notion of universality for the weak interactions. But there always arises the question how to explain effects like the consistent suppression of strangeness changing leptonic decays of mesons and baryons, which seem to violate at least a naive universality principle. If we assume that the currents which enter into the weak Lagrangian have well defined transformation properties with respect to the approximate symmetries of strong interactions, then we may ask whether the breaking of these symmetries could have something to do with deviations from universality. We want to examine this possibility here in connection with the commutation relations of the current densities and certain assumptions about the divergence of the axial vector current.

Let us assume that the current densities $V^{(i)}(x)$ and $A^{(i)}(x)$ transform like components of octets under SU_3 , and that they satisfy the corresponding commutation relations ¹⁾. Then we consider the following models for the weak current involving the strongly interacting particles :

A)

$$J = \cos \theta \left(J^{(1)} + i J^{(2)} \right) + \sin \theta \left(J^{(4)} + i J^{(5)} \right), \quad (1)$$

where $J = V, A$, and where the angle $\theta = \theta_A = \theta_V$ has nothing to do with strong interactions ^{2),3)}. For some unspecified reason the weak coupling selects this component, which remains essentially unchanged, if the symmetry breaking part of the strong interaction is switched on or off. In this model θ is small and describes the suppression of $\Delta S = 1$ leptonic decays, except for renormalization corrections which are presumed to be negligible.

B)

$$J = \left(J^{(1)} + i J^{(2)} \right) + \left(J^{(4)} + i J^{(5)} \right), \quad (2)$$

2.

with

$$\begin{aligned}\bar{J}^{(1)} + i \bar{J}^{(2)} &= \cos \theta (\hat{\bar{J}}^{(1)} + i \hat{\bar{J}}^{(2)}) \\ \bar{J}^{(4)} + i \bar{J}^{(5)} &= \sin \theta (\hat{\bar{J}}^{(4)} + i \hat{\bar{J}}^{(5)})\end{aligned}$$

In this case we assume that the angle θ is due to the symmetry breaking part of the strong interactions, and in such a way that $\theta \rightarrow 0$ in the SU_3 limit⁴⁾. The caret denotes the currents in this limit.

c)

$$\bar{J} = (\bar{J}^{(1)} + i \bar{J}^{(2)}) + (\bar{J}^{(4)} + i \bar{J}^{(5)})$$

but now the $\Delta S = 0$ current does not change in the limit of SU_3 symmetry, i.e.,

$$\bar{J}^{(1)} + i \bar{J}^{(2)} = \hat{\bar{J}}^{(1)} + i \hat{\bar{J}}^{(2)},$$

whereas the strangeness-changing current has an approximately universal damping factor $\lambda \approx \lambda_V \approx \lambda_A$ such that

$$\bar{J}^{(4)} + i \bar{J}^{(5)} = \lambda (\hat{\bar{J}}^{(4)} + i \hat{\bar{J}}^{(5)}) \quad (3)$$

In a rough approximation we have $\lambda \approx \frac{m_\pi}{m_K}$, and hence $\lambda \rightarrow 1$ in the SU_3 limit^{1),5)}. The parameter λ is a "large" manifestation of the "small" symmetry breaking interaction, just like the pion-kaon mass difference.

Phenomenologically, the models A) and B) are equivalent, whereas model C) differs from the others only in so far as it does not affect the present situation in the ^{14}O problem.

We now want to consider these models in connection with commutation relations and with the assumption that the divergence of the axial vector currents is approximately proportional to the appropriate meson operators ²⁾. These field operators project out single pion or kaon states. Writing the strong Hamiltonian density in the form

$$H = H_0 + S^{(0)} + \gamma S^{(8)}, \quad (4)$$

and using the relation

$$\partial_\alpha A_{\alpha(x)}^{(j)} = -i \left[\int d^3y A_0^{(j)}(y), H(x) \right]_{x_0=y_0},$$

we find

$$\begin{aligned} \partial_\alpha (A^{(1)} + i A^{(2)})_\alpha &= - \left(\left(\frac{2}{3}\right)^{1/2} + \gamma \left(\frac{1}{3}\right)^{1/2} \right) (P^{(1)} + i P^{(2)}) \\ \partial_\alpha (A^{(4)} + i A^{(5)})_\alpha &= - \left(\left(\frac{2}{3}\right)^{1/2} - \gamma \left(\frac{1}{12}\right)^{1/2} \right) (P^{(4)} + i P^{(5)}) \end{aligned} \quad (5)$$

Here $S^{(0)}$ is the symmetric mass term, $\gamma S^{(8)}$ the symmetry breaking interaction, and $S^{(j)}(x)$, $P^{(i)}(x)$ are scalar and pseudoscalar densities respectively. Using Eq. (5), we obtain two relevant sets of commutation relations. The first relations are generated by I spin and K spin ; they are

$$\begin{aligned}
& \left[V_4^{(1)}(x) - i V_4^{(2)}(x), \partial_\beta (A^{(1)}(x') + i A^{(2)}(x'))_\beta \right]_{x_0 = x'_0} = \\
& = 2i \delta(x - x') \left(\left(\frac{2}{3}\right)^{1/2} + \gamma \left(\frac{1}{3}\right)^{1/2} \right) P^{(3)}(x), \\
& \left[V_4^{(4)}(x) - i V_4^{(5)}(x), \partial_\beta (A^{(4)}(x') + i A^{(5)}(x'))_\beta \right]_{x_0 = x'_0} = \\
& = 2i \delta(x - x') \left(\left(\frac{2}{3}\right)^{1/2} - \gamma \left(\frac{1}{12}\right)^{1/2} \right) \frac{1}{2} \{ P^{(3)}(x) + \sqrt{3} P^{(8)}(x) \},
\end{aligned} \tag{6}$$

and similar ones with V replaced by A . We note that these formulae do not involve any strangeness changing neutral currents or densities. They are analogous to the familiar commutators $[I_+, I_-] = 2I_3$ and $[K_+, K_-] = I_3 + \frac{3}{2}Y$ ⁶⁾. In contrast, our second set of commutation relations is related to the mixed commutators $[I_-, K_+] = L_+$ and $[K_-, I_+] = L_-$, where $L_\pm = F_6 \pm iF_7$, and hence they involve neutral $\Delta S = 1$ terms. We find for these relations

$$\begin{aligned}
& \left[V_4^{(4)}(x) - i V_4^{(5)}(x), \partial_\beta (A^{(4)}(x') + i A^{(5)}(x'))_\beta \right]_{x_0 = x'_0} = \\
& = i \delta(x - x') \left(\left(\frac{2}{3}\right)^{1/2} + \gamma \left(\frac{1}{3}\right)^{1/2} \right) (P^{(6)}(x) - i P^{(7)}(x)), \\
& \left[V_4^{(1)}(x) - i V_4^{(2)}(x), \partial_\beta (A^{(4)}(x') + i A^{(5)}(x'))_\beta \right]_{x_0 = x'_0} = \\
& = i \delta(x - x') \left(\left(\frac{2}{3}\right)^{1/2} - \gamma \left(\frac{1}{12}\right)^{1/2} \right) (P^{(6)}(x) + i P^{(7)}(x)),
\end{aligned} \tag{7}$$

and again similar formulae with V replaced by A .

Let us now consider matrix elements of Eqs. (6) and (7) with respect to meson and vacuum states. Using the assumed properties of the divergence of the axial currents, we obtain relations between the following matrix elements :

$$\begin{aligned}
 \langle \pi^0 | V_{\alpha}^{(1)} - i V_{\alpha}^{(2)} | \pi^+ \rangle &= (\pi_{\alpha} + \pi_{\alpha}^0) \sqrt{2} F_{\pi\pi} , \\
 \langle K^0 | V_{\alpha}^{(1)} - i V_{\alpha}^{(2)} | K^+ \rangle &= (K_{\alpha} + K_{\alpha}^0) F_{KK} , \\
 \langle \pi^0 | V_{\alpha}^{(4)} - i V_{\alpha}^{(5)} | K^+ \rangle &= \left\{ (K_{\alpha} + \pi_{\alpha}) + (K_{\alpha} - \pi_{\alpha}) \xi \right\} \frac{1}{\sqrt{2}} F_{\pi K} , \\
 \langle 0 | A_{\alpha}^{(1)} - i A_{\alpha}^{(2)} | \pi^- \rangle &= i \pi_{\alpha} B_{\pi} , \quad \text{etc.}
 \end{aligned} \tag{8}$$

Here the invariant form factors are, of course, functions of the appropriate invariants, but in the following we ignore this dependence. We also have assumed that the vector currents are F type only. From Eq. (6), and the corresponding ones involving the axial vector currents, we obtain the approximate relations ⁵⁾

$$\begin{aligned}
 \frac{F_{\pi K} (1 + \xi) B_{\pi}}{F_{\pi\pi} B_{\pi}} &\approx \frac{m_{\pi}^2}{m_K^2} \frac{1 - \gamma/2\sqrt{2}}{1 + \gamma/\sqrt{2}} , \\
 \frac{B_K^2}{B_{\pi}^2} &\approx \frac{m_{\pi}^2}{m_K^2} \frac{1 - \gamma/2\sqrt{2}}{1 + \gamma/\sqrt{2}} \frac{1 - \rho/2\sqrt{2}}{1 + \rho/\sqrt{2}} ,
 \end{aligned} \tag{9}$$

where $\rho = \langle S^{(8)} \rangle_0 / \langle S^{(0)} \rangle_0$. On the other hand, from the mixed commutators (7) we find a formula like

$$\frac{F_{\pi K} (1 - \xi) B_{\pi}}{F_{KK} B_K} \approx \frac{m_K^2}{m_{\pi}^2} \frac{1 + \gamma/\sqrt{2}}{1 - \gamma/2\sqrt{2}} . \tag{10}$$

We do not consider the corresponding commutator with the axial current, because if we use our approximation consistently, it gives relations involving also matrix elements like $\langle K^- | A^{(4)} - iA^{(5)} | \pi^+ \pi^- \rangle$. Since we assume that $\gamma \ll 1$, and hence also $\rho \ll 1$, and since $F_{KK}(0) = F_{\pi\pi}(0) = 1$ as a consequence of the conserved vector current, we see that Eqs. (9) and (10) are consistent only in the limit of SU_3 symmetry. In this limit we have $\gamma = \rho = 0$, $m_\pi = m_K$ and $\xi = 0$. Our assumptions concerning the divergence $\partial_\alpha A_\alpha^{(i)}$, and their use in the commutation relations of the currents, are, of course, rather restrictive. Hence it may not be surprising that we are forced into the SU_3 limit if we use all relevant commutation relations. On the other hand, if we find some reason that only the first set of commutation relations should be used in connection with the "almost conserved" axial current, then the formulae (9) indicate an universal suppression factor $\lambda \approx \lambda_A \approx \lambda_V \approx m_\pi/m_K$. We note that these commutation relations are related to those of the I spin and K spin operators, which generate the corresponding SU_2 subgroups of SU_3 .

Let us now consider these results in view of the models A) - C). The first model is certainly consistent with the SU_3 limits of (9) and (10), but in addition we must assume that changes due to $\gamma S^{(8)}$ are small *). The second model requires that the $\Delta S = 1$ current vanishes in the limit $\gamma \rightarrow 0$. But if the currents satisfy commutation relations like (6) and/or (7), we would expect rather $\tan \theta \rightarrow 1$ than $\theta \rightarrow 0$. However, $\tan \theta \rightarrow 1$ in the SU_3 limit may be difficult to reconcile with CVC. The third model is very suggestive if we consider only the relations (9), but we must add some further assumptions in order to justify the omission of the mixed commutation relations (7), at least within the approximate scheme we have used here. At present, we can only speculate about this, but it may be of interest in connection with the question of neutral currents in weak interactions.

We would like to thank Professor L. Van Hove for his kind hospitality at CERN.

*) There may also be other arguments which favour this model. See, for example, Ref. 7).

R E F E R E N C E S

- 1) M. Gell-Mann, Phys. Rev. 125, 1067 (1962).
- 2) M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).
- 3) N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).
- 4) N. Cabibbo, Phys. Rev. Letters 12, 62 (1964).
- 5) R. Oehme, Phys. Rev. Letters 12, 550 (1964), 12, 604 (E) (1964).
- 6) Compare J.J. de Swart, Rev. Mod. Phys. 35, 916 (1963).
- 7) M. Gell-Mann, Proceedings of the Tenth Annual International Conference on High Energy Physics, 1960, (Interscience Publishers, New York, 1960).