CM-P00057004

CHARGE CONSERVATION AND THE MASS OF THE PHOTON

B. Zumino *)

CERN - Geneva

*) National Science Foundation Senior Post-doctoral Fellow, on sabbatical leave from New York University.

It is very natural to try to understand the conservation of nucleon number (and possibly other conservation laws) in analogy with charge conservation, by postulating the existence of an associated vector field capable of undergoing gauge transformations. Attempts to build the theory along these lines would encounter a very serious difficulty if the vector field were to have zero mass as a consequence of the gauge invariance of the theory. That this need not be the case was observed by Schwinger and others ¹⁾⁻³⁾ who have pointed out, in a qualitative way, how a mass could arise in a gauge invariant theory. Schwinger ²⁾ has also considered a concrete model, quantum electrodynamics with vanishing spinor mass in a two-dimensional space time. He has given an exact solution for the propagators in the Landau gauge, from which one can see that the "photon" acquires a mass in presence of interaction, although the theory is gauge invariant. Clearly the model should not be taken too seriously, its main value being that of a simple mathematical example, which shows how the various propagators develop the desired analytic behaviour. Nevertheless, since this is the only case known where a gauge particle actually has a non-vanishing mass, it may be worth while to point out some difficulties which one encounters in this model.

In order to study the properties of the Heisenberg fields as they operate in the physical Hilbert space, one must employ the Coulomb gauge. In this gauge the model has been studied by Brown ⁴⁾ who has given an exact solution for the propagators ; his solution could also be obtained by transforming, in a formal way, the solution in the Landau gauge. The Hamiltonian of the theory shows that there are no quantized degrees of freedom belonging to the photon field, only a c number "Coulomb" interaction of the spinor with itself. Although this fact decreases the significance of this particular model for the general problem of mass and gauge invariance, one may still wonder if the long range Coulomb potential (it increases linearly with distance) will actually generate long range forces in the theory. One discovers easily that the forces (at least between external charge distributions) are short range. However, one finds the following difficulty in connection with the total charge. It is easy to see ⁴⁾ that the current density vector J_μ satisfies the operator equations

2.

$$\partial_0 J^0 + \partial_1 J^1 = 0$$

$$\partial_0 J^1 + \partial_1 J^0 = \frac{e}{\pi} E$$

$$\partial_1 E = e J^0$$

$$\partial_0 E = -e J^1$$

where $E = F_{01} = -F_{10}$ is the "electric field". Combining these equations one obtains

$$\left(\partial_0^2 - \partial_1^2 + \frac{e^2}{\pi} \right) J_\mu = 0, \quad (1)$$

which exhibits the finite mass occurring in the theory. However, from (1), by integration, one sees that the total charge

$$Q = e \int J^0 dx_1$$

satisfies

$$\left(\partial_0^2 + \frac{e^2}{\pi} \right) Q = 0$$

This equation shows that Q cannot be constant unless it vanishes. Similarly one sees that Q cannot be independent of the particular Lorentz frame chosen unless it vanishes. On the other hand, the existence in the theory of the operator ψ satisfying

$$[Q, \psi] = -e \psi$$

prevents the operator Q from vanishing identically. We conclude that the Coulomb gauge formulation of the theory is not truly covariant, unless one is willing to restrict oneself to states of zero charge. A study of the propagators in the Coulomb gauge confirms the above results. The difficulty stems from the fact that the operators (and propagators) of the theory cannot all vanish at infinity in space-like direction and this is undoubtedly due to the vanishing of the spinor mass as much as to the one-dimensionality of space ⁵⁾. However, these are also the reasons why the vector particle has a mass in the model ³⁾.

The above results should be compared with the situation in the Landau gauge which is manifestly covariant. Since we cannot use operators, we must consider propagators involving the current density, e.g. ⁶⁾,

$$\begin{aligned} \langle (\psi(x) J_\mu(\xi) \psi^*(y))_+ \rangle &= \left[-\alpha \varepsilon_{\mu\nu} \partial^\nu (\Delta_c(\xi-x) - \Delta_c(\xi-y)) + \right. \\ &\quad \left. + \partial_\mu (D_c(\xi-x) - D_c(\xi-y)) \right] \langle (\psi(x) \psi^*(y))_+ \rangle. \end{aligned} \quad (2)$$

No difficulty occurs with the total charge. From (2) one obtains, upon space integration,

$$\begin{aligned} \langle (\psi(x) Q(\xi_0) \psi^*(y))_+ \rangle &= \\ &= \frac{e}{2} \left[\varepsilon(x^0 - \xi^0) + \varepsilon(\xi^0 - y^0) \right] \langle (\psi(x) \psi^*(y))_+ \rangle, \end{aligned}$$

which shows that the total charge behaves as expected. However (2) exhibits a zero-mass singularity in its ξ dependence; this singularity is not present in the Coulomb gauge operators or propagators. It is required to ensure the

4.

validity of the identity

$$\partial^\mu \langle (\psi J_\mu \psi^*)_+ \rangle = [\delta(\xi - y) - \delta(\xi - x)] \langle (\psi(x) \psi^*(y))_+ \rangle$$

and would be present, in the Landau gauge, irrespective of the number of dimensions of space time.

REFERENCES AND FOOTNOTES

- 1) J. Schwinger, Phys. Rev. 125, 397 (1962) ;
D.G. Boulware and W. Gilbert, Phys. Rev. 126, 1563 (1962).
- 2) J. Schwinger, Phys. Rev. 128, 2425 (1962).
- 3) B. Zumino, 1962 Eastern Theoretical Physics Conference, edited by M.E. Rose,
Gordon and Breach Publishers, New York (1963).
- 4) L.S. Brown, Nuovo Cimento 29, 617 (1963) ;
See also Ref. ³⁾.
- 5) Because of this difficulty, inherent in the boundary conditions, we must
conclude that the formal proof of covariance in the Coulomb gauge is
incorrect. Similarly one cannot trust here the usual proof of equivalence
between the Landau and the Coulomb gauge.
- 6) The matrix α is Pauli's σ_3 , $\epsilon_{\mu\nu}$ is the antisymmetric tensor in
two-dimensional space time and Δ_c the causal propagator relative to
the mass $e/\sqrt{\pi}$, while D_c is the propagator for zero mass.