CM-P00057168

THE T VIOLATING DECAY OF $\pi^0 \rightarrow 3\gamma$

F.A. Berends ^{*)}

CERN - Geneva

^{*)} On leave of absence from : Instituut-Lorentz, Leiden,
The Netherlands.

In this letter we calculate the angular distribution for $\pi^0 \rightarrow 3\gamma$ decay, using the simplest possible parity conserving effective Hamiltonian. Such a decay would be possible in a theory which ascribes a charge conjugation violating part to the strong interactions ¹⁾. A very rough estimate gives a branching ratio of order 10^{-6} to $\pi^0 \rightarrow 2\gamma$ decay.

Looking for a Lorentz, gauge invariant, and parity conserving effective Hamiltonian, we find two possibilities, involving five derivatives. The matrix elements vanish, however, due to the Bose statistics of the photons. As it turns out, effective Hamiltonians containing seven derivatives give either vanishing matrix elements or the same ones as the density ^{*)}

$$H = \frac{1}{4} (\partial_\alpha \pi^0) \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} (\partial_\delta \partial_\beta F_{\rho\sigma}) (\partial^\delta F^{\alpha\beta}) \quad (1)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$$

In the centre-of-mass system, it is convenient to introduce polarization vectors $\vec{E}_i$, making an angle of 90° with the momentum $\vec{q}_i$ and lying in the plane of the momenta, and a vector $\vec{E}'_i$ perpendicular to this plane. As independent kinematical variables, we take q_1 and Θ , the energy of the first photon and the angle between $\vec{q}_1$ and $\vec{q}_2$. The non-vanishing matrix elements of (1) are :

$$M(\vec{q}_1, \vec{E}'_1, \vec{q}_2, \vec{E}_2, \vec{q}_3, \vec{E}_3) = -im q_1^2 q_2^2 (m - q_1 b)(2q_2 b - m) \sin \Theta$$

$$M(\vec{q}_1, \vec{E}_1, \vec{q}_2, \vec{E}'_2, \vec{q}_3, \vec{E}_3) = im q_1^2 q_2^2 (m - q_2 b)(2q_1 b - m) \sin \Theta$$

$$M(\vec{q}_1, \vec{E}_1, \vec{q}_2, \vec{E}_2, \vec{q}_3, \vec{E}'_3) = im^2 q_1^2 q_2^2 b (q_2 - q_1) \sin \Theta$$

*) We use $g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$.

2.

where $b = 1 - \cos\theta$, and

$$q_2 = \frac{m \left(\frac{m}{2} - q_1 \right)}{m - q_1 b} \quad (2)$$

is the energy of the second photon. The sum of $|M|^2$ over all polarizations becomes :

$$\begin{aligned} \sum |M|^2 &= m^2 q_1^4 q_2^4 \sin^2\theta \left[(m - q_1 b)^2 (2 q_2 b - m)^2 + \right. \\ &\quad \left. + (m - q_2 b)^2 (2 q_1 b - m)^2 + m^2 (q_2 - q_1)^2 b^2 \right] \\ &= 2(q_1 q_2)(q_2 q_3)(q_3 q_1) \left[(q_1 q_2)^2 ([q_1 - q_2] q_3)^2 + \right. \\ &\quad \left. + (q_1 q_3)^2 ([q_1 - q_3] q_2)^2 + (q_2 q_3)^2 ([q_2 - q_3] q_1)^2 \right] \end{aligned}$$

The last expression is given in terms of invariants, namely the inner products of the four-momenta of the photons, which is useful for the laboratory system.

For the angular distribution in the centre-of-mass system, we find, leaving out inessential constants :

$$\begin{aligned}
F(0, q_1) &= \frac{\partial^2 \sigma}{\partial q_1 \partial \cos \theta} = \int \frac{\sum |M|^2}{m q_1 q_2 q_3} \delta(f(q_2)) q_1^2 q_2^2 dq_2 = \\
&= \frac{\sum |M|^2 q_1 q_2}{m(m - q_1 b)} = \\
&= m^3 q_1^5 \frac{\left[m\left(\frac{m}{2} - q_1\right) \right]^5}{[m - q_1 b]^8} \sin^2 \theta \cdot \\
&\quad \cdot \left[(m - q_1 b)^2 (b m - b q_1 - m)^2 + m^2 (2 q_1 b - m)^2 \left(1 - \frac{b}{2}\right)^2 \right. \\
&\quad \left. + b^2 \left(\frac{m^2}{2} - 2 m q_1 + q_1^2 b \right)^2 \right]
\end{aligned}$$

where

$$\begin{aligned}
f &= q_1 + q_2 + \sqrt{q_1^2 + q_2^2 + 2 q_1 q_2 \cos \theta} - m \\
0 &\leq q_1 \leq \frac{m}{2}
\end{aligned}$$

The behaviour of this function for some values of q_1 is sketched in Fig. 1 in an arbitrary scale.

In the laboratory system, where we use the variables indicated in Fig. 2, we find, again leaving out constants :

$$\begin{aligned}
\frac{\partial^4 \sigma}{\partial q_1 \partial \cos \alpha \partial \cos \beta \partial \varphi} &= \frac{q_1^5 \left(\frac{m^2}{2} - q_1 c \right)^5}{P_0 (c - q_1 b)^8} b \left(c^2 - \frac{m^2}{2} b \right) \cdot \\
&\cdot \left[b^2 \left(q_1 c^2 + q_1 c d - q_1^2 c b - \frac{m^2}{2} d \right)^2 + \left(c^2 - \frac{m^2}{2} b \right)^2 (2 q_1 b - d)^2 + \right. \\
&\quad \left. + (d - q_1 b)^2 (m^2 b - q_1 c b - c^2)^2 \right]
\end{aligned}$$

4.

where

$$b = 1 - \cos \theta = 1 - \sin \alpha \sin \beta \cos \varphi - \cos \alpha \cos \beta$$

$$c = p_0 - p \cos \alpha, \quad d = p_0 - p \cos \beta, \quad p = |\vec{p}|$$

$$0 \leq \alpha, \beta \leq \pi, \quad 0 \leq \varphi \leq 2\pi$$

$$0 \leq q_1 \leq \frac{m^2}{2} \frac{1}{p_0 - p \cos \alpha}$$

$p_0, \vec{p}$ are energy, momentum of π^0 . Although we derived the distributions for the Hamiltonian (1), some properties are more general :

$$i) \quad M = 0 \quad \text{for} \quad q_1 = \frac{m}{3}, \quad \theta = 120^\circ$$

$$ii) \quad \frac{F(\theta_{12}, q_1)}{F(\theta_{13}, q_1)} = \left(\frac{m - q_1 b_{13}}{m - q_1 b_{12}} \right)^2$$

$$iii) \quad \frac{F(\theta, q_1)}{F(\theta, q_2)} = \frac{m - q_2 b}{m - q_1 b}$$

Here θ_{13} is the angle between first and third photon for a given θ_{12} , q_1 , and q_2 is given by (2). The first relation follows by use of the symmetry of a matrix element in at least two photons, the last two follow from the symmetry of $\sum |M|^2$ in three photons. They hold for both P or T conserving interactions ²⁾, the difference being that in the first case all or one polarization vectors are perpendicular to the plane, in the second case, two or none.

Finally, we want to make a remark about the branching ratio of 3γ decay to 2γ decay. One could try to use a model in which π^0 decays virtually into a ρ^0 and a η meson, these decaying into γ and 2γ , respectively. The transversality of photons, however, gives a

vanishing product of the $\pi^0 \eta e^0$ and $e^0 \gamma$ vertices. Therefore, we only compare the phase space integrals (made dimensionless by some factor containing the pion mass ³⁾) of the 2 γ and 3 γ cases :

$$I_{2\gamma} = (2\pi)^4 m \int \frac{1}{2^3 m q_1 q_2} \delta(m - q_1 - q_2) \delta(\vec{q}_1 + \vec{q}_2) \frac{d\vec{q}_1}{(2\pi)^3} \frac{d\vec{q}_2}{(2\pi)^3}$$

$$= \frac{1}{16\pi}$$

$$I_{3\gamma} = \frac{(2\pi)^4}{m} \int \frac{1}{2^4 m q_1 q_2 q_3} \delta(m - q_1 - q_2 - q_3) \delta(\vec{q}_1 + \vec{q}_2 + \vec{q}_3) \frac{d\vec{q}_1}{(2\pi)^3} \frac{d\vec{q}_2}{(2\pi)^3} \frac{d\vec{q}_3}{(2\pi)^3}$$

$$= \frac{1}{8(4\pi)^3}$$

The ratio $I_{2\gamma}/I_{3\gamma} \approx 315$, combined with a 10% C violation of strong interactions and one order higher in electromagnetic interactions, gives as a rough estimate a branching ratio of order 10^{-6} .

It may be noted that the occurrence of seven derivatives in the Hamiltonian necessitates the introduction of a factor m^{-7} , where m is some mass, for dimensional reasons. For any choice of m equal or higher than the pion mass, this implies a further strong suppression.

ACKNOWLEDGEMENTS

We are grateful to Drs. M. Veltman and J. Prentki for suggesting this problem and for helpful discussions. We wish to thank Professor Van Hove for the kind hospitality extended to us at CERN.

R E F E R E N C E S

- 1) J. Prentki and M. Veltman, Phys. Letters 15, 88 (1965).
- 2) The case of a P violating $\pi^0 \rightarrow 3\gamma$ decay has been considered by :
J. Schechter, Phys.Rev. 132, 841 (1963).
- 3) J.D. Jackson, Brandeis lectures, Vol.1 (1962) (ed. Benjamin).

FIGURE CAPTIONS

- Figure 1 : The angular distribution $F(\theta, q_1)$ in an arbitrary scale.
- Figure 2 : Spherical co-ordinates which are chosen such that $\vec{p}$ lies along the 3 axis, and $\vec{q}_1$ lies in the 1,3 plane.

FIG.1

FIG. 2