# RELATION BETWEEN TWO-BODY ANNIHILATION AND BACKWARD ELASTIC SCATTERING AT HIGH ENERGY

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9 November 1964

9831/TH. 486

It was shown recently [1),2)] that the analyticity of the scattering amplitude together with the assumption that the scattering amplitude does not oscillate at high energy (in the physical region) imply simple relations between the differential cross-sections of crossed reactions in the high energy limit. We would like to discuss here some possibilities of the experimental checking of the relations of this type.

Let us first remind the main results obtained in the paper by Logunov et al. [2)]. We will present them on a simple example which was not treated in the latter paper. Consider the pair of crossed reactions :

(a) backward elastic scattering of pions on protons

$$p + \pi^{+} \longrightarrow \pi^{+} + p \tag{a}$$

and

(b) annihilation of antiprotons into two pions

$$p + \bar{p} \longrightarrow \pi^{+} + \pi^{-} \tag{b}$$

In the high-energy limit the processes of the type (a) and type (b) are highly peripheral. In our notation, equation $A+B \rightarrow C+D$ means that the particle $C$ goes in the direction of the particle $A$, and $D$ in the direction of $B$. <u>Reaction $A+B \rightarrow C+D$ is therefore different from $A+B \rightarrow D+C$.</u>

The processes (a) and (b) are described by the scattering amplitudes $\mathcal{T}_a(s,t)$ and $\mathcal{T}_b(s,t)$ which are functions of $s$ and $t$ [3)], $s$ being the c.m. energy squared and $t = -$ (c.m. momentum transfer)$^2$. We define the $t$ variables in such a way that both the reactions (a) and (b) are described by small values of $(-t)$ (see figure).

We make the following assumptions on the behaviour of $\mathcal{T}_a(s,t)$ and $\mathcal{T}_b(s,t)$ for fixed $t$ [4) ]:

(i) $\mathcal{T}_a(s,t)$ has the usual analyticity properties postulated for the scattering amplitudes, i.e., $\mathcal{T}_a(s,t)$ is analytic and bounded by a polynomial in $s$, in the upper half $s$ plane ; apart from a finite interval $\mathcal{T}_a(s,t)$ is continuous along the real axis.

(ii) for $s \to \infty$ along the real axis, the amplitudes $\mathcal{T}_a(s,t)$ and $\mathcal{T}_b(s,t)$ do not oscillate. One can take, e.g.,

$$\mathcal{T}_{a,b}(s,t) \xrightarrow[t\ \text{fixed}]{s\to\infty} s^{\alpha(t)} (\ln s)^{\beta(t)}\, C_{a,b}(t) \tag{1}$$

where $\alpha(t)$, $\beta(t)$ are real and both $c_a(t) \neq 0$ and $c_b(t)$ can be complex.

It follows from these assumptions and from crossing symmetry that there exist an interval of $t$, $t_0 \leqslant t \leqslant t_1$ and a value $s_0$ $(-t_0, -t_1 \ll s_0)$ such that, for any $s \geqslant s_0$ and $t_0 \leqslant t \leqslant t_1$, a simple relation holds between the differential cross-sections for the reactions (a) and (b) :

$$\frac{d\sigma_a(s,t)}{dt} \simeq \lambda(s)\,\frac{d\sigma_b(s,t)}{dt} \tag{2}$$

where

$$\lambda(s) = \frac{s(s-4M^2)}{(s+M^2-m_\pi^2)-4M^2 s} \approx \frac{s(s-4M^2)}{(s-M^2)^2} \xrightarrow[s\to\infty]{} 1 \tag{3}$$

$M$ and $m_\pi$ denote the nucleon and pion mass, respectively. The region of $s$ and $t$ variables where formula (2) holds is the same in which formula (1) represents the correct approximation of the amplitudes $\mathcal{T}_a(s,t)$ and $\mathcal{T}_b(s,t)$.

The same kind of argument applies to any pair of reactions of the type

$$A + B \rightarrow C + D \tag{4}$$

$$A + \bar{D} \rightarrow C + \bar{B} \tag{5}$$

The cross-sections for reactions (4) and (5) obey the relation

$$\frac{d\sigma_4(s,t)}{dt} \simeq \mu(s)\, \frac{d\sigma_5(s,t)}{dt} \tag{6}$$

with $\mu(s)$ given by

$$\mu(s) = \frac{[s + M_A^2 - M_D^2]^2 - 4M_A^2 s}{[s + M_A^2 - M_B^2]^2 - 4M_A^2 s} \xrightarrow[s\to\infty]{} 1 \tag{7}$$

We would like to emphasize that the assumptions made in this discussion are currently used by people working on the theory of strong interactions and are considered as rather fundamental ones. It seems, therefore, that detailed experimental study of reactions of types (4) and (5) may be very useful in establishing the fundamental analyticity properties of the scattering amplitudes. The processes we consider in this paper have vanishing cross-sections in the high-energy limit. For such processes the asymptotic relations have yet to be investigated experimentally.

We will discuss now the existing experimental evidence concerning the reactions (a) and (b) and also another pair of crossed reactions, namely charge-exchange scattering of protons on neutrons and charge-exchange scattering of protons on antiprotons :

$$p + n \rightarrow n + p \tag{c}$$

$$p + \bar{p} \rightarrow n + \bar{n} \tag{d}$$

Reaction (a), i.e., backward peak in $\pi^+p$ elastic scattering was observed by Kulakov et al.[5] at 4.6 GeV/c in a counter experiment and confirmed at 4 GeV/c in a hydrogen bubble-chamber experiment by Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.) and München collaboration[6]. In the latter experiment it was found that the cross-section for backward scattering is about 40 $\mu$b and that the angular distribution shows a pronounced backward peak. Similar cross-section for backward $\pi^+p$ scattering in this energy range was found by Alikhanov et al.[7]. Some evidence of the backward peak in this reaction at 8 GeV/c was obtained by Aachen-Berlin-CERN collaboration[8]. The corresponding cross-section, based however on two events, is about 7 $\mu$b.

Two-pion annihilation of antiprotons of the type (b) with $\pi^+$ going in the direction of proton in the c.m. system was observed in a hydrogen bubble-chamber at 3 GeV/c and at 4 GeV/c[9),10)]; cross-sections of $10 \pm 10$ $\mu$b (1 event) and $7 \pm 5$ $\mu$b (2 events), respectively, were found.

It is interesting to note that no events of the reaction $p + \bar{p} \to \pi^- + \pi^+$ were observed. This agrees with the fact that in contrast to $\pi^+$-p elastic scattering, no evidence exists for a backward peak in the $\pi^-$-p elastic scattering at 4 and 10 GeV/c[8),11),12)].

Charge exchange of pn and $p\bar{p}$ was studied at similar momenta, pn at 2.85 and 3.68 GeV/c by Palevsky et al.[13], $p\bar{p}$ at 3 and 3.6 GeV/c by a CERN group[14),9)]. At 2.85 GeV/c $\sigma_{ch.ex.}(pn) = 0.65 \pm 0.15$ mb, at 3.68 GeV/c $\sigma_{ch.ex.}(pn) = 0.43 \pm 0.16$ mb. The differential cross-sections in the lab. system at zero degrees are $26 \pm 5$ mb/sr and $30 \pm 7$ mb/sr at 2.85 and 3.68 GeV/c, respectively. For $p\bar{p}$ charge exchange, the combined data at the two energies give $\sigma_{ch.ex.}(p\bar{p}) = 2 \pm 0.6$ mb, $\frac{d\sigma}{d\Omega}(0^o)_{lab.} \approx 20$ mb/sr. Charge exchange cross-sections for pn and $p\bar{p}$ are of the same order of magnitude and for small $t$ become identical within the errors[14].

In conclusion, the experimental evidence does not contradict formulae (6) and (7). Clearly, however, one needs more precise measurements before drawing quantitative conclusions. It would be interesting to look at these reactions at higher energy.

Finally, let us remind that the same argument applies to any pair of crossed reactions of the two-body type [2]. It seems that it may be possible to study reactions of the type $p+\bar{p} \to X^{\pm}+\pi^{\mp}$ and $p+\pi^{\pm} \to X^{\pm}+p$ where X is a pion resonant state (e.g., $\rho$ meson). Another possible example is a backward scattering of hyperons on protons together with the annihilation into pair of hyperons, like $p+\Lambda \to \Lambda+p$, $p+\bar{p} \to \Lambda+\bar{\Lambda}$.

The authors would like to thank Professor L. Van Hove, Drs. V.T. Cocconi and D.R.O. Morrison for helpful discussions and encouragement. One of the authors (A.B.) wishes to thank Professor L. Van Hove for the hospitality at the CERN Theoretical Study Division, and the other (O.C.) expresses his gratitude to Professor Ch. Peyrou for the hospitality extended to him at the CERN Track Chamber Division.

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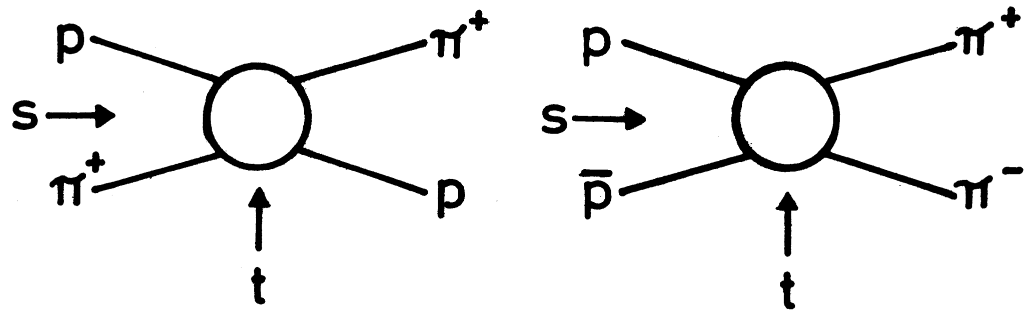


$p + \pi \longrightarrow \pi^+ + p$
(a)

$p + \bar{p} \longrightarrow \pi^+ + \pi^-$
(b)

Fig. 1