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ON MUON CAPTURE IN HEAVY NUCLEI

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A B S T R A C T

The theory of the Primakoff plot is developed in a simple way. Corrections advocated by Klein and Wolfenstein are shown to be absent when reasonable correlation functions, such as those of the ideal Fermi gas, are used. However, development of the gas model throws doubt on the accuracy of the theory. The excellent agreement with experiment must be in part accidental.

I. INTRODUCTION

The theory of muon capture in heavy nuclei has been developed especially by Primakoff ¹⁾. His predictions have been confirmed by experiment ^{2),3)}, both as regards absolute capture rates (calculated on the universal Fermi interaction hypothesis) and their dependence on fractional neutron excess. However, Klein and Wolfenstein ⁴⁾ have found that certain correction terms in the Primakoff theory, ignored by him, are in fact large and spoil the agreement. We argue here that these terms are absent if ideal Fermi gas correlation functions are used and perhaps should be absent with any physically reasonable correlations. Using the ideal gas functions largely restores Primakoff's original results. But further development of the gas model throws doubt on the qualitative validity of two of the basic approximations. These are a linear approximation in fractional neutron excess and a procedure (including a non-relativistic approximation) which we shall refer to briefly as the closure approximation. We conclude that the excellence of the fit with experiment must be in part accidental.

II. THE PRIMAKOFF PLOT

We will only summarize the initial steps of the argument, fully treated by Primakoff ¹⁾ and by Luyten, Rood and Tolhoek ⁵⁾. The general Lorentz invariant matrix element is constructed for the reaction between free particles

An effective interaction Hamiltonian is found which, in a non-relativistic description of the nucleons, correctly reproduces the matrix element to terms of first order in the nucleon momenta divided by nucleon mass. This effective Hamiltonian is then used in the nuclear problem. The procedure omits possible many body terms in the capture interaction, including for example those which might arise when in the calculation of "small components" of the spinors the nucleons are not considered to be free. There results the following formula for the capture rate, treating the muon momentum and the initial nucleon momentum as negligible

$$\Lambda(a \rightarrow b) = \frac{V_{ab}^2}{2\pi} \int \frac{d\hat{U}_{ab}}{4\pi} \left\{ G_V^2 |\int \psi_1|^2 + G_A^2 |\int \sigma_{\hat{U}}|^2 + (G_P^2 - 2G_P G_A) \left| \hat{U} \cdot \int \sigma_{\hat{U}} \right|^2 \right\}$$

The initial and final nuclear states are a and b , V_{ab} is the energy of the neutrino and $\hat{U}$ is a unit vector in the direction in which it is emitted, and the following notation is used for matrix elements :

$$\int \psi_1 = \langle b | \sum_i \tau_i^{(-)} \exp(-i\hat{U} \cdot \mathbf{r}_i) \phi(\mathbf{r}_i) | a \rangle$$

$$\int \sigma_{\hat{U}} = \langle b | \sum_i \tau_i^{(-)} \exp(-i\hat{U} \cdot \mathbf{r}_i) \phi(\mathbf{r}_i) \sigma_{\hat{U}i} | a \rangle$$

Here ϕ is the muon wave function, and $\nu = \sum_{ab} \hat{\nu}_{ab}$. With the conventional universal (V-A) theory and reasonable assumptions about form factors Fujii and Primakoff⁵⁾ obtain the following values for the effective coupling constants

$$G_V = 1.01 G$$

$$G_A = -1.38 G$$

$$G_P = -0.58 G$$

where G is the basic universal weak coupling constant

$$G = 1.01 \times 10^{-5} / (\text{proton mass})^2$$

The effective coupling constants actually depend, very slightly, on the value of ν , and those quoted are for $\nu \approx 85$ MeV.

The total capture rate is obtained by summing over all states b , a being the nuclear ground state. The so-called "closure approximation" is then made; in this ν_{ab} is replaced by a constant value ν . If at the same time the muon wave function product $\phi^+(r_i) \phi(r_j)$ is replaced by an average value $|\phi|_{av}^2$, we have finally for the total capture rate

$$\Lambda = \frac{\nu^2}{2\pi} |\phi|_{av}^2 \int \frac{d\Omega}{4\pi} \langle a | \sum_{ij} \tau_i^{(+)} \tau_j^{(-)} \quad (1)$$

$$e^{i\nu \cdot (\vec{r}_i - \vec{r}_j)} \left\{ G_V^2 + G_A^2 \vec{\sigma}_i \cdot \vec{\sigma}_j + (G_P^2 - 2 G_P G_A) \vec{\sigma}_i \cdot \frac{\nu}{\nu} \frac{\nu}{\nu} \cdot \vec{\sigma}_j \right\} | a \rangle$$

Plausible values for $|\phi|_{av}^2$ have been computed by Sens²⁾ by averaging over nuclear density distributions. It remains to discuss the ground state matrix element in (1).

The terms with $i = j$ in the summation give at once

$$\begin{aligned} \langle a | \sum_{i=j} \tau_i^{(+)} \tau_j^{(-)} \dots | a \rangle \\ = Z (G_V^2 + 3 G_A^2 + G_P^2 - 2 G_P G_A) \end{aligned}$$

The remaining ($i \neq j$) contribution contains Fourier transforms of certain correlation functions which can be expected to have finite range. Then at least in very large nuclei it may be reasonable to use the correlation functions calculated for homogeneous nuclear matter with the appropriate density and proton to neutron ratio. Therefore we assume the form

$$\begin{aligned} \int \frac{d\mathbf{U}}{4\pi} \langle a | \sum_{i,j} \tau_i^{(+)} \tau_j^{(-)} e^{i\mathbf{U} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \{ G_V^2 \dots \} | a \rangle \\ = -\frac{1}{2} A \left\{ G_V^2 F_V(\nu, \rho, \alpha) + 3 G_A^2 F_A(\nu, \rho, \alpha) \right. \\ \left. + (G_P^2 - 2 G_P G_A) F_P(\nu, \rho, \alpha) \right\} \quad (2) \end{aligned}$$

where ρ is the nuclear matter density and α the fractional neutron excess $(N-Z)/A$. Now the functions F are even in α . For we may interchange neutrons and protons by a rotation in isospace; on the l.h.s. of (2) this merely interchanges $\tau^{(+)}$ and $\tau^{(-)}$, which has the same effect as interchanging $\mathbf{r}_i$ and $\mathbf{r}_j$, or changing the sign of $\mathbf{U}$, which does not affect the integral. Assuming the F 's to depend smoothly on α near $\alpha = 0$ (which is at least true in the ideal gas model) and ignoring terms of order α^2 , the dependence of F on α can be ignored. Then

$$\Lambda = \frac{\nu^2}{2\pi} |\phi|_{av}^2 Z (G_V^2 + 3G_A^2 + G_P^2 - 2G_P G_A) \left(1 - \frac{A}{2Z} F(\nu, \eta)\right) \quad (3)$$

where F is the appropriate coupling constant weighted average of $(F_V(\nu, \eta, 0), F_A(\nu, \eta, 0),$ and $F_P(\nu, \eta, 0)$. To first order in α

$$\frac{A}{2Z} = \frac{4(A-Z)}{2A}$$

So we have the result of Primakoff

$$\Lambda / \langle \eta \rangle = C \left(\frac{\nu}{m}\right)^2 \left\{ 1 - \frac{A-Z}{2A} \delta \right\} \quad (4)$$

where

$$\begin{aligned} \langle \eta \rangle &= Z |\phi|_{av}^2 \\ C &= \frac{m^2}{2\pi} (G_V^2 + 3G_A^2 + G_P^2 - 2G_P G_A) \\ \delta &= 4F(\nu, \eta) \end{aligned}$$

If we take ν and η to be constants, then δ also is a constant. Sens²⁾ and Telegdi³⁾ find that this formula fits the data remarkably well, even up to the nuclei with largest α . Telegdi's fit is shown in Fig. 1. The parameters obtained are

$$\begin{aligned} \delta &= 3.13 \\ C \left(\frac{\nu}{m}\right)^2 &= 9.55 \times 10^{-30} \text{ sec}^{-1} \text{ cm}^3 \end{aligned}$$

The latter corresponds, with the values of the G 's already quoted to

$$\left(\frac{v}{m}\right) = .80 \quad (5)$$

This is in accord with the predictions of Primakoff ¹⁾, based on empirical excitation energies, $(v/m) \approx .75 \pm .07$. We discuss below the theoretical value of δ .

III. FERMI GAS AND SHELL MODELS WITH CLOSURE

To discuss the theoretical value of δ it is necessary to discuss the correlation functions of which F is the averaged Fourier transform. It is sufficient, as we have seen, to consider the case $N = Z$. The obvious model to consider first is that of a non-interacting infinite Fermi gas. Instead of calculating the correlation functions and then Fourier transforming, it is quicker to evaluate the matrix element in (1) directly. The condition that a proton of initial momentum p should contribute is simply that the momentum transfer $\underline{p}$ is sufficient for the neutron into which it is changed to lie outside the occupied Fermi sphere. The fraction of neutrons F which do not contribute is just the fraction of a sphere overlaps with another of equal size with a suitably displaced centre. The result is well known

$$\begin{aligned}(1-F) &= \left(1 - \frac{1}{4} \delta\right) \\ &= \frac{3}{2} \left(\frac{\nu}{2K_f}\right) - \frac{1}{2} \left(\frac{\nu}{2K_f}\right)^3\end{aligned}$$

where K_f is the Fermi momentum. We use the standard value

$$K_f r_0 = 1.52 \quad r_0 = 1.12 \times 10^{-13} \text{ cm.}$$

$$K_f = 1.36 \times 10^{13} \text{ cm}^{-1} = 268 \text{ Mev/c.}$$

The experimental value (5) for δ , $1 - F = .22$, corresponds to $(\nu/2K_f) = .146$ or

$$\nu = .40 \text{ fermi}^{-1} = 78 \text{ Mev} = .74 m \quad (6)$$

which is not too different from the value (5).

The largest real nuclei are only about six mean nucleon spacings in diameter. One might well be uneasy therefore about the case of the infinite gas model. We have compared the Fermi gas value of $\{1 - F(\nu)\}$ with evaluations of the same matrix element for the small nuclei ^{16}O and ^{40}Ca . For the latter oscillator shell model wave functions were used, with mean square radii fitted to electron scattering. The formulae are given by Luyten et al. (their equations (4.27) and (4.28)) and we used their preferred size parameters $b = 1.80, 2.03$. The three curves are plotted in Fig. 2. In the region $1 - F \approx .22$ ($\xi \approx 3.13$) they are remarkably close. For this reason we are not tempted to try to allow for finite size effects by, for example ^{1),5)}, multiplying infinite gas correlation functions by finite nucleus density distributions.

IV. CORRELATION FUNCTIONS

The above development is not subject to a criticism which has been made, of the original treatment of Primakoff, by Klein and Wolfenstein⁴⁾. To discuss this let us for simplicity follow Primakoff in assuming that the function F in (3) does not depend on the relative magnitudes of the various coupling constants. This is correct for the ideal gas. Then we may write

$$F(\nu) = - \int d\underline{r} G(\underline{r}) \frac{\sin \nu |\underline{r}|}{\nu |\underline{r}|} \quad (7)$$

where the correlation function G is given by

$$G(\underline{r}) = \frac{2}{A} \langle a | \sum_{i \neq j} \tau_i^{(+)} \tau_j^{(-)} \delta(\underline{r}_i - \underline{r}_j - \underline{r}) | a \rangle \quad (8)$$

For the ideal Fermi gas with $N = Z$

$$\begin{aligned} G(\underline{r}) &= - \int_{|\underline{q}| < 2K_f} \frac{d\underline{q}}{(2\pi)^3} \left\{ 1 - \frac{3}{2} \left| \frac{\underline{q}}{2K_f} \right| + \frac{1}{2} \left| \frac{\underline{q}}{2K_f} \right|^3 \right\} e^{i\underline{q} \cdot \underline{r}} \\ &= \frac{\varrho}{4} \left\{ \frac{3 \cos K_f r}{(K_f r)^2} - \frac{3 \sin K_f r}{(K_f r)^3} \right\}^2 \end{aligned} \quad (9)$$

where $\varrho (= 2K_f^3/3\pi^2)$ is the nuclear matter density.

Primakoff did not use the gas function (9) but a simple box

$$\begin{aligned} G_p(\underline{r}) &= \varrho/4, \quad |\underline{r}| < d \\ &= 0, \quad |\underline{r}| > d \end{aligned} \quad (10)$$

This gives

$$1 - F_p(0) = 1 - (2/9\pi)(K_f d)^3 (1 - v^2 d^2/10 + \dots) \quad (11)$$

The experimental fit gives $1 - F = .22$ or

$$\begin{aligned} d/r_0 &\approx [.78 \times 9\pi / 2]^{1/3} / 1.52 \\ &\approx 1.47 \end{aligned}$$

which is the value obtained by Primakoff from a parallel study of Coulomb energies.

The function (11) is to be compared with the gas model function

$$1 - F(v) = \frac{3}{2} \left(\frac{v}{2K_f} \right) - \frac{1}{2} \left(\frac{v}{2K_f} \right)^3 \quad (12)$$

This behaves quite differently at the origin, going linearly to zero where (11) goes to a finite value not much different from that for $v \approx m$. That the two functions should agree at the point relevant in muon capture, their respective parameters being otherwise determined, seems merely coincidental.

Now for $v = 0$ we have what Tolhoek calls a "detailed normalization" condition. From (7) and (8)

$$\begin{aligned} F(0) &= - \int d\mathbf{r} G(\mathbf{r}) \\ &= - \frac{2}{A} \langle 0 | \sum_{i \neq j} \tau_i^{(+)} \tau_j^{(-)} | 0 \rangle \\ &= - \frac{2}{A} \langle 0 | \sum_i \tau_i^{(+)} \sum_j \tau_j^{(-)} | 0 \rangle + 1 \end{aligned}$$

But for $N = Z$ we may suppose the total isospin to be zero or small, and for a state of zero isospin

$$\sum_j \tau_j^{(-)} |0\rangle = 0$$

So

$$1 - F(0) = 1 + \int d\epsilon G(\epsilon) = 0 \quad (13)$$

The gas function (12) has this property, but the function (11) does not. This is the difficulty raised by Klein and Wolfenstein. They resolved it by restoring certain terms originally present in the Primakoff development, but later neglected by him.

Primakoff did not work directly with the correlation function (8), but preferred to split it into a combination of others. For each of the latter he made an assumption involving the box function (10), as well as the density distribution in finite nuclei and an over-all constant to secure the appropriate "detailed normalization". The reason for this detour through these other functions, each of them no better known than (8) itself (and more complicated since of long range), is obscure to us. However, if the process is taken very seriously the result for G is (10) only for nuclear matter ($A \rightarrow \infty$) and is more accurately (we label the expression by the initials of Klein and Wolfenstein)

$$G_{KW} = G_p(\epsilon) - A^{-1} C(r/R)$$

where $R(\propto A^{1/3})$ is the radius of the particular nucleus considered and the function C approaches zero for $(r/R) \gg 1$. Despite the decrease for given r

of the extra term with A , because of its increasing range it can maintain the detailed normalization (13). For $V \neq 0$ the extra term would make a negligible contribution to the Fourier transform for very large A . However, actual nuclei are not in fact very large, so Klein and Wolfenstein find that muon capture rates are very much reduced by the extra term.

The ideal gas function already satisfies the condition (13). With it there is no occasion for long range renormalization, and the predictions (in the closure approximation and for small $(N - Z)/A$) are essentially those which Primakoff originally made. Moreover there are a number of arguments which, although inconclusive, support the gas function rather than the box function :

- 1) it has some dynamical basis ; it is correct for non-interacting particles.
- 2) it is a reasonable conjecture that

$$\int d\mathbf{r} \lim_{A \rightarrow \infty} G(\mathbf{r}) = -1$$

holds for nuclear matter even when interactions are allowed for. We suppose $N = Z$ and that the nuclear matter ground state is isospin invariant. Then the isospin operators in (8) may be replaced as follows

$$\begin{aligned} \tau^{(+)} \tau^{(-)} &= \frac{1}{4} (\tau_1 + i\tau_2)(\tau_1 - i\tau_2) \\ &\rightarrow \frac{1}{4} (\tau_1\tau_1 + \tau_2\tau_2) \\ &\rightarrow \frac{1}{2} (\tau_3\tau_3) \\ &\rightarrow \frac{1}{2} (\tau_p - \tau_n)(\tau_p - \tau_n) \\ &\rightarrow \tau_p\tau_p - \tau_p\tau_n \end{aligned}$$

where $\gamma_{p,n} = \frac{1}{2}(1 \pm \gamma_3)$. Thus the relevant quantity is the difference between proton-proton and proton-neutron correlation functions. By standard arguments of thermodynamics ^{7),8)}, treating sub-regions of the system as grand canonical ensembles, we then obtain

$$1 - \int d\mathbf{r} \lim_{A \rightarrow \infty} G(\mathbf{r}) = kT \left(\frac{\partial \rho_p}{\partial \mu_p} - \frac{\partial \rho_p}{\partial \mu_n} \right)$$

where T is temperature, ρ_p (ρ_n) is proton (neutron) density, and μ_p (μ_n) is the proton (neutron) chemical potential. As there is no reason to expect the coefficient of kT to be singular at $T = 0$

$$\lim_{T \rightarrow 0} \int d\mathbf{r} \lim_{A \rightarrow \infty} G(\mathbf{r}) = 1$$

This is almost what we want. To get from it exactly what we want

$$\int d\mathbf{r} \lim_{A \rightarrow \infty} \lim_{T \rightarrow 0} G(\mathbf{r}) = 1$$

would require a delicate discussion.

3) It can be argued ⁹⁾ that the long wave length Fourier components of such correlation functions are dominated by collective oscillations. A semiclassical argument, following Feynman and Cohen, then shows that for the ground state of nuclear matter

$$\int G(\mathbf{r}) d\mathbf{r} = 1$$

It also shows that the Fourier transform, like (12), varies linearly near the origin. However, the momentum of actual interest ($\nu \approx m$) is probably too large for such a purely collective approach.

V. IDEAL GAS WITH LARGE NEUTRON EXCESS

Although we have argued in favour of Primakoff's original result, rather than those of Wolfenstein and Klein, it remains that the theory contains numerous approximations of unverified accuracy. Some of these can be considered within the gas model. In this section we remove the linear approximation in fractional neutron excess, and in the next we dispense with the closure approximation.

In the ideal gas model, with closure, we can of course do the calculation for $N \neq Z$.

This gives incidentally a check on the general argument of chapter 2. The mathematical problem is that of the overlap of two unequal spheres. This was calculated by Berman⁶⁾, and the following result is equivalent to his.

$$\begin{aligned} \Lambda \langle g \rangle &= C \left(\frac{\nu}{m} \right)^2 \left\{ 1 - \frac{A}{2Z} F \right\} \\ F &= 1 - \frac{3x}{4} \left[(1+\alpha)^{2/3} + (1-\alpha)^{2/3} \right] \\ &\quad + \frac{x^3}{2} - \frac{3}{32x} \left[(1+\alpha)^{4/3} - (1-\alpha)^{4/3} \right]^2 \end{aligned} \tag{14}$$

where $\alpha = (N - Z)/A$ $x = \nu/2K_F$; K_F is the Fermi momentum corresponding to the total density, taken to be independent of α . Near $\alpha = 0$, F has no part linear in α , confirming the argument leading to (3). In Fig. 2 the result (14) is plotted together with the linear approximation used previously. There is considerable divergence for the largest nuclei. It is therefore quite surprising that the experimental points follow the straight line almost to the zero of the latter.

Also given in Fig. 2 is a curve in which F has been computed with a value of ν reduced by some 17% from the value (6) but $C(\nu/m)^2$ has been calculated with ν increased by some 8% from (5). This gives a better over-all fit to the data, but can hardly be considered satisfactory because of the use of very different ν 's at different points, a difference of about 30%.

VI. GAS MODEL WITHOUT CLOSURE

An unfortunate feature of the closure approximation is the sensitivity of the result to the value used for V . This quantity appears both explicitly in (3) and in the quantity F . The capture rate is roughly proportional to V^3 . It is largely for this reason that despite the fair agreement in Fig. 1 between gas and shell models Luyten et al.⁵⁾, in their shell model studies of ^{16}O and ^{40}Ca , were unable to reconcile the observed capture rates with the universal coupling strength. Using also shell model energies they were led to adopt for V a rather higher value (89 MeV) than either (5) or (6). Additional factors were their inclusion of some relativistic effects and the difference between shell and gas models. We are not so much concerned here with the theoretical (V/m) for given nuclei, but much concerned with the possibility that a variation of it with A might straighten up the curve of Fig. 2. In order to discuss such a variation of V we would have to define it rather carefully, which means in effect going beyond the closure approximation. We have therefore performed, using the gas model, a calculation which dispenses with the closure approximation, and also in fact with the non-relativistic approximation.

Consider the reaction

$$\mu(k) + p(\underline{k}) \rightarrow \nu(k') + n(\underline{k}')$$

between free particles of the momenta indicated. With the usual current-current interaction the covariant matrix element has the form

$$\mathcal{M} = \frac{G}{\sqrt{2}} i \bar{\nu} \gamma_{\alpha} (1 + \gamma_5) \mu (p | J_{\alpha} | n)$$

where in terms of the usual form factors

$$(p | J_\alpha | n) = i \bar{p} \left\{ f_1 \gamma_\alpha - f_2 \sigma_{\alpha\beta} q_\beta + f_A \gamma_\alpha \gamma_5 - i f_P q_\alpha \gamma_5 \right\} n$$

The f 's are functions of

$$q^2 = (k - k')^2 = -m^2 + 2m|\underline{k}'|$$

Squaring, averaging over initial spins, and summing over final spins

$$|M|^2 = \sum \bar{\sum} |m|^2 = \frac{G^2 \{ A(q^2) + (s-u) B(q^2) + (s-u)^2 C(q^2) \}}{16 |\underline{k}'| (M^2 + \underline{k}^2)^{1/2} (M^2 + \underline{k}'^2)^{1/2} (M^2 + \underline{k}^2)^{1/2}}$$

where

$$(s-u) = -(k+K)^2 + (k-K')^2 = 2m((M^2 + \underline{k}^2)^{1/2} + (M^2 + \underline{k}'^2)^{1/2})$$

and

$$\begin{aligned} A(q^2) = (q^2 + m^2) \{ & 4m^2(f_A^2 - f_1^2) + 4Mq^2 f_1 f_2 + q^4 f_2^2 \\ & + (q^2 - m^2) [f_A^2 + (f_1 + 2Mf_2)^2] \\ & + m^2(q^2 f_P^2 - 4M f_A f_P) \} \end{aligned}$$

$$B(q^2) = 4q^2 (f_1 + 2Mf_2) f_A$$

$$C(q^2) = f_1^2 + f_A^2 + q^2 f_2^2$$

The capture rate per unit volume is

$$\int \frac{d\mathbf{k}'}{(2\pi)^3} |M|^2 2\pi \delta(E_\mu + E_p - E_n - E_\nu)$$

where

$$\mathbf{k}' = \mathbf{k} + \mathbf{K} - \mathbf{K}'$$

and E_μ etc., are the energies of the particles indicated. For the present problem we put $\mathbf{k} = 0$, average over the protons of the Fermi sphere, $|\mathbf{K}| < K_{fp}$ and require $|\mathbf{K}'| > K_{fn}$. Thus finally the capture rate per proton for unit muon density is

$$\frac{\Lambda}{\langle \epsilon \rangle} = \frac{3}{2} \left(\frac{1}{2\pi} \right)^3 \left(\frac{1}{K_{fp}} \right)^3 \iint_{\substack{|\mathbf{K}| < K_{fp} \\ |\mathbf{K}'| > K_{fn}}} d\mathbf{k} d\mathbf{k}' |M|^2 \delta(E_\mu + E_p - E_n - E_\nu) \quad (16)$$

We have, of course, $E_\nu = |\mathbf{k}'|$. For E_p and E_n , which because of the exclusion principle refer to particles not far below and not far above the Fermi surface, an appropriate effective mass approximation will be used, and for E_μ we take the muon mass m less the muon binding energy B . The provisions of the last sentence strictly speaking go beyond the approximation of free particles. The question then arises as to whether some similar modifications should be made of the energies entering into the calculation of the matrix element M . This is a question of physics to which we do not know the answer. Probably the effects in doubt are small. In any case, in order not to depart without good reason

from convention, we continue to compute M as a function of 3 momenta as if the particles were free. For the nucleon energies we take specifically

$$E_n - E_p = \frac{1}{2M^*} \left\{ (\underline{k}'^2 - K_{fn}^2) - (\underline{K}^2 - K_{fp}^2) \right\}$$

where

$$\frac{K_{fn}^3}{N} = \frac{K_{fp}^3}{Z} = \frac{2 K_f^3}{A}$$

Thus, in order to allow for Coulomb forces as well as one can in homogeneous nuclear matter, we assume that the energies of protons and neutrons at their respective Fermi surfaces are equal.

The integration over $|\underline{k}'|$ in (16) can be performed. Let Θ be the angle between $\underline{k}'$ and $\underline{K}$. Then

$$\Delta \langle \Theta \rangle =$$

$$\frac{3}{2\pi} \left(\frac{1}{K_{fp}} \right)^3 \int_{K_{min}}^{K_{fp}} K^2 dK \int_{-1}^{\cos \Theta'} d \cos \Theta \cdot \frac{|\underline{k}'|^2 |M|^2}{1 + (|\underline{k}'| - |\underline{K}| \cos \Theta)/M^*}$$

in which $|\underline{k}'|$ is determined by

$$|\underline{k}'| = m - B - \frac{1}{2M^*} \left(|\underline{K}|^2 - K_{fn}^2 + |\underline{k}'|^2 - 2|\underline{K}||\underline{k}'| \cos \Theta \right) + \frac{1}{2M^*} \left(|\underline{K}|^2 - K_{fp}^2 \right)$$

or explicitly

$$|k'| = - (M^* - |k| \cos \theta) + [(M^* - |k| \cos \theta)^2 + \delta]^{1/2}$$

where

$$\delta = 2 M^* (m - B) + K_{fn}^2 - K_{fp}^2$$

and

$$K_{\min} = -M^* + [(M^* + K_{fn})^2 - \delta]^{1/2}$$

$$\cos \theta' = (|k|^2 + \gamma^2 - K_{fn}^2) / 2 |k| \gamma$$

where

$$\gamma = (\delta + |k|^2 - K_{fn}^2) / 2 M^*$$

Taking the standard value for the nuclear matter density the only parameter in this approach is the effective mass. Results for $M^* = M$ and $M^* = 0.5 M$ are given in Fig. 4, and compared with the empirical straight line.

The form factors used are

$$f_1 = f = (1 + 1.25 q^2 / m^2)^{-2}$$

$$f_2 = 3.71 f / 2 M$$

$$f_A = 1.25 f$$

$$f_P = (8 f_A / m) (1 + m^2 / m_\pi^2) (1 + q^2 / m_\pi^2)^{-1}$$

However the way in which these vary away from $q^2 = 0$ is not very important here. The slight scatter in the theoretical results comes from the use of empirical muon binding energies. It is seen that the theoretical lines curve in the opposite sense from the closure results of Fig. 2 ; such a reversed curvature could be obtained in the closure calculation only by varying γ slightly with $(A - Z)/2A$. With $M = M^*$ the theoretical results have the right slope but lie too high. With lower values of M^* the decay rates are reduced but the slope becomes incorrect.

The situation might be improved somewhat by, for example, varying the strength of the induced pseudoscalar coupling constant. However, we do not wish to take the gas model too seriously when the closure approximation is not made. For light nuclei, with large excitation energies, the significance of the gas model excitation energies is very dubious. What we do wish to stress is that for this model the closure treatment with constant γ is not very good. There is at present no ground for confidence that it should be better for more realistic models.

CONCLUSION

We find that the difficulties of Klein and Wolfenstein arise from the use originally of unreasonable correlation functions. They disappear with the use of ideal Fermi gas functions. However, further development of the gas model gives grounds for doubting the accuracy of two of the basic approximations in the theory. These are the linear approximation in fractional neutron excess (within the closure approximation) and the closure approximation with constant neutrino energy. There are of course many other approximations involved. When all are considered the errors may be found to cancel. However, the excellence of the agreement with experiment of the present very crude theory can only be accidental.

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FIG. 1

FIG. 2

FIG. 3

FIG. 4