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SYMPLECTIC SYMMETRY OF HADRONS

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A B S T R A C T

A model is proposed for SU_3 symmetry in which the known SU_3 multiplets of hadrons (strongly interacting particles) are considered as bound states of basic particles t of spin $1/2$, baryon number 1 and electric charge $0, \pm 1$, and of their antiparticles $\bar{t}$. These basic particles form SU_3 triplets and are regarded to be heavy compared to the proton mass. The known meson octets and singlet are supposed to result from binding of a t and a $\bar{t}$, giving in terms of SU_3 symmetry the structure $\bar{3} \times 3 = 1 + 8$. The baryon octet, decuplet (and possibly singlet) are given the composition $\bar{t}tt$, with the SU_3 structure $3 \times 3 \times 3 = 1 + 8 + 8 + 10$. These requirements, and the condition to minimize the number of basic particles, lead uniquely to the introduction of 6 basic particles t , one 3 triplet of charges $+, 0, 0$ and one $\bar{3}$ triplet of charges $0, +, +$. They are associated with the values $1/3$ and $-1/3$ of a new quantum number Z . For the known particles Z is equal to the baryon number.

One then introduces a higher symmetry group incorporating the new quantum number as well as SU_3 . This is done within the framework of simple Lie groups, and we are led to select the symplectic group in six dimensions Sp_6 , whose lowest dimensional representation exactly accommodates the six basic particles t . With this higher symmetry the model predicts a SU_3 triplet of new pseudoscalar mesons, a SU_3 sextet of new vector mesons, as well as their antiparticles, with mass separations calculable from the masses of the known mesons. For the known pseudoscalar mesons and baryons one finds the usual Gell-Mann Okubo mass formulae, whereas for the known vector mesons one derives a mass relation identical to the formula obtained in Schwinger's W_3 symmetry model. Our model predicts in addition many new baryon resonances of high mass and large width. All mass predictions are obtained by taking a mass splitting operator of form $\Delta m_1 + \Delta m_2$, where Δm_1 breaks Sp_6 symmetry but preserves SU_3 , while Δm_2 breaks SU_3 . Δm_1 is regarded as larger than Δm_2 and is taken to second order, while for Δm_1 only first order effects are included. We are led to this assumption of a symmetry violation which is larger for Sp_6 than for SU_3 by the circumstance that the first order mass splitting effects leave the new mesons to be degenerate in mass with the known ones, a situation which is unattractive in several respects.

1. INTRODUCTION

The present paper discusses in some detail a composite model of SU_3 symmetry proposed in a recent letter ¹⁾. This model gives a construction of the known SU_3 multiplets of hadrons (strongly interacting particles) by means of two basic triplets of spin $\frac{1}{2}$ particles with integral charge and baryon number unity.

The unitary symmetry scheme of Gell-Mann ²⁾ and Ne'eman ³⁾ groups the hadrons in multiplets formed by the single-valued representations of the group SU_3/Z_3 , where Z_3 , the centre of SU_3 , is the multiplicative group of the three numbers $\exp(2\pi mi/3)$, $m = 0, 1, 2$. These single-valued representations are in order of increasing dimension the representations $1, 8, 10, \bar{10}, 27, \dots$. In addition SU_3/Z_3 has three-valued representations like $3, \bar{3}, 6, \bar{6}, \dots$. All are single-valued representations of the universal covering group SU_3 .

It is tempting to speculate about the possible existence and properties of hitherto unknown particles which would belong to the lowest dimensional representations 3 and $\bar{3}$ of SU_3 , and to try to construct the known hadron multiplets as combinations of such basic triplets. It is therefore not surprising that many attempts have been made recently in this direction ^{4), 5)}. If in addition the basic triplets have masses much higher than those of the known hadrons, which are then bound states with very strong binding, one may understand better the success of mass-splitting formulae established to lowest order of perturbation. The mass differences within a multiplet, while appreciable when compared with the actual masses of the multiplet members, would be quite small in comparison with the masses of the basic triplets. The latter would set the fundamental mass scale of the theory and might be as heavy as 5 or 10 proton masses.

Gell-Mann and Zweig ⁴⁾ have proposed a very elegant basic triplet model which postulates one basic triplet of spin $\frac{1}{2}$ particles A belonging to the representation 3 and the triplet $\bar{3}$ of the anti-particles $\bar{A}$. The known meson multiplets (8 for the pseudoscalar mesons π, K, η ; 1 and 8 for the vector mesons ρ, K^*, ω, ϕ) are constructed by the combination $\bar{A}A$ ($\bar{3} \times 3 = 1+8$), whereas the combination AAA ($3 \times 3 \times 3 = 1+8+8+10$) gives the known baryon multiplets $\sqrt{8}$ for the baryons N, Λ, Σ, Ξ , 10 for the decuplet formed by the isobars $N^*(1237 \text{ MeV})$, $Y_1^*(1385 \text{ MeV})$, $\Xi^*(1533 \text{ MeV})$ and the Ω^- particle of mass 1680 MeV ⁶⁾, and perhaps 1 for the isobar $Y_0^*(1405 \text{ MeV})$. This implies that the "quarks" or "aces" A are particles with the unusual values $\frac{1}{3}$ for the baryon number N and $\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}$ for the electric charge Q .

The composite model we have proposed ¹⁾ attempts a similar construction in terms of basic spin $\frac{1}{2}$ particles with $N = \pm 1$ and $Q = 0, \pm 1$. If the meson singlet and octets have to be obtained from two basic particles through the product $\bar{3} \times 3$, and the baryon octet and decuplet from three basic particles through the product $3 \times 3 \times 3$, the minimum number of basic particles with $N = 1$ is 6, they form a triplet T belonging to the representation 3 and one Θ belonging to $\bar{3}$, and their electric charges are entirely fixed (1,0,0 for T ; 0,1,1 for Θ). As indicated earlier ¹⁾ and discussed in more detail in the present paper, this trion model leads naturally to the consideration of a symplectic generalization of SU_3 symmetry, based on the group Sp_6 .

If on the other hand, one is willing to construct the baryon octet out of three basic particles but the baryon decuplet out of five, it is sufficient to introduce 4 basic particles with $N = 1$ and spin $\frac{1}{2}$, which form a triplet T belonging to 3 and a singlet S . While S is neutral the charges of T can now be either 1,0,0 or 0,-1,-1. The baryon octet is obtained through $\bar{3} \times 3 \times 1$, the decuplet through $\bar{3} \times 3 \times \bar{3} \times 3 \times 1$. This model leads naturally to embedding SU_3 into a higher symmetry characterized by the group SU_4 ⁷⁾. Of course many other models are possible if some of the basic particles are bosons or have $N \neq 1$ ⁵⁾.

The following two sections describe the properties of the trion model and the arguments which lead to the consideration of the symplectic symmetry group Sp_6 . The consequences of this higher symmetry are presented in sections 4 and 5. The paper ends with some general remarks.

4.

2. THE TRION MODEL

The trion model introduces two basic triplets T and Θ of particles with spin $\frac{1}{2}$ and unit baryon number. T belongs to the representation 3 of SU_3 , Θ to the conjugate representation $\bar{3}$. The mesons are supposed to be bound states $\bar{T}T$, or $\bar{\Theta}\Theta$, or combinations thereof. The baryons and baryon isobars are obtained as bound states $\bar{\Theta}TT$. If we require the trions T and Θ to have charges $Q = 0, \pm 1$, their actual charges can be uniquely determined. This is done by adopting for the basic triplets a generalized Gell-Mann Nishijima relation

$$Q = I_3 + \frac{Y}{2} + \frac{D}{3} \quad (2.1)$$

where $\frac{D}{3} = \langle Q \rangle$ is the average charge of the triplet ⁸⁾ Y and I_3 are as usual the hypercharge and the third component of the isospin. The fact that $\bar{\Theta}TT$ must reproduce the charges of the known baryons and isobars requires that

$$2D_T - D_\Theta = 0.$$

Taking into account that we allow only charges $0, \pm 1$ for the particles in T and Θ , which implies

$$D_T = 1 \text{ or } -2, \quad D_\Theta = -1 \text{ or } 2,$$

we are left with the only possibility

$$D_T = 1, \quad D_\Theta = 2.$$

The resulting charges of the basic trions are given in Tables 1 and 2.

TABLE 1

T-trions ($N=1$, $D=1$, spin $s=\frac{1}{2}$, SU_3 repr. 3)			
STATES	T^+	T^0	T'^0
charge Q	1	0	0
hypercharge Y	$1/3$	$1/3$	$-2/3$
isospin I	$1/2$	$1/2$	0
I_3	$1/2$	$-1/2$	0

TABLE 2

Θ -trions ($N=1$, $D=2$, spin $s=\frac{1}{2}$, SU_3 repr. $\bar{3}$)			
STATES	Θ^0	Θ^+	Θ'^+
Q	0	1	1
Y	$-1/3$	$-1/3$	$2/3$
I	$1/2$	$1/2$	0
I_3	$-1/2$	$1/2$	0

The model assumes that all known hadrons which have been successfully assigned to SU_3 multiplets are bound states of trions with the value zero for the new quantum number D . As mentioned in the introduction, one is tempted to speculate that the trions have quite a large mass, perhaps of the order of 5 to 10 proton masses ⁹⁾, and it is also very natural to assume that the condition $D = 0$ characterizes the lightest and therefore most stable combinations of trions. All $D = 0$ combinations of 2 and 3 trions are listed in Table 3. They are supposed to give the known meson singlet and octets, as well as the known baryon and antibaryon singlets, octets and decuplets. One easily shows that all $D = 0$ combinations of 4 or more trions can be obtained by combining together the four combinations $\bar{T}T$, $\bar{e}e$, $\bar{e}TT$ and $\bar{T}T\bar{e}$ mentioned in Table 3.

TABLE 3

Number of trions in composite particles	Representations	Baryonic number N	Spin and parity
2	$\bar{T}T = 1+8$	0	0^- or 1^-
	$\bar{e}e = 1+8$	0	0^- or 1^-
3	$\bar{e}TT = 1+8+8+10$	1	$1/2$ or $3/2$
	$e\bar{T}\bar{T} = 1+8+8+\bar{10}$	-1	$1/2$ or $3/2$
spin and parity have been given for s-state binding			

We now discuss the question of conservation of the new quantum number D . In contrast with the hypercharge Y in the conventional SU_3 symmetry scheme, conservation of D can be violated not only by weak interactions, but also by electromagnetic, and even strong interactions ¹⁰⁾.

If, however, we choose to assume exact conservation of Q and I_3 in strong and electromagnetic transitions, non-conservation of D is subject to the following two restrictions. Firstly it requires non-conservation of Y so as to ensure conservation of the quantity $Y + \frac{2}{3} D$ which is a function of Q and I_3 by virtue of the generalized Gell-Mann Nishijima relation (2.1). This of course implies conservation of Y for the known hadrons which are characterized by $D = 0$. Secondly, the violation of D and Y by strong interactions must be sufficiently weak not to affect the success of SU_3 symmetry and of the mass formulae for the known hadrons.

The violations of D and Y conservation just mentioned have an important consequence. If, as we proposed before, the $D \neq 0$ particles are heavier than the $D = 0$ ones, they will decay into the latter with conservation of $Y + \frac{2}{3} D$ at a rate at least as fast as allowed by electromagnetic interactions. All new hadrons will then appear as resonances. They will be distinguishable from the ordinary $D = 0$ hadrons by the fact that they form SU_3 multiplets where the Gell-Mann Nishijima formula holds only after adding a non-vanishing constant $\frac{1}{3} D$ to its right-hand side.

3. THE HIGHER SYMMETRY GROUP

The occurrence of the new quantum number D , and of the resulting new hadrons characterized by $D \neq 0$, suggests the consideration of possible higher symmetries as a means to predict what their properties might be. It is natural to study this problem by postulating the existence of a higher symmetry group which would incorporate SU_3 as a subgroup and contain at the same time the additive quantum number D . We have chosen to restrict ourselves to simple Lie groups of rank three, the three commuting elements of the Lie algebra corresponding to the additive quantum numbers I_3, Y and D . Table 4 lists all simple Lie algebras of rank three and the corresponding simple Lie groups ¹¹⁾.

TABLE 4

Simple Lie groups and algebras of rank 3			
Algebra (dimension)	Group	Basic representation	SU_3 submultiplets
A_3 (15)	SU_4	4	$1 + 3$
		$\bar{4}$	$1 + \bar{3}$
	$SO_6 = SU_4/Z_2$	$6 = \bar{6}$	$3 + \bar{3}$
	SU_4/Z_4	$15 = \bar{15}$	$1 + 3 + \bar{3} + 8$
B_3 (21)	C	8	$1 + 1 + 3 + \bar{3}$
	$SO_7 = C/Z_2$	7	$1 + 3 + \bar{3}$
C_3 (21)	Sp_6	6	$3 + \bar{3}$
	Sp_6/Z_2	14	$3 + \bar{3} + 8$

The conventional symbols are used for the algebras and the groups, except for C which denotes the covering group of SO_7 . The basic representation is defined as the lowest dimensional single-valued representation out of which all single-valued representations can be obtained by multiplication and reduction. All representations of B_3 and C_3 are self-conjugate.

In view of the fact that our basic trions T, Θ are six in number we concentrate our attention on the groups SO_6 and Sp_6 ; their basic representation has dimension 6 and can accommodate exactly the basic particles of our model. That these groups occur naturally in the present context can be seen by considering the following bilinear form

$$f = \sum_{j=1,2,3} t_j t_{-j} \quad (3.1)$$

where $t_{1,2,3}$ denote the trion fields T^+, T^0, T'^0 respectively, and $t_{-1,-2,-3}$ the fields $\Theta^0, \Theta^+, \Theta'^+$. Clearly the transformations of SU_3 leave f invariant, since under them the t_i ($i = 1, 2, 3$) transform among themselves like the representation 3, while the t_{-i} do this like $\bar{3}$. Let us consider now six dimensional unitary transformations mixing all six t fields

$$t'_\beta = \sum_{\alpha} u_{\beta\alpha} t_{\alpha} ; \quad \alpha, \beta = \pm 1, \pm 2, \pm 3 \quad (3.2)$$

and let us restrict ourselves to the group G of those unitary transformations which have determinant one and leave f invariant:

$$\sum_{j=1,2,3} t'_j t'_{-j} = \sum_{j=1,2,3} t_j t_{-j}. \quad (3.3)$$

If, in calculating the right-hand side of (3.3) from insertion of (3.2) in the left-hand side, one treats the products $t_{\alpha} t_{\beta}$ as commutative ($t_{\alpha} t_{\beta} = t_{\beta} t_{\alpha}$ for all α, β) the resulting group G is the group SO_6 (12). If the products $t_{\alpha} t_{\beta}$ are taken as anticommutative ($t_{\alpha} t_{\beta} + t_{\beta} t_{\alpha} = 0$ for all α, β), one obtains the group Sp_6 of unitary symplectic transformations in six dimensions (for the latter case invariance of f implies automatically that the determinant of the transformation is one, see next section).

Our further discussion is entirely devoted to exploring the consequences of Sp_6 symmetry for the trion model. The main argument we can put forward to justify the choice of Sp_6 over SO_6 is that, the trions being spin $\frac{1}{2}$ particles, it is more natural to treat their fields as anticommuting than as commuting. In addition Sp_6 is simply connected whereas SO_6 has the universal covering group SU_4 with a basic representation of dimension 4. We realize, however, that such arguments have only limited significance and, in connection with a detailed investigation of SU_4 by D. Amati, J. Prentki and two of the present authors ⁷⁾, the implications of SO_6 symmetry have been considered.

4. THE Sp_6 MULTIPLICETS OF MESONS AND BARYONS

The group Sp_6 can be defined as the group of six dimensional unitary transformations (3.2) which verify (3.3) for anticommuting t_α , or, equivalently, which verify

$$\sum_{\alpha\beta} h_{\alpha\beta} u_{\alpha\alpha'} u_{\beta\beta'} = h_{\alpha'\beta'} \quad (4.1)$$

where h is the antisymmetric matrix

$$h = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \quad (4.2)$$

0 and I denote the three dimensional null and unit matrices. Rows and columns in (4.2) are labelled in the order $1, 2, 3, -1, -2, -3$. One can prove ¹³⁾ that for all transformations of the group

$$\det u = 1. \quad (4.3)$$

The infinitesimal transformations of the group are of the form

$$u = 1 + i \epsilon \lambda \quad (4.4)$$

where ϵ is an infinitesimal real number and λ a Hermitian matrix verifying

$$\lambda^t h + h \lambda = 0. \quad (4.5)$$

(λ^t = transposed of λ). There are 21 linearly independent generators λ . They can be chosen to be the matrices listed in Appendix A, where $\lambda_1, \dots, \lambda_8$ correspond to the usual generators of the subgroup SU_3 . In particular the operators for the quantum numbers I_3 and Y are

$$I_3 = \frac{1}{2} \lambda_3, Y = \frac{1}{\sqrt{3}} \lambda_8. \quad (4.6)$$

The third generator commuting with λ_3 and λ_8 is λ_{21} . It is connected to the new quantum number D by the equations

$$\frac{D}{3} = \frac{N-Z}{2}, Z = \sqrt{\frac{2}{3}} \lambda_{21}. \quad (4.7)$$

N is the baryon number. Z will be used henceforth instead of D . The known particles are characterized by $Z = N$, and the generalized Gell-Mann Nishijima relation becomes

$$Q = I_3 + \frac{1}{2}(N + Y - Z). \quad (4.8)$$

Comparison with the Gell-Mann Nishijima relation in its original form

$$Q = I_3 + \frac{1}{2}(N + S) \quad (4.9)$$

suggests to regard $Y-Z$ as a generalized strangeness. It has to be conserved by strong and electromagnetic interactions whereas conservation of Y and Z can be violated separately, although strong interaction violation must be sufficiently moderate not to affect the success of SU_3 symmetry and SU_3 mass formulae (see the end of section 2).

The structure constants of Sp_6 are given in Appendix B. Appendix C lists the lowest representations with their Young diagrams and their contents in SU_3 representations; the value of Z for each SU_3 submultiplet is indicated. The multiplication table of low dimensional representation of Sp_6 is given in Appendix D.

The basic rule of multiplication is

$$6 \times 6 = 1 + 14 + 21. \quad (4.10)$$

The left-hand side represents the transformation law of a general covariant tensor $m_{\alpha\beta}$ of rank two

$$m'_{\alpha\beta} = \sum_{\alpha' \beta'} u_{\alpha\alpha'} u_{\beta\beta'} m_{\alpha' \beta'} \quad (4.11)$$

The reduction in the right-hand side of (4.10) is obtained by separating $m_{\alpha\beta}$ in its symmetric and antisymmetric parts

$$\frac{1}{\sqrt{2}}(m_{\alpha\beta} + m_{\beta\alpha}) = s_{\alpha\beta}, \quad \frac{1}{\sqrt{2}}(m_{\alpha\beta} - m_{\beta\alpha}) = a_{\alpha\beta} \quad (4.12)$$

The invariant

$$\sum_{\alpha\beta} h_{\alpha\beta} m_{\alpha\beta} = \frac{1}{\sqrt{2}} \sum_{\alpha\beta} h_{\alpha\beta} a_{\alpha\beta} = g \quad (4.13)$$

gives representation 1, the symmetric tensor $s_{\alpha\beta}$ gives representation 21¹⁴), and representation 14 is the remaining part

$$a_{\alpha\beta} - \frac{\sqrt{2}}{6} gh_{\alpha\beta}.$$

We note also that all representations of Sp_6 are self-conjugate. This follows from the fact that the 6 dimensional representation

$$t'_{\alpha} = \sum_{\beta} u_{\alpha\beta} t_{\beta} \quad (4.14)$$

and its conjugate

$$\bar{t}'_{\alpha} = \sum_{\beta} u_{\alpha\beta}^* \bar{t}_{\beta} \quad (4.15)$$

are equivalent, as is seen by noting that $\sum_{\beta} h_{\alpha\beta} \bar{t}_{\beta}$ transforms like t_{α} (use (4.1) in the proof).

The mesons are obtained as combinations $\bar{t}_\alpha t_\beta$, where $t_{1,2,3}$ are identified with the trions T^+, T^0, T'^0 and $t_{-1,-2,-3}$ with the trions e^0, e^+, e'^+ . The Hermitian tensor $\bar{t}_\alpha t_\beta$ which is once covariant, once contravariant can be reduced either directly or by considering the corresponding twice covariant tensor

$$m_{\alpha\beta} = \sum_{\alpha'} h_{\alpha\alpha'} \bar{t}_{\alpha'} t_\beta \quad (4.16)$$

and carrying out its reduction by means of (4.12). One thus obtains the invariant $\sum_{\alpha} \bar{t}_{\alpha} t_{\alpha}$, the 21 representation given by the symmetric part $s_{\alpha\beta}$ of $m_{\alpha\beta}$, and the 14 representation given by the anti-symmetric part $a_{\alpha\beta}$ (see (4.12) for the normalization). It is natural to put the vector mesons in 21, because it is the regular representation and also because it contains a singlet and an octet as SU_3 submultiplets with $Z = 0$. The full set of 21 vector mesons is given in the following symmetric matrix,

$\sqrt{2} \rho'^{-}$	ρ'^0	$\overline{K'^*0}$	$\frac{1}{\sqrt{2}}(\omega^0 + \rho^0)$	ρ^+	K^{*+}
	$\sqrt{2} \rho'^{+}$	$K'^*{-}$	ρ^{-}	$\frac{1}{\sqrt{2}}(\omega^0 - \rho^0)$	K^{*0}
	$\sqrt{2} \phi'^{-}$	K^{*-}	$\overline{K^{*0}}$	ϕ^0	
		$-\sqrt{2} \rho'^{-}$	$-\rho'^0$	$-\overline{K'^*0}$	
			$-\sqrt{2} \rho'^{+}$	$-\overline{K'^*+}$	
				$-\sqrt{2} \phi'^{+}$	

(4.17)

The unwritten matrix elements are determined by the symmetry condition. Upper indices $+, 0, -$ refer to the electric charge. The $Z = 0$ mesons are in the upper right submatrix. The ω^0 and ϕ^0 are combinations of the SU_3 singlet ϕ_0^0 and the isosinglet member ω_0^0 of the SU_3 octet :

$$\omega^0 = \frac{1}{\sqrt{3}}(\omega_0^0 + \sqrt{2}\phi_0^0), \quad \phi^0 = \frac{1}{\sqrt{3}}(\phi_0^0 - \sqrt{2}\omega_0^0). \quad (4.17')$$

The mixing angle is given by

$$\tan \theta = 1/\sqrt{2} \quad \text{or} \quad \theta = 35^\circ 16' \quad (4.18)$$

quite close to the value derived from the observed masses of the ω^0 and ϕ^0 particles ¹⁵⁾. The new vector mesons are an isotriplet $\rho^{+, 0, -}$, an isodoublet $K^{*, 0, +}$, an isosinglet ϕ^{+} , and their antiparticles. The ρ', K'^* and ϕ' form representation $\bar{6}$ of SU_3 and have $Z = -\frac{2}{3}$, while $\bar{\rho}', \bar{K}'^*, \bar{\phi}'$ have $Z = \frac{2}{3}$ and form representation 6 of SU_3 .

Since the pseudoscalar mesons established up to now are eight in number and form a SU_3 octet, we choose to assign them to the Sp_6 representation 14 whose $Z = 0$ elements form precisely a SU_3 octet. The full set of 14 pseudoscalar mesons is given in the following anti-symmetric matrix

$$\begin{array}{c|ccc|ccc} 0 & \overline{\pi}^{+0} & \overline{K}^{+0} & \frac{\eta^0}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ & \\ & 0 & K^{+0} & \pi^- & \frac{\eta^0}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 & \\ & & 0 & K^- & K^0 & -\sqrt{\frac{2}{3}}\eta^0 & \\ \hline & & & 0 & \pi^{+0} & K^{+0} & \\ & & & & 0 & K^{+0} & \\ & & & & & 0 & \end{array} \quad (4.19)$$

Antisymmetry fixes the unwritten matrix elements. The known particles are again in the upper right submatrix. The new particles are the isosinglet π'^0 and the isodoublet $K'^0, ^+$, which have $Z = -\frac{2}{3}$ and form the SU_3 representation 3, as well as their antiparticles $\bar{\pi}', \bar{K}'$. A further discussion of vector and pseudoscalar mesons will be given in the next section on the basis of the mass formulae. Appendix E shows the trion combinations $\bar{t}t$ which correspond to the various mesons.

The baryons and $p_{3/2}$ baryon isobars are to be obtained from trion combinations $\bar{t}_\alpha t_\beta t_\gamma$, which reduce into the following Sp_6 representations

$$6 \times 6 \times 6 = 6 + 6 + 6 + 14' + 56 + 64 + 64. \quad (4.20)$$

The known particles should have $Z = N$, i.e., $Z = 1$. The corresponding SU_3 submultiplets are 1 in $14'$, 10 in 56 and 8 in 64 (see Appendix C). The baryon octet should therefore be placed in the representation 64, and the isobar decuplet in 56. As we shall see in the next section the many new particles contained in these representations can be expected to have appreciably larger masses and to be highly unstable for decay into $Z = 1$ baryons and $Z \neq 0$ (i.e., new) mesons. Most of them would therefore be very difficult to observe, even as resonances.

We end this section by giving the group theoretical properties of the electromagnetic current operator in our symplectic model. In terms of the basic trions the current has the same Sp_6 transformation properties as

$$\begin{aligned} j &\propto \bar{T}^+ T^+ + \bar{e}^+ e^+ + \bar{e}'^+ e'^+ = \\ &= \frac{1}{2} (\bar{T}^+ T^+ + \bar{T}^0 T^0 + \bar{T}'^0 T'^0 + \bar{e}^0 e^0 + \bar{e}^+ e^+ + \bar{e}'^+ e'^+) \\ &- \frac{1}{6} (\bar{T}^+ T^+ + \bar{T}^0 T^0 + \bar{T}'^0 T'^0 - \bar{e}^0 e^0 - \bar{e}^+ e^+ - \bar{e}'^+ e'^+) \\ &+ \frac{1}{3} (2\bar{T}^+ T^+ - \bar{T}^0 T^0 - \bar{T}'^0 T'^0 - 2\bar{e}^0 e^0 + \bar{e}^+ e^+ + \bar{e}'^+ e'^+). \end{aligned} \quad (4.21)$$

As can be seen from Appendix E, the Sp_6 and SU_3 multiplets to which the three lines in the right-hand side belong are $(1,1)$, $(21,1)$ and $(21,8)$ respectively. The last one has the same SU_3 transformation property as the current in the octet model. From what has been said above, the new ($Z \neq 1$) baryons are expected to be highly unstable, so that we are mainly interested in the consequences of (4.21) for the $Z = 1$ baryons. The presence of the terms $(1,1)$ and $(21,1)$ implies that the predictions of SU_3 are modified by addition of a SU_3 singlet term to the current. While electromagnetic mass effects remain unaffected, a common constant is added to the magnetic moments of all members of the baryon multiplets. In particular, for the baryon octet, the predictions are

$$\begin{aligned} \mu(p) &= \mu(\Sigma^+), \quad \mu(\Xi^0) = \mu(n), \quad \mu(\Xi^-) = \mu(\Sigma^-) \\ \mu(\Lambda) &= \frac{1}{3} [\mu(\Sigma^0) + 2\mu(n)], \quad \mu(\Sigma^0) = \frac{1}{2} [\mu(\Sigma^+) + \mu(\Sigma^-)] \\ \langle \Sigma | \mu | \Lambda \rangle &= \frac{\sqrt{3}}{2} [\mu(\Lambda) - \mu(\Sigma^0)] \end{aligned} \quad (4.22)$$

where $\langle \Sigma | \mu | \Lambda \rangle$ is the transition moment for $\Sigma \rightarrow \Lambda + \gamma$. As is well known, the octet model of SU_3 symmetry imposes in addition the restriction that the sum of all eight moments vanishes, which implies $\mu(\Lambda) = \frac{1}{2} \mu(n)$.

5. THE MASS FORMULAE

Following Okubo's method so successfully applied in the case of SU_3 ¹⁶⁾, we suppose the mass splitting operator to transform like the $I=Y=Z=0$ components of the product 6×6 of two basic representations. This amounts to giving it the transformation properties of

$$\begin{aligned} \Delta m \propto m_T (\bar{T}^+ T^+ + \bar{T}^0 T^0) + m'_T \bar{T}'^0 T'^0 \\ + m_e (\bar{e}^0 e^0 + \bar{e}^+ e^+) + m'_e \bar{e}'^+ e'^+ \end{aligned} \quad (5.1)$$

with arbitrary values for m_T, m'_T, m_e, m'_e . From the reduction formula (4.10) it follows that

$$\Delta m = \Delta m(1,1) + \Delta m(21,1) + \Delta m(21,8) + \Delta m(14,8) \quad (5.2)$$

where $\Delta m(a,b)$ belongs to representation a of Sp_6 and b of SU_3 . Since the well-known mass formulae of the octet model are based on a mass splitting operator of form $\Delta m(1) + \Delta m(8)$, the predictions derived from (5.2) will be compatible, within any SU_3 multiplet, with the familiar Gell-Mann Okubo mass formula. As in the octet model, mass formulae can be written for the masses themselves or their squares. For the heavier particles like baryons and vector mesons, the difference between the two formulations is rather small. For the pseudoscalar mesons the mass formula is much better satisfied by the squares of the masses, which may be understood by supposing that the masses are practically zero in absence of splitting ¹⁷⁾.

The first order effects of (5.2) on the masses of the mesons are surprisingly stringent. The products 14×14 and 21×21 each contain once the representations 1, 14 and 21. Hence the effects of $\Delta m(21,1)$ and $\Delta m(21,8)$ are of the form

$$\begin{aligned}
\langle \chi' | \Delta m(21,1) | \chi \rangle &\propto \langle \chi' | \lambda_{21} | \chi \rangle \propto z_\chi \langle \chi' | \chi \rangle \\
\langle \chi' | \Delta m(21,8) | \chi \rangle &\propto \langle \chi' | \lambda_8 | \chi \rangle \propto y_\chi \langle \chi' | \chi \rangle
\end{aligned} \quad (5.3)$$

where χ, χ' are meson states and y_χ, z_χ are the values of Y and Z for χ . Since masses are even for particle-antiparticle conjugation while Y and Z are odd, the terms (5.3) do not contribute to the mass splittings. The latter originate entirely from $\Delta m(14,8)$ whose transformation properties are the same as

$$\begin{aligned}
\Delta m(14,8) &\propto \overline{T^+}T^+ + \overline{T^0}T^0 + \overline{e^0}e^0 + \overline{e^+}e^+ \\
&\quad - 2(\overline{T^+}T^0 + \overline{e^+}e^0).
\end{aligned} \quad (5.4)$$

Clearly this is invariant for all transformations of Sp_6 which leave the two subspaces $(t_1=T^+, t_2=T^0, t_{-1}=e^0, t_{-2}=e^+)$ and $(t_3=T'^0, t_{-3}=e'^+)$ invariant. These transformations form a subgroup $Sp_4 \times Sp_2$. The following mass relations obtain immediately (we drop charge indices since all members of an isospin multiplet have the same mass)

$$m_{\eta'} = m_\eta, \quad m_{K'} = m_K \quad (5.5)$$

$$m_{\rho'} = m_\rho, \quad m_{K'^*} = m_{K^*}, \quad m_{\phi'} = m_\phi \quad (5.6)$$

$$m_\omega = m_\rho, \quad \langle \phi | \Delta m | \omega \rangle = 0. \quad (5.7)$$

The following additional relations hold for the $Z = 0$ mesons (we write them for the masses squared)

$$m_\eta^2 + 3m_\eta^2 = 4m_K^2 \quad (5.8)$$

$$m_\rho^2 + 3 \langle \omega_0 | \Delta m | \omega_0 \rangle^2 = 4m_{K^*}^2 \quad (5.9)$$

$$m_\phi^2 = 2m_{K^*}^2 - m_\rho^2. \quad (5.10)$$

For the known mesons these relations go beyond the predictions of SU_3 by giving through (4.17) and (5.7) the masses and mixing parameter of ω and ϕ . The second equation (5.7) shows that in the present approximation the states given by (4.17') are the physical states of the ω and ϕ mesons. It is therefore satisfactory that the value (4.18) of the mixing angle agrees with experiment. As is already implied by this fact the theory gives in (5.7) and (5.10) rather good predictions for the masses of ω and ϕ ¹⁸⁾.

As to the $Z \neq 0$ mesons, we obtain the surprising result that they are degenerate in mass with their $Z = 0$ counterparts. If this is valid, the decay modes are easily predicted

- (i) $\pi'_1 \equiv \frac{1}{\sqrt{2}}(\pi' + \bar{\pi}') \rightarrow 2\gamma$ and $\pi'_2 \equiv \frac{1}{\sqrt{2}}(\pi' - \bar{\pi}') \rightarrow 3\gamma$ electromagnetically (Y-Z conserved),
- (ii) $K' \rightarrow K + 2\gamma$ electromagnetically (Y-Z conserved; we suppose the K' a little heavier than the K due to higher order mass splittings),
- (iii) $\rho' \rightarrow \pi' + \pi$, $K'^* \rightarrow K' + \pi$ and $\phi' \rightarrow K' + K$ by strong interactions (Y and Z conserved).

While the existence of such new mesons would probably not be in contradiction with known facts if we assume that their production is not very abundant (perhaps a few percent of pion production in high energy collisions, which would be about a factor 5 less abundant than strange particle production), their masses do not satisfy the general expectation expressed in section 2 that the new particles characterized by $Z \neq N$ would be heavier, and therefore more unstable than the $Z = N$ particles. An additional difficulty results from the existence of π' with the same mass as the ordinary pion ¹⁹⁾. Indeed, if we assume that the interaction responsible for the decay of the vector mesons is Sp_6 invariant, we find that the branching ratio

$$\Gamma(\omega^0 \rightarrow \pi^0 + \overline{\pi^0}) / \Gamma(\rho^0 \rightarrow \pi^+ + \pi^-) \quad (5.11)$$

is unity, predicting for the ω a neutral width two orders of magnitude larger than observed. Such a situation would require that the vector mesons be essentially stable for the very strong interactions which satisfy Sp_6 invariance, so that their finite width would be due to a semi-strong symmetry breaking interaction. Taken to first order, none of the terms occurring in the symmetry breaking interaction (5.2) used in the mass splitting calculation is able to change the branching ratio (5.11), but $\Delta m(21,1)$ taken to second order changes it to an arbitrary value. It is remarkable that the second order effect of $\Delta m(21,1)$ on the meson masses produces also a mass shift of all $Z \neq 0$ mesons, retaining nevertheless valid predictions for the masses of the $Z = 0$ mesons.

This suggests the following proposal for taking care of the two difficulties created by our first order mass predictions. Among the symmetry breaking interactions we assume that those which preserve SU_3 symmetry are stronger than those which violate it, and we treat the second order effects of the former as comparable to the first order effects of the latter. For the meson masses this amounts to adding to the mass splittings already calculated from (5.2) the effect of an additional term $[\Delta m(21,1)]^2$ in Δm . The predictions for pseudoscalar mesons then reduce to (in terms of the squares)

$$m_{K^0}^2 - m_{\pi^0}^2 = m_K^2 - m_{\pi}^2 \quad (5.12)$$

while the Gell-Mann Okubo relation (5.8) remains unchanged. For the $Z \neq 0$ vector mesons one obtains instead of (5.6)

$$m_{\phi}^2 - m_{K^{*0}}^2 = m_{K^{*+}}^2 - m_{\rho^+}^2 = m_{K^{*-}}^2 - m_{\rho^-}^2. \quad (5.13)$$

The $Z = 0$ vector mesons are affected in such a way that $\langle \phi_0 | \Delta m | \phi_0 \rangle$ becomes a free parameter while $\langle \phi_0 | \Delta m | \omega_0 \rangle$ and (5.9) are unchanged. This modifies the mixing and the masses m_ϕ, m_ω of the physical ϕ and ω particles in such a way that the following equation is satisfied

$$(m_\phi^2 - m_\rho^2)(m_\omega^2 - m_\rho^2) = \frac{4}{3}(m_{K^*}^2 - m_\rho^2)(m_\phi^2 + m_\omega^2 - 2m_{K^*}^2). \quad (5.14)$$

This relation agrees remarkably well with the experimental values²⁰⁾. Furthermore, since the first order mass relations (5.7), (5.10) are satisfied with a fairly good approximation by the experimental masses, we conclude that the physical ω and ϕ particles are not very different from the states (4.17'), i.e., the mixing angle is not very different from the value (4.18).

Thanks to the $[\bar{\Delta} m(21,1)]^2$ symmetry breaking term we have now achieved our goal of allowing the $Z \neq 0$ mesons to become heavier than the known mesons, eliminating of course at the same time the difficulty related to $\omega \rightarrow \eta' + \bar{\eta}'$ decay. We would certainly expect the mass shift of $Z \neq 0$ mesons produced by $[\bar{\Delta} m(21,1)]^2$ to be appreciable, maybe of order 500 GeV, and the new mesons will now be able to decay through a greater variety of modes. Of course, as before, at least one of them will decay with violation of Y and Z but conservation of $Y-Z$, and this decay can be electromagnetic or strong, but not as strong as the SU_3 symmetric interactions (it, of course, might be weak if Y and Z happen to be conserved by electromagnetic and strong interactions).

In this form of our model, the mass splitting term $\Delta m(21,1)$ must be regarded as quite appreciable, and this will have an important effect on the baryon masses. Indeed, for the latter and in contrast with mesons, $\Delta m(21,1)$ contributes mass splittings to first order, and we can therefore expect the splitting between baryon SU_3 submultiplets belonging to different values of Z to be much larger than the mass separations inside SU_3 submultiplets. Since the $Z \neq 0$ mesons will

not all be extremely heavy, the $Z \neq 1$ baryons will have many possibilities to decay through the very strong Sp_6 invariant interaction into several lighter particles, and their detection as broad resonances emerging above a large background may be extremely difficult. If this is the case the multiplets 64 and 56 in which we accommodate the baryon octet and decuplet may well turn out to contain only a few SU_3 submultiplets manifesting themselves as recognizable resonances, all others being lost in a large continuous background.

6. CONCLUDING REMARKS

While we have at present no argument indicating that the particular model of basic SU_3 triplets discussed in the present paper is more likely to be realized in nature than many of the other models proposed up till now, we regard its study as instructive because it illustrates various points which may have more general validity.

In contrast with some other models, the new particles predicted by the symplectic model may all be very short-lived because their decay may proceed through electromagnetic or (if they are sufficiently heavy) semi-strong interactions. In particular the heaviest among the new particles may well have so many decay channels open that their width would be quite large, making their observation as resonances very difficult. This would certainly be true for the basic particles of the model, the trions T and Θ , which are estimated to have masses of perhaps 5 to 10 GeV. Thus the new particles most likely to be detected are not the fundamental triplets, nor the many higher baryon resonances predicted in the high dimensional representations 56 and 64, but rather the new pseudoscalar and vector mesons. The latter might have masses in the GeV range. They could be produced either singly by semi-strong interactions (ordinary strange particles being produced at the same time to conserve $Y-Z$) or in pairs by very strong interactions. The production cross-sections should be rather low, probably still appreciably smaller than for associated production of ordinary strange particles. The most characteristic property of these new mesons would of course be that they form SU_3 triplets or sextets satisfying (2.1) with $D \neq 0$, their mass separations being related to the masses of the known mesons as given in Eqs. (5.12) and (5.13).

Whereas the latter facts are specific predictions of Sp_6 symmetry and of our assignment of pseudoscalar and vector mesons to representations 14 and 21, the more general feature that the least unstable among the new particles are relatively light objects belonging to SU_3 triplets or sextets is undoubtedly common to many similar models. It should be regarded as a general indication that basic triplets exist, and the properties of these new particles may reveal whether any higher symmetry than SU_3 is obeyed by very strong interactions. The question as to what the basic triplets are would of course be much more difficult to answer and may even be a theoretically ill-defined problem.

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A P P E N D I X A

Infinitesimal generators of the basic representation of Sp_6

$$\lambda_1 = \begin{vmatrix} \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} \quad \lambda_2 = \begin{vmatrix} \cdot & -i & \cdot & \cdot & \cdot & \cdot \\ i & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & -i & \cdot \\ \cdot & \cdot & \cdot & i & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} \quad \lambda_3 = \begin{vmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

$$\lambda_4 = \begin{vmatrix} \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -1 & \cdot & \cdot \end{vmatrix} \quad \lambda_5 = \begin{vmatrix} \cdot & \cdot & -i & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ i & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -i \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & i & \cdot & \cdot \end{vmatrix} \quad \lambda_6 = \begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -1 \\ \cdot & \cdot & \cdot & \cdot & -1 & \cdot \end{vmatrix}$$

$$\lambda_7 = \begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -i & \cdot & \cdot & \cdot \\ \cdot & i & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -i \\ \cdot & \cdot & \cdot & \cdot & i & \cdot \end{vmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{vmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 2 \end{vmatrix} \quad \lambda_9 = \begin{vmatrix} \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

(continued)

A P P E N D I X B

Structure constants of Sp_6

The completely antisymmetric structure constants f_{ijk} are defined by

$$[\lambda_i, \lambda_j] = i f_{ijk} \lambda_k.$$

We list only the non-vanishing f_{ijk}

i	j	k	f_{ijk}	i	j	k	f_{ijk}	i	j	k	f_{ijk}
1	2	3	2	4	5	8	$\sqrt{3}$	7	9	11	1
1	4	7	1	4	9	14	1	7	10	12	1
1	5	6	-1	4	10	13	-1	7	13	17	$-\sqrt{2}$
1	9	16	$\sqrt{2}$	4	11	16	$\sqrt{2}$	7	13	19	$\sqrt{2}$
1	9	18	$\sqrt{2}$	4	11	20	$\sqrt{2}$	7	14	18	$-\sqrt{2}$
1	10	15	$-\sqrt{2}$	4	12	15	$-\sqrt{2}$	7	14	20	$\sqrt{2}$
1	10	17	$-\sqrt{2}$	4	12	19	$-\sqrt{2}$	8	9	10	$2/\sqrt{3}$
1	11	14	1	5	9	13	1	8	11	12	$-1/\sqrt{3}$
1	12	13	-1	5	10	14	1	8	13	14	$-1/\sqrt{3}$
2	4	6	1	5	11	15	$-\sqrt{2}$	8	15	16	$2/\sqrt{3}$
2	5	7	1	5	11	19	$\sqrt{2}$	8	17	18	$2/\sqrt{3}$
2	9	15	$-\sqrt{2}$	5	12	16	$-\sqrt{2}$	8	19	20	$-4/\sqrt{3}$
2	9	17	$\sqrt{2}$	5	12	20	$\sqrt{2}$	9	10	21	$4/\sqrt{6}$
2	10	16	$-\sqrt{2}$	6	7	8	$\sqrt{3}$	11	12	21	$4/\sqrt{6}$
2	10	18	$\sqrt{2}$	6	9	12	1	13	14	21	$4/\sqrt{6}$
2	11	13	1	6	10	11	-1	15	16	21	$4/\sqrt{6}$
2	12	14	1	6	13	18	$\sqrt{2}$	17	18	21	$4/\sqrt{6}$
3	4	5	1	6	13	20	$\sqrt{2}$	19	20	21	$4/\sqrt{6}$
3	6	7	-1	6	14	17	$-\sqrt{2}$				
3	11	12	1	6	14	19	$-\sqrt{2}$				
3	13	14	-1								
3	15	16	2								
3	17	18	-2								

A P P E N D I X C

Lowest dimensional representations of Sp_6 ,
with their Young diagrams, their contents
in SU_3 representations and the values of Z
corresponding to the various SU_3 submultiplets

Representations of Sp_6	1	6	14				14'				21			
Young diagram
Submultiplets SU_3	1	3	$\bar{3}$	$\bar{3}$	8	3	1	$\bar{6}$	6	1	6	1	8	$\bar{6}$
Z	0	$1/3$	$-1/3$	$2/3$	0	$-2/3$	1	$1/3$	$-1/3$	-1	$2/3$	0	0	$-2/3$

Representations of Sp_6	56								64						
Young diagram
Submultiplets SU_3	10	3	15	$\bar{15}$	$\bar{3}$	$\bar{10}$	8	3	$\bar{6}$	15	$\bar{15}$	6	$\bar{3}$	8	
Z	1	$1/3$	$1/3$	$-1/3$	$-1/3$	-1	1	$1/3$	$1/3$	$1/3$	$-1/3$	$-1/3$	$-1/3$	-1	

A P P E N D I X D

Multiplication table for the representations of Sp_6 up to 64×64

6	14	14'	21	56	64	
1+14+21	6+14'+64	14+70	6+56+64	21+126' +189	14+21+70 +90+189	6
	1+14+21 +70+90	6+64+126	14+21+70 +189	56+64+216 +448	6+14'+56 +64+64+126 +216+350	14
		1+21+84 +90	14'+64 +216	70+189 +525	14+21+70 +90+189 +512	14'
			1+14+21 +90+126' +189	6+56+64 +252+350 +448	6+14'+56 +64+64+126 +216+350 +448	21
				1+14+21 +90+126' +189+385 +462+924 +924'	14+21 +70+90 +126'+189 +189+512 +525+924 +924'	56
					1+14+14 +21+21+70 +70+70+84 +90+90+126' +189+189 +189+385 +512+512 +525+924	64

A P P E N D I X E

Mesons in terms of the trion combinations $\bar{t}t$

$$6 \times 6 = 1 + 14 + 21$$

Representation 1

Name of part.	Z	Rep. SU_3	Y	I	I_3	trion combination
	0	1	0	0	0	$\frac{1}{\sqrt{6}} \left[\bar{e}^+ e^+ + \bar{e}^0 e^0 + \bar{e}^- e^- + \bar{T}^+ T^+ + \bar{T}^0 T^0 + \bar{T}^- T^- \right]$

(continued)

Representation 14

Name of part.	Z	Rep. of SU ₃	Y	I	I ₃	trion combination
$\overline{K}^{\prime 0}$	2/3	$\overline{3}$	-1/3	1/2	1/2	$\frac{1}{\sqrt{2}} [\overline{e}^0 T^{\prime 0} - \overline{e}^{\prime +} T^+]$
$K^{\prime -}$	2/3	$\overline{3}$	-1/3	1/2	-1/2	$\frac{1}{\sqrt{2}} [\overline{e}^+ T^{\prime 0} - \overline{e}^{\prime +} T^0]$
$\overline{\pi}^{\prime 0}$	2/3	$\overline{3}$	2/3	0	0	$\frac{1}{\sqrt{2}} [\overline{e}^0 T^0 - \overline{e}^{\prime +} T^+]$
K^+	0	8	1	1/2	1/2	$\frac{1}{\sqrt{2}} [\overline{e}^0 e^{\prime +} + \overline{T}^{\prime 0} T^+]$
K^0	0	8	1	1/2	-1/2	$\frac{1}{\sqrt{2}} [\overline{e}^+ e^{\prime +} + \overline{T}^{\prime 0} T^0]$
π^+	0	8	0	1	1	$\frac{1}{\sqrt{2}} [\overline{e}^0 e^+ + \overline{T}^0 T^+]$
π^0	0	8	0	1	0	$\frac{1}{2} [\overline{e}^0 e^0 - \overline{e}^+ e^+ + \overline{T}^+ T^+ - \overline{T}^0 T^0]$
π^-	0	8	0	1	-1	$\frac{1}{\sqrt{2}} [\overline{e}^+ e^0 + \overline{T}^+ T^0]$
η	0	8	1	0	0	$\frac{1}{2\sqrt{3}} [\overline{e}^+ e^+ + \overline{e}^0 e^0 + \overline{T}^0 T^0 + \overline{T}^+ T^+ - 2\overline{T}^{\prime 0} T^{\prime 0} - 2\overline{e}^{\prime +} e^{\prime +}]$
$\overline{K}^0$	0	8	-1	1/2	1/2	$\frac{1}{\sqrt{2}} [\overline{e}^{\prime +} e^+ + \overline{T}^0 T^{\prime 0}]$
K^-	0	8	-1	1/2	-1/2	$\frac{1}{\sqrt{2}} [\overline{e}^{\prime +} e^0 + \overline{T}^+ T^{\prime 0}]$
$K^{\prime +}$	-2/3	3	1/3	1/2	1/2	$\frac{1}{\sqrt{2}} [\overline{T}^{\prime 0} e^+ - \overline{T}^0 e^{\prime +}]$
$K^{\prime 0}$	-2/3	3	1/3	1/2	-1/2	$\frac{1}{\sqrt{2}} [\overline{T}^{\prime 0} e^0 - \overline{T}^+ e^{\prime +}]$
$\pi^{\prime 0}$	-2/3	3	-2/3	0	0	$\frac{1}{\sqrt{2}} [\overline{T}^0 e^0 - \overline{T}^+ e^+]$

(continued)

Name of part.	Z	Rep. of SU ₃	Y	I	I ₃	trion combination
$\overline{\rho}^{1-}$	2/3	6	2/3	1	1	$\overline{e}^0 T^+$
$\overline{\rho}^{10}$	2/3	6	2/3	1	0	$\frac{1}{\sqrt{2}} [\overline{e}^+ T^+ + \overline{e}^0 T^0]$
$\overline{\rho}^{0+}$	2/3	6	2/3	1	-1	$\overline{e}^+ T^0$
$\overline{K}^{10*}$	2/3	6	-1/3	1/2	1/2	$\frac{1}{\sqrt{2}} [\overline{e}^0 T^{10} + \overline{e}^{1+} T^+]$
$\overline{K}^{1*-}$	2/3	6	-1/3	1/2	-1/2	$\frac{1}{\sqrt{2}} [\overline{e}^+ T^{10} + \overline{e}^{1+} T^0]$
ϕ^{1-}	2/3	6	-4/3	0	0	$\overline{e}^{1+} T^{10}$
K^{*+}	0	8	1	1/2	1/2	$\frac{1}{\sqrt{2}} [\overline{e}^0 e^{1+} - \overline{T}^{10} T^+]$
K^{*0}	0	8	1	1/2	-1/2	$\frac{1}{\sqrt{2}} [\overline{e}^+ e^{1+} - \overline{T}^{10} T^0]$
ρ^+	0	8	0	1	1	$\frac{1}{\sqrt{2}} [\overline{e}^0 e^+ - \overline{T}^0 T^+]$
ρ^0	0	8	0	1	0	$\frac{1}{2} [\overline{e}^0 e^0 - \overline{e}^+ e^+ + \overline{T}^0 T^0 - \overline{T}^+ T^+]$
ρ^-	0	8	0	1	-1	$\frac{1}{\sqrt{2}} [\overline{e}^+ e^0 - \overline{T}^+ T^0]$
ω_o^0	0	8	0	0	0	$\frac{1}{2\sqrt{3}} [\overline{e}^0 e^0 + \overline{e}^+ e^+ - 2\overline{e}^{1+} e^{1+} - \overline{T}^+ T^+ - \overline{T}^0 T^0 + 2\overline{T}^{10} T^{10}]$
$\overline{K}^{*0}$	0	8	-1	1/2	1/2	$\frac{1}{\sqrt{2}} [\overline{e}^{1+} e^+ - \overline{T}^0 T^{10}]$
$\overline{K}^{*-}$	0	8	-1	1/2	-1/2	$\frac{1}{\sqrt{2}} [\overline{e}^{1+} e^0 - \overline{T}^+ T^{10}]$
ϕ_o^0	0	1	0	0	0	$\frac{1}{\sqrt{6}} [\overline{e}^+ e^+ + \overline{e}^0 e^0 + \overline{e}^{1+} e^{1+} - \overline{T}^+ T^+ - \overline{T}^0 T^0 - \overline{T}^{10} T^{10}]$
ϕ^{1+}	-2/3	$\overline{6}$	4/3	0	0	$\overline{T}^{10} e^{1+}$
$\overline{K}^{1+*}$	-2/3	$\overline{6}$	1/3	1/2	1/2	$\frac{1}{\sqrt{2}} [\overline{T}^{10} e^+ + \overline{T}^0 e^{1+}]$
$\overline{K}^{10*}$	-2/3	$\overline{6}$	1/3	1/2	-1/2	$\frac{1}{\sqrt{2}} [\overline{T}^+ e^{1+} + \overline{T}^{10} e^0]$
ρ^{1+}	-2/3	$\overline{6}$	-2/3	1	1	$\overline{T}^0 e^+$
ρ^{10}	-2/3	$\overline{6}$	-2/3	1	0	$\frac{1}{\sqrt{2}} [\overline{T}^+ e^+ + \overline{T}^0 e^0]$
ρ^{1-}	-2/3	$\overline{6}$	-2/3	1	-1	$\overline{T}^+ e^0$

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- 8) D is related to the triality number considered by L.C. Biedenharn and E.C. Fowler, preprint (December 1963).
- 9) As noted by many people, this order of magnitude is suggested by estimating the relative size of 1st order and 2nd order perturbation effects in the mass splittings, and estimating from there what would be the magnitude of the 0th order.

- 10) We are grateful to Professors V.F. Weisskopf and D. Amati for having drawn our attention on this point.
- 11) As reference on the theory of Lie groups we quote :
 G. Racah - Group theory and spectroscopy, CERN 61-8, reprint of lectures delivered at the Institute for Advanced Study, Princeton (spring 1951) ;
 M. Hamermesh - Group theory (Addison-Wesley Publ. Comp. (1962) U.S.A.).
- 12) This is the group of real rotations in six dimensions, with determinant one, as is readily seen by going over to the variables

$$\begin{aligned} \tau_j &= t_j + t_{-j} \\ \tau_{-j} &= i(t_j - t_{-j}) \end{aligned}$$

- 13) The proof follows easily from (3.2) and (3.3) if the t_α are treated as anticommuting. One notices that $\prod_\alpha t'_\alpha = (\det u) \prod_\alpha t_\alpha$, and that raising (3.3) to the third power gives $\prod_\alpha t'_\alpha = \prod_\alpha t_\alpha$.
- 14) The 21 dimensional representation is the regular representation also realized by the infinitesimal generators of the group. These generators are the matrices λ verifying (4.5). The equivalence with symmetric tensors is given through $s = h\lambda$, the symmetry following immediately from (4.5) and from antisymmetry of h .
- 15) The significance of this point will appear later when we shall find that ω^0 and ϕ^0 are very close to the physical ω and ϕ mesons. For the experimental values of θ see :
 J.J. Sakurai, Phys. Rev. 132, 434 (1963).
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- 17) See, F. Gürsey et al., Ref. ⁵⁾ (section I).

- 18) These conclusions on ω , ϕ masses and mixing are identical to those derived from another model by Gürsey, Lee and Nauenberg ⁵⁾.
- 19) We are very grateful to Professor J. Prentki for raising this important point.
- 20) Equation (5.14) has also been obtained by Schwinger ⁵⁾ in the framework of another model. Its form shows clearly that it is a generalization of (5.7) and (5.10).