CM-P00057071

TWO-PARTICLE AND MANY-PARTICLE CHANNELS
IN HIGH-ENERGY COLLISIONS

A. Bia~~l~~as^{*)} and L. Van Hove

CERN, Geneva

ABSTRACT

This paper continues previous investigations of high-energy collisions of hadrons in the asymptotic region of constant cross-sections. Two models proposed earlier, one in which inelastic collisions give rise to many particles with a minimum of correlation between them, the other in which they consist entirely of isobar excitation of one or both incident particles, are combined together and the resultant shadow scattering is compared with the experimental characteristics of elastic scattering. Excellent agreement is obtained if most of the inelastic collisions are of the uncorrelated many-particle type, only a small fraction of the inelastic cross-section corresponding to isobar excitation without further particle creation. The method used exploits unitarity of the S matrix and crossing symmetry of the isobar excitation channels which are assumed to be dominated by $C = 1$ exchange. The isobar excitation channels are analyzed in terms of a simple K matrix. The effect of the uncorrelated many-particle channels is described by means of an "overlap function" which determines the angular distribution of elastic scattering and isobar excitation. Its effect on the latter is found to be a simple generalization of the distorted wave Born approximation.

*) On leave from Jagellonian University, Cracow, Poland.

1. INTRODUCTION

This paper continues previous model studies of elastic scattering of hadrons as shadow scattering of inelastic collisions in the asymptotic high-energy region where cross-sections are constant. In earlier publications, complementary and in fact opposite situations were discussed^{1,2)}. Thus, in Ref. 1, the dominant inelastic collisions were assumed to give rise to production of many particles with a minimum of correlations between them (to be called hereafter uncorrelated model). The resultant elastic scattering was found to be in excellent agreement with the experimental facts for the shape of the diffraction peak as well as for the ratio σ_{el}/σ_{tot} ^{3,4)}. Ref. 2, on the other hand, deals with a model where all high-energy inelastic collisions are of two-body type and consist in excitation of one or both of the incident particles A,B to an isobar state A*,B*. (Ref. 2 in fact specializes this model further by assuming $A = B = \bar{A}$, $A^* = B^* = \bar{A}^*$). Using unitarity and crossing symmetry it expresses all collisions in terms of a single function $a(\rho)$ where a is the K-matrix element for $A+B \rightarrow A^*+B^*$ and ρ is the impact parameter. For reasonable shapes of $a(\rho)$ and for moderate strength of this K-matrix element one found that the experimental shape of the diffraction peak is well reproduced, but that the ratio σ_{el}/σ_{tot} of elastic to total cross-section is much too large⁵⁾.

The present paper introduces two new elements in these model studies and develops their consequences. Firstly, it improves on Ref. 2 by allowing the incident particles A,B and their isobars A*, B* to be all different, and by allowing to replace the assumption of selfconjugation of all particles ($A = \bar{A}$ etc.) by the assumption that $C = 1$ exchange completely dominates $C = -1$ exchange at high energy. The latter assumption is well supported by experiment. Secondly, the present paper combines the isobar excitation model with the uncorrelated many particle model of Ref. 1, in the sense that it

incorporates the following inelastic processes

$$A+B \rightarrow A^* + B, A+B^*, A^* + B^* \quad (1.1)$$

$$A+B \rightarrow \text{many uncorrelated particles} \quad (1.2)$$

all of which are allowed to have non-vanishing cross-sections at high energy.

The main qualitative results derived from the combined model are the following:

- (i) the experimental facts about diffraction peak shape and value of σ_{el}/σ_{tot} allow at most a small amount of two-particle processes of type (1.1); at least about two thirds of the inelastic cross-section must be due to (1.2);
- (ii) whereas the cross-section for double isobar excitation $A+B \rightarrow A^* + B^*$ can remain non-vanishing at high energy (although its value should be small) single isobar excitation should become vanishingly rare.

We believe conclusion (i) to be rather insensitive to the special assumptions made in our model study. Conclusion (ii), in contrast, may well be modified if more than one isobar state of A or B were available for excitation with $C = 1$ exchange. Experimental tests of both conclusions would be most welcome to suggest further improvements of the model.

Sections 2 and 3 describe the general formalism which we use to combine two-particle channels (1.1) and many particle channels (1.2) in a single S matrix. The effect of the many particle channels

is incorporated in an overlap function of the type used in Ref. 1, whereas the basic unknown interaction between two body channels, i.e. the isobar excitation effect, is expressed in terms of a K matrix. Using crossing symmetry, the asymptotic properties of the elastic and two-body inelastic amplitudes at high energy are derived in Section 4. In particular the conclusion (ii) mentioned above is obtained. The detailed predictions of the combined model are worked out in Section 5. Their comparison with experiment leads to limitations on the free parameters, especially to conclusion (i). Section 6 compares our formalism to other investigations of two-particle inelastic collisions at high energy and of their modification by the existence of many-particle channels. Our model turns out to combine the main features of the distorted wave Born approximation and of the K-matrix formalism. A summary of conclusions is found in Section 7.

2. THE UNITARITY CONDITION IN THE MANY-CHANNEL PROBLEM

In terms of the T matrix

$$iT = S - 1 \quad (2.1)$$

the unitarity condition for the S matrix can be written as

$$i(T^\dagger - T) = T^\dagger T. \quad (2.2)$$

As we are interested here mainly in the implications of the unitarity condition for the two-particle channels we will exploit only a part of Eq. (2.2), namely the one which contains two-body channels:

$$i\{\langle i|T^+|j\rangle - \langle i|T|j\rangle\} = \sum_n \langle i|T^+|n\rangle \langle n|T|j\rangle \quad (2.3)$$

Here $|i\rangle$ and $|j\rangle$ are states containing two particles, $\{|n\rangle\}$ runs over the complete set of states. The sum over the complete set of states can be separated into the sum over the complete set of two-particle states and the complete set of other (many-particle) states. We will denote the many particle states by $|x\rangle$. Taking out the delta function describing the energy-momentum conservation:

$$\begin{aligned} \langle i|T|j\rangle &= \delta^{(4)}(P_i^\mu - P_j^\mu) \langle i|t|j\rangle \\ \langle i|T|x\rangle &= \delta^{(4)}(P_i^\mu - P_x^\mu) \langle i|t|x\rangle \end{aligned} \quad (2.4)$$

we get

$$\begin{aligned} i\{\langle i|t^+|j\rangle - \langle i|t|j\rangle\} &= \sum_{j'} \langle i|t^+|j'\rangle \delta^{(4)}(P_{j'}^\mu - P_j^\mu) \langle j'|t|j\rangle + \\ &+ \sum_x \langle i|t^+|x\rangle \delta^{(4)}(P_x^\mu - P_j^\mu) \langle x|t|j\rangle. \end{aligned} \quad (2.5)$$

For given channels i and j the second term in the RHS of Eq. (2.5) depends on the c.m. energy $\sqrt{s}$ of the colliding particles, and the angle Θ between the momenta of particles in the states i and j . We introduce an "overlap matrix"

$$F_{ij}(s, \cos \Theta) \equiv 2\pi \sqrt{\frac{k_i k_j \epsilon_i^{(1)} \epsilon_i^{(2)} \epsilon_j^{(1)} \epsilon_j^{(2)}}{s}} \sum_x \langle i | t^+ / x \rangle \delta^{(4)}(P_x - P_j) \langle x | t / j \rangle. \quad (2.6)$$

Here $\epsilon_i^{(1,2)}$ are the c.m. energies of the particles in the channel i , k_i is the c.m. momentum in the channel i .

The overlap matrix $F_{ij}(s, \cos \Theta)$ represents a generalization of the overlap function introduced before by one of us¹⁾ in the discussion of a model in which there exists only one two-particle channel. To see this let us observe that

$$\begin{aligned} \sum_x \langle i | t^+ / x \rangle \delta^{(4)}(P_x - P_j) \langle x | t / j \rangle &= \\ &= \sum_n \langle i | t^+ (1-Q) / n \rangle \delta^{(4)}(P_n - P_j) \langle n | (1-Q) t / j \rangle = \\ &= \langle f_i(\vec{k}_i) | \delta^{(4)}(\hat{P} - P_j) | f_j(\vec{k}_j) \rangle \end{aligned} \quad (2.7)$$

where Q is the projection operator on the two-particle states, $\hat{P}$ is the total energy-momentum and the state $|f_j(\vec{k}_j)\rangle$ is defined by the equation

$$\delta^{(4)}(\hat{P} - P_j) |f_j(\vec{k}_j)\rangle \equiv (1-Q) S |j, \vec{k}_j, -\vec{k}_j\rangle. \quad (2.8)$$

We see from Eq. (2.8) that the vector $|f_j(\vec{k}_j)\rangle$ is the sum of all many-particle components which enter into the final state resulting from the collision of two particles with momenta $\vec{k}_j$ and $-\vec{k}_j$ in the channel j .

To simplify the unitarity condition we will write it down in terms of the partial wave amplitudes. We denote

$$\tau_{ij}(s, \cos \theta) \equiv \frac{\langle i | t | j \rangle}{\sqrt{\epsilon_i^{(1)} \epsilon_i^{(2)} \epsilon_j^{(1)} \epsilon_j^{(2)}}} = \frac{1}{4\pi} \sqrt{\frac{s}{k_j k_i}} i \sum_l (2l+1) \eta_{ij}^l(s) P_l(\cos \theta) \quad (2.9)$$

$$F_{ij}(s, \cos \theta) = \sum_l (2l+1) f_{ij}^l(s) P_l(\cos \theta). \quad (2.10)$$

Furthermore, we assume time reversal invariance of the S matrix:

$$\tau_{ij}(s, \cos \theta) = \tau_{ji}(s, \cos \theta). \quad (2.11)$$

Introducing the formulae (2.6) and (2.9) - (2.11) into the unitarity

condition (2.5) one gets

$$\operatorname{Re} \eta_{ij}^L - \frac{1}{2} \sum_k \eta_{ik}^L (\eta_{jk}^L)^* = f_{ij}^L. \quad (2.12)$$

This equation can be simplified by introducing instead of η_{ij}^L the matrices $S^L = s_{ij}^L$ defined by

$$S^L \equiv s_{ij}^L = \delta_{ij} - \eta_{ij}^L. \quad (2.13)$$

One obtains easily from Eqs. (2.12) and (2.13)

$$S^{L\dagger} S^L = I - 2F^L \quad F^L \equiv f_{ij}^L \quad (2.14)$$

where I is a unit matrix. Eq. (2.14) is the basis of our further discussion.

3. THE UNITARITY CORRECTIONS TO THE TWO-PARTICLE CHANNEL AMPLITUDES

In this section we apply our formalism to the problem of the "unitarity corrections" in models where the two-particle amplitudes are given a priori, as for example the one-particle exchange model. This problem has been studied by many authors⁶⁻¹⁹⁾. Our formulae suggest very directly a simple solution for it.

Suppose we want to choose a set of transition amplitudes between the two-particle states. These transition amplitudes should obey the unitarity condition in presence of the many-particle channels, as expressed e.g. by Eq. (2.14). Obviously, when a set of two-particle amplitudes is given, it will, in general, violate Eq. (2.14). Our task is to find how to "correct" the given set of amplitudes in such a way that the corrected set would obey the unitarity condition. The answer to this question is, of course, not unique.

As seen from Eq. (2.14), the many-particle channels affect two-body processes via the overlap matrix $F_{ij}(s, \cos \theta)$. It is rather difficult, and perhaps useless, to find the most general S^e verifying Eq. (2.14) for arbitrary overlap matrix. Therefore, in the absence of any experimental information, we prefer to make a simplifying assumption which allows a simple solution of Eq. (2.14) and which seems physically reasonable.

Expressed in physical terms, our main assumption is that the many-particle states generated in collisions with different two-particle initial states i, j are sufficiently different to be treated as approximately orthogonal. Mathematically, our assumption means that the overlap matrix is diagonal:

$$F_{ij}(s, \cos \theta) = F_i(s, \cos \theta) \delta_{ij} \quad (3.1)$$

A similar equation holds for the partial-wave decomposition of the matrix F_{ij} :

$$F^L \equiv f_{ij}^L = f_i^L \delta_{ij} \quad (3.2)$$

We do not claim that the condition (3.1) is strictly valid in the real physical situation. We think, however, that it represents a reasonable first approximation.

Eq. (2.14) shows that the matrices F^ℓ are Hermitian and therefore real because of the condition (3.2). It follows from this, from the time-reversal invariance (2.11) and from the unitarity condition that the matrices S^ℓ commute with F^ℓ :

$$S^\ell F^\ell = F^\ell S^\ell . \quad (3.3)$$

Written in full Eq. (3.3) reads

$$\sum_k S_{ik}^\ell f_{\kappa}^\ell \delta_{\kappa j} = \sum_n f_i^\ell \delta_{in} S_{\kappa j}^\ell \quad (3.4)$$

or

$$S_{ij}^\ell (f_i^\ell - f_j^\ell) = 0 . \quad (3.5)$$

We see that the condition $S_{ij}^\ell \neq 0$ implies $f_i^\ell = f_j^\ell$. If we consider only the group of states which are coupled with each other (directly or via intermediate channels) we have

$$F^\ell = f^\ell I . \quad (3.6)$$

The unitarity condition (2.14) takes now the very simple form

$$S^{e+} S^e = (1 - 2f^e) I. \quad (3.7)$$

One can work out easily the most general solution for Eq. (3.7).

It is written in the form:

$$S^e = \sqrt{1 - 2f^e} \Omega^e \quad (3.8)$$

where the Ω^e are unitary and symmetric matrices

$$\Omega^{e+} \Omega^e = I \quad \Omega^e = \Omega^{e+}. \quad (3.9)$$

The conditions (3.9) imply that the Ω^e can be expressed in terms of K matrices K^e

$$\Omega^e = \frac{1 + i K^e}{1 - i K^e}. \quad (3.10)$$

The matrices Ω^e obey the conditions (3.9) if the matrices K^e are real and symmetric. There are no other constraints on K^e due to the unitarity condition. The formulae (3.8), (3.10) express therefore the most general matrix S^e satisfying the unitarity condition (3.7) in terms of the $\frac{1}{2}n(n+1)$ independent elements of

K^ℓ (n is the number of two-body channels).

Suppose now that we have given a set of real two-particle amplitudes $B_{ij}(s, \cos \theta)$, ($i, j = 1 \dots n$) which satisfy time reversal invariance:

$$B_{ij}(s, \cos \theta) = B_{ji}(s, \cos \theta). \quad (3.11)$$

These may be for example Born amplitudes, with or without form factor corrections. The corresponding partial-wave amplitudes we denote by $b_{ij}^\ell = B^\ell$.

$$B_{ij}(s, \cos \theta) = \frac{1}{4\pi} \sqrt{\frac{s}{k_i k_j}} \sum_{\ell} (2\ell+1) b_{ij}^\ell P_{\ell}(\cos \theta). \quad (3.12)$$

It is clear that, for a given ℓ , the number of independent partial-wave amplitudes b_{ij}^ℓ is $\frac{1}{2}n(n+1)$, i.e. the same as the number of the independent elements of the K matrix. Moreover, the b_{ij}^ℓ are supposed real. This suggests to write

$$k_{ij}^\ell = \frac{1}{2} b_{ij}^\ell \quad (3.13)$$

and calculate the S -matrix elements according to formulae (3.10) and (3.8). The K matrix defined by (3.13) is real and symmetric, therefore the corresponding S matrix (3.8) will obey the unitarity condition (3.7). The "corrected" S matrix is now given by

$$S^L = \sqrt{1-2f^L} \frac{1 + \frac{i}{2} B^L}{1 - \frac{i}{2} B^L}. \quad (3.14)$$

The "uncorrected" S matrix would be

$$S^L = 1 + i B^L. \quad (3.15)$$

The form (3.15) should hold if the influence of many-body channels is negligible ($f^L \approx 0$), and if the "Born" terms B^L are small. One sees that (3.14) reduces to (3.15) in this approximation. This explains why we introduced the factor $1/2$ in (3.13).

The use of the K-matrix formalism in order to correct the Born amplitudes was first proposed by Chernavskii^{a)} in the study of pp and p $\bar{p}$ interactions. Since then the method has been discussed and generalized by Arnold¹⁶⁾ and by Dietz and Pilkuhn¹⁷⁾. The underlying idea being essentially the same, the approach proposed in the present paper differs from those of Ref. 16 and Ref. 17, mainly in the treatment of the channels with many particles. In the paper by Arnold the many-particle channels are simply neglected. In the paper by Dietz and Pilkuhn the many-particle channels are introduced into the K matrix similarly to the two-particle channels.

In Section 6 we will compare these three approaches for the simple model which we are now going to introduce.

4. FOUR-CHANNEL PROBLEM IN THE HIGH-ENERGY LIMIT

In this Section we would like to discuss a simple generalization of the model of high-energy scattering which we discussed in a preceding paper²). We consider only two particles A,B and their isobars A^* , B^* . Furthermore, we assume that the particles A,B, A^* , B^* are spinless, and that they are selfconjugate or equivalently that only $C = 1$ is exchanged in the high-energy two-particle process (a property known to be valid experimentally). The possible two-particle channels are denoted as follows:

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ AB & A^*B^* & A^*B & AB^* \end{array} \quad (4.1)$$

We now consider the consequences of the crossing symmetry and analyticity properties of the S matrix. These consequences are of an asymptotic nature for $s \rightarrow +\infty$. We make the following assumptions concerning the high energy behaviour of the amplitudes and overlap functions:

- (i) The backward elastic scattering vanishes at high energy.
- (ii) For fixed t , the elastic scattering amplitudes have the following asymptotic behaviour for $s \rightarrow +\infty$:

$$s^{-1} \tau_{ii}(s,t) \xrightarrow[s \rightarrow \infty]{t \text{ fixed}} a_i(t) \quad (4.2)$$

and $a_1(t)$ being different from zero at least in an interval $t_0 \leq t \leq 0$ and tending to zero for $t \rightarrow -\infty$ (this corresponds to constant non-vanishing forward elastic cross-sections).

- (iii) The generalized overlap function has a similar behaviour for $s \rightarrow +\infty$, t fixed:

$$s^{-1} f(s, t) \xrightarrow[s \rightarrow \infty]{t \text{ fixed}} f(t) \quad (4.3)$$

$f(t)$ may or may not vanish. (This corresponds to constant total cross-section for all many-particle channels).

In terms of partial wave amplitudes, the conditions (4.2), (4.3) read:

$$\eta_{ii}^{\ell}(s) = \eta_{ii}(\rho) \quad f^{\ell}(s) = f(\rho) \quad (4.4)$$

for $s \rightarrow +\infty$, where $\rho = \frac{\ell}{k}$ is the impact parameter. The functions $\eta_{ii}(\rho)$ do not vanish identically.

If we now make the additional assumption that at very high energy the off-diagonal partial-wave amplitudes $\eta_{ij}^{\ell}(s)$ $i \neq j$ are continuous functions of ρ and s , we can prove from unitarity and from the conditions (4.4) that for $s \rightarrow +\infty$

$$\eta_{ij}^{\ell}(s) \equiv \eta^{ij}(\rho) \quad i \neq j. \quad (4.5)$$

Consequently, the off-diagonal elements of τ have at fixed t an asymptotic behaviour of the type

$$s^{-1} \tau_{ij}(s, t) \xrightarrow[t \text{ fixed}]{s \rightarrow \infty} a_{ij}(t) \quad (4.6)$$

$i \neq j, i=1,2,3,4$

where $a_{ij}(t)$ may or may not vanish.

The consequences of crossing symmetry can now be obtained by the method used in Ref. 2. Using the fact that the particles A, B, A^*, B^* are selfconjugate, or equivalently the fact that only $C = 1$ is exchanged, one finds

$$a_i(t) = a_i^*(t) \quad (4.7)$$

$$a_{13}(t) = a_{13}^*(t) \quad a_{14}(t) = a_{14}^*(t) \quad a_{23}(t) = a_{23}^*(t) \quad a_{24}(t) = a_{24}^*(t) \quad (4.8)$$

$$a_{12}(t) = a_{34}^*(t) \quad (4.9)$$

The conditions (4.7) - (4.9) together with the assumption (i) imply

$$\eta^{ii}(p) = \eta^{*ii}(p) \quad (4.10)$$

$$\eta_{13}(p) = \eta_{13}^*(p) \quad \eta_{14}(p) = \eta_{14}^*(p) \quad \eta_{23}(p) = \eta_{23}^*(p) \quad \eta_{24}(p) = \eta_{24}^*(p) \quad (4.11)$$

$$\eta_{12}(p) = \eta_{34}^*(p) \quad (4.12)$$

Taking into account time reversal invariance we end up with an S matrix of the form

$$S_{ij}(p) = \begin{pmatrix} r_1 & \alpha + i\beta & R_{13} & R_{14} \\ \alpha + i\beta & r_2 & R_{23} & R_{24} \\ R_{13} & R_{23} & r_3 & \alpha - i\beta \\ R_{14} & R_{24} & \alpha - i\beta & r_4 \end{pmatrix} \quad (4.13)$$

where r_i , R_{ij} , ($i, j = 1 \dots 4$), α and β are real functions of the impact parameter.

Now we impose the unitarity condition in the form (3.8) - (3.9). One can show in a way very similar to that used in Ref. 2 that it implies

$$S_{ij}(p) = \begin{pmatrix} 1 - \eta(p) & i Y(p) & 0 & 0 \\ i Y(p) & 1 - \eta(p) & 0 & 0 \\ 0 & 0 & 1 - \eta(p) & -i Y(p) \\ 0 & 0 & -i Y(p) & 1 - \eta(p) \end{pmatrix} \quad (4.14)$$

with

$$\begin{aligned}\eta(\rho) &= \frac{2 a^2(\rho)}{1+a^2(\rho)} \sqrt{1-2f(\rho)} + 1 - \sqrt{1-2f(\rho)} = \\ &= 1 - \frac{1-a^2(\rho)}{1+a^2(\rho)} \sqrt{1-2f(\rho)}\end{aligned}\quad (4.15)$$

$$\gamma(\rho) = \frac{2 a(\rho)}{1+a^2(\rho)} \sqrt{1-2f(\rho)}.\quad (4.16)$$

The corresponding K matrix is given by

$$K_{ij}(\rho) = \begin{pmatrix} 0 & a(\rho) & 0 & 0 \\ a(\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & -a(\rho) \\ 0 & 0 & -a(\rho) & 0 \end{pmatrix}\quad (4.17)$$

We see that our model contains two arbitrary functions of the impact parameter: the overlap function $f(\rho)$ and the K-matrix element $a(\rho)$. In the next Section we will discuss the scattering properties for a specific choice of $f(\rho)$ and $a(\rho)$. The form (4.14) obtained for the S matrix suggest that when $C = 1$ exchange dominates the high-energy two-body reactions, the cross-sections for single isobar excitation

become negligible at high energy. The only two-body processes which persist at high energy are elastic scattering and double

isobar excitation

It remains an open question whether this result is valid under more general assumptions, and whether it is supported experimentally.

5. THE MODEL FOR THE K MATRIX AND OVERLAP FUNCTION

The model for high-energy scattering developed in the preceding Section is, in some sense, a combination of the two models worked out before. For $a(\rho) = 0$ we obtain a model described in Ref. 1. It contains no two-particle inelastic channels. For $f(\rho) = 0$ we get a model which is very similar to that of Ref. 2, the difference being in the number of channels considered and in the fact that in the present model we consider the scattering of non-identical particles.

We discuss here the combined model obtained when both $a(\rho)$ and $f(\rho)$ are different from zero. The shape of these functions will be chosen on the basis of the analysis of the two limiting cases just mentioned. As was proved in Ref. 1, the assumption that correlations between particles in the many-particle states are as weak as possible implies a Gaussian form of the overlap function:

$$f(\rho) = f_0 e^{-\frac{\rho^2}{r^2}}. \quad (5.1)$$

From the unitarity condition (3.7) we have the condition

$$0 \leq f_0 \leq \frac{1}{2}. \quad (5.2)$$

For the K-matrix element $a(\rho)$ we will assume, following Ref. 2, that its t dependence is given by the diagrams in Fig. 1 in which the object exchanged is taken to have a given mass m , $C = 1$ and no spin nor change, and in which no t dependence is considered for the vertices. For the partial-wave decomposition at high energy this implies

$$a(\rho) = D K_0(m\rho) \quad (5.3)$$

The dimensionless constant D gives the strength of the interaction. It was shown in Ref. 2 that the detailed shape of $a(\rho)$ has little effect on the properties of the scattering. This holds even more strongly in the present model.

From (4.14) one obtains the formulae for the differential elastic and inelastic (two-body) cross-sections in terms of the functions $\eta(\rho)$ and $Y(\rho)$ given by Eqs. (4.15) and (4.16).

$$\frac{d\sigma_{el}}{dt} = \pi \left| \int_0^\infty \rho d\rho \eta(\rho) J_0(\rho\sqrt{-t}) \right|^2 \quad (5.4)$$

$$\frac{d\sigma_{1 \rightarrow 2}}{dt} = \pi \left| \int_0^\infty \rho d\rho Y(\rho) J_0(\rho\sqrt{-t}) \right|^2. \quad (5.5)$$

Eq. (5.4) applies to all two-body channels. The cross-section (5.5) refers to the processes AB (channel 1) $\leftrightarrow A^*B^*$ (channel 2) and $A+B^* \leftrightarrow A^*+B$. From the optical theorem we can calculate the total cross-section in terms of the function $\eta(\rho)$:

$$\sigma_{tot} = 4\pi \int_0^\infty \rho \eta(\rho) d\rho. \quad (5.6)$$

Eqs. (5.4) - (5.6) enable us to calculate the elastic, two-body inelastic and total cross-sections in terms of three dimensionless parameters f_0 , $D, \gamma = mr$ and one length r which we have at our disposal.

We first calculated the ratios

$$\alpha = \frac{\sigma_{el}}{\sigma_{tot}} \quad (5.7)$$

and

$$\beta = \frac{\sigma_{1 \rightarrow 2}}{\sigma_{tot}} \quad (5.8)$$

where $\sigma_{1 \rightarrow 2}$ is the total cross-section for the reaction $1 \rightarrow 2$.

The ratios α and β are dimensionless and therefore they do not depend on r . The dependence on other parameters is shown in Tables 1 and 2. They give the values α and β , respectively, for different values of f_0 , D and γ .

TABLE 1

α		$f_0 = 0.1$				$f_0 = 0.5$			
D	γ	0.5	$1/\sqrt{2}$	1	1.5	0.5	$1/\sqrt{2}$	1	1.5
0		0.026	0.026	0.026	0.026	0.185	0.185	0.185	0.185
0.2		0.077	0.074	0.063	0.052	0.187	0.195	0.194	0.191
0.5		0.193	0.193	0.187	0.158	0.216	0.230	0.229	0.214
1		0.317	0.316	0.314	0.296	0.290	0.292	0.289	0.264

TABLE 2

β		$f_0 = 0.1$				$f_0 = 0.5$			
D	γ	0.5	$1/\sqrt{2}$	1	1.5	0.5	$1/\sqrt{2}$	1	1.5
0		0	0	0	0	0	0	0	0
0.2		0.67	0.51	0.33	0.20	0.21	0.094	0.031	0.011
0.5		0.74	0.68	0.59	0.45	0.50	0.32	0.16	0.054
1		0.66	0.64	0.60	0.52	0.59	0.48	0.32	0.14

One can see from these Tables that

- (i) for small values of D , α increases with f_0 . This feature was found also in the uncorrelated model of Ref. 1 (which corresponds to $D = 0$). For larger D the situation is reversed: α decreases when f_0 increases. This effect can be understood if one remembers that in the pure two-particle model ($f_0 = 0$) $\alpha \rightarrow 1$ for $D \rightarrow \infty$, whereas for the uncorrelated model one has, for all f_0 , $\alpha \leq \alpha(f_0 = 0.5) = 0.185$.
- (ii) When the strength D and/or the range γ^{-1} of two-body interaction increases, the ratio α also increases.
- (iii) The ratio β decreases if f_0 increases, which simply means that if f_0 increases, the two-body processes become less important in the total cross-section. This is, of course, due to the increasing absorption.
- (iv) If the range γ^{-1} of the two-body interaction decreases the ratio β decreases. This is also quite clear.
- (v) The dependence of β on D is more complicated. If D increases from zero then β also increases from zero to some maximal value (which depends on γ and f_0) and after this it starts to decrease. The decrease of β for $D \rightarrow \infty$ can be understood if we observe that, as could be seen from formula (4.16), the two-body inelastic partial wave tends to zero if the coupling (i.e. the K-matrix element) for this partial wave tends to infinity. This is due to the self-damping effect imposed by the unitarity condition.

To conclude, we see that in our model we can get for the ratios α and β any value between 0 and 1. There exists no natural bound as in the case of the uncorrelated model ($\alpha \leq 0.185$). The experimental values of the ratio α are rather small for all reactions measured up to now. Thus, $\alpha \approx 0.16$ for πp at 17 GeV/c and $\alpha \approx 0.24$ for pp at 20 GeV/c. Recent results suggest that, for pp , α will become smaller than 0.185 at sufficiently high energies where the elastic amplitude becomes purely imaginary⁴). This suggests that the coupling in the two-particle channels is small ($D < 1$) or, if it is big, the range of two-body interaction should be small ($\gamma > 1$) so that the two-body inelastic scattering will be strongly suppressed by absorption. Whereas β is small in the latter case, it could be large in the former. But this is excluded by the fact that $D < 1$ and weak absorption would give a diffraction peak with a shape characteristic for weak coupling (see Ref. 2), which contradicts experiment. Therefore, β must be rather small in all cases.

In the further analysis we restricted ourselves to the range

$$0.15 \leq \alpha \leq 0.25 \quad (5.9)$$

$$\beta \leq 1 - \alpha - \beta. \quad (5.10)$$

The condition (5.10) means that at least half of the inelastic cross-section corresponds to many-particle final states. This seems to be in agreement with experimental evidence. The condition (5.9) is also satisfied by all reactions measured up to now.

The next point we discuss is whether one can obtain in our model reasonable values of the total cross-section for reasonable values of the mass m of the exchanged particle. To this end we calculated the total cross-section for $m = m_\pi$ and for $m = m_\rho$, other parameters varying in the limits given by Eqs. (5.9) and (5.10). To fix r we put the additional limitation

$$20 \text{ mb} \leq \sigma_{tot} \leq 40 \text{ mb} \quad (5.11)$$

corresponding to $r \approx (2m_\pi)^{-1}$. Together with the conditions (5.9) and (5.10) it implies that the damping due to many-body channels cannot be too small :

$$f_0 \geq 0.3. \quad (5.12)$$

Furthermore, the conditions (5.9) - (5.11) imply that for $m = m_\pi$ the coupling in the two-body channel should be rather weak

$$D \leq 0.5. \quad (5.13)$$

The corresponding value of β is ≤ 0.3 . For $m = m_\rho$ stronger coupling is admissible [$D \leq 2$], but because of the self-damping effect the two-body inelastic cross-section is in this case very small ($\beta \leq 0.1$).

Concerning the shape of the diffraction peak, we found that the elastic scattering in our model, for all values of parameters satisfying the conditions (5.9) - (5.12), can be very

well approximated by the formula

$$\frac{d\sigma_{el}}{dt} = \left(\frac{d\sigma_{el}}{dt} \right)_{t=0} e^{at+bt^2} \quad \frac{b}{a^2} \ll 1 \quad (5.14)$$

where, because of the optical theorem,

$$\left(\frac{d\sigma_{el}}{dt} \right)_{t=0} = \frac{\sigma_{tot}^2}{16\pi} \quad (5.15)$$

In particular, one obtains the shape (5.14) in the limit of both weak ($D \rightarrow 0$) and strong ($D \rightarrow \infty$) couplings. [Of course, if $D \rightarrow \infty$ then also $\gamma \rightarrow \infty$ in order to satisfy the conditions (5.9) and (5.10)].

This great stability of the shape of the diffraction peak seems to provide a possible explanation of the fact that the formula (5.14) applies to all elastic processes measured up to now, despite the different kinds of interactions involved.

The inelastic two-body scattering cross-section $d\sigma_{1 \rightarrow 2}/dt$ can be calculated from Eq. (5.5), the inelastic partial wave amplitudes being given by Eq. (4.16). These partial wave amplitudes are "corrected for unitarity" and differ appreciably from the simple one-particle exchange partial waves given by Eq. (5.3). Let us observe that in formula (4.16) the one-particle-exchange partial wave amplitude $2a(\rho)$ is damped by two factors: (a) the factor $1/1+a^2(\rho)$ coming from a self-damping of the process $1 \rightarrow 2$, and (b) the factor $\sqrt{1-2f(\rho)}$ which represents damping due to competing inelastic processes of many-particle type. The unitarity corrections to the one-particle exchange models have been discussed recently by several

authors⁶⁻¹⁹). We postpone the comparison with the other treatments to Section 6.

We calculated the inelastic two-body scattering from formula (5.5) for several values of the parameters. A typical example of the function $d\sigma_{1\rightarrow 2}/dt$ is plotted in Fig. 2. In the same figure we plot also the uncorrected one-particle exchange cross-section.

From the calculations one can draw the following conclusions:

- (i) The inelastic scattering, as calculated from Eq. (5.5), is much steeper than the uncorrected one-particle exchange cross-section.
- (ii) The function $d\sigma_{1\rightarrow 2}/dt$ shows, in general, more than one diffraction maximum. The amplitude of the oscillations is strongly damped, however. The ratio of the first to the second maximum is of the order 10^3 .
- (iii) The inelastic scattering is strongly peaked in the forward direction. For $m = m_\pi$ the peaking is much stronger than in elastic scattering. For $m = m_\rho$ the forward peak is broader than the elastic one. This last feature is due to the existence of a big amount of many-particle processes (without them, the cases $m = m_\pi$ and $m = m_\rho$ would, of course, only differ by a change of t scale).

6. COMPARISON OF OUR UNITARITY CORRECTIONS WITH THE TREATMENT OF OTHER AUTHORS

The "unitarity corrections" to the one-particle exchange model were considered recently by a number of authors. There exist essentially three different approaches to this problem:

- (a) The methods based on the analytic properties of the scattering amplitude : N/D equations with the left-hand cut given by the one-particle exchange forces^{6,19}).
- (b) The use of the distorted wave Born approximation (DWBA), which amounts to introducing a **phenomenological** absorption due to competing channels. This absorption is calculated directly from the elastic scattering^{7, 11-13}), or assumed to be of the black sphere type^{9, 10}).
- (c) The use of the K-matrix formalism^{8, 16, 17}).

The approach proposed in the present paper is a combination of ideas expressed in (b) and (c). It is interesting, therefore, to compare these different methods in the frame of the simple 4-channel model which we discussed in the preceding Sections.

The corrections due to unitarity are expressed in our model by formula (5.5):

$$Y(\rho) = \frac{2 a(\rho)}{1 + a^2(\rho)} \sqrt{1 - 2f(\rho)} \quad (6.1)$$

the uncorrected amplitude being $2a(\rho)$. We recall also the

expression for the elastic phase shift:

$$e^{2i\delta(\rho)} = 1 - \eta(\rho) = \frac{1 - a^2(\rho)}{1 + a^2(\rho)} (1 - 2f(\rho))^{\frac{1}{2}}. \quad (6.2)$$

It is easy to see that the formula used in the DWBA approach^{7, 11, 13}) can be derived as a special case of Eqs. (6.1) and (6.2). Assuming that $a(\rho)$ is very small, and keeping only linear terms in $a(\rho)$ we get

$$\eta(\rho) = 2 a(\rho) (1 - 2f(\rho))^{\frac{1}{2}} = 2 a(\rho) e^{2i\delta(\rho)} \quad (6.3)$$

which is identical with the DWBA formula, provided the elastic scattering for the initial and final states is the same (which is the case in our model).

To see what is the difference in the cross-sections obtained from formulae (6.1) and (6.3) we calculated the cross-sections for a few values of the parameters. We restricted ourselves to small values of D which, as we have shown in the preceding Section, correspond better to the real physical situation. It appears that in this case there is no essential difference between the results of DWBA and our approach. The angular distribution in DWBA is slightly broader and the total two-body cross-section is slightly larger (5 - 10%). The first feature can be seen in Fig. 2, where we plotted the function $d\sigma_{1 \rightarrow 2}/dt$ versus t . The parameters are given in the figure.

The next model we would like to discuss is that proposed by Arnold¹⁶). In this model the many-particle channels are neglected. This corresponds in our model to the assumption $f(\rho) \equiv 0$.

One obtains then

$$\gamma(p) = \frac{2 a(p)}{1 + a^2(p)} \quad (6.4)$$

$$e^{2i\delta(p)} = \frac{1 - a^2(p)}{1 + a^2(p)}. \quad (6.5)$$

These formulae are essentially the same as those we discussed in some detail in the preceding paper²⁾. Here we would like to repeat only that it seems rather difficult to obtain from the formula (6.5) both the correct value of the ratio of elastic to total cross-sections, and the correct shape of the diffraction peak. This difficulty was realized also by Arnold¹⁶⁾.

We calculated the elastic and inelastic cross-section in Arnold's model for $D = 0.5$, and $m = m_\pi$. The results are plotted in Fig. 3, together with the corresponding curves for our model (as given in Fig. 2). One can see that the elastic cross-sections behave similarly for small momentum transfers. The inelastic cross-section in Arnold's model is much less peaked than in our model. This effect is due to the fact that we consider a rather small coupling ($D \simeq 0.5$).

Finally, we discuss a model proposed by Dietz and Pilkuhn¹⁷⁾. Assuming that the part of the K matrix describing two-particle channels is of the form (6.17), the corrected Born term in the Dietz-Pilkuhn model is

$$\gamma(\rho) = \frac{2a(\rho)}{[1+b(\rho)]^2 + a^2(\rho)} \quad (6.6)$$

The partial wave elastic amplitude is given by

$$\eta(\rho) = 2 \frac{a^2(\rho) + b(\rho)[1+b(\rho)]}{[1+b(\rho)]^2 + a^2(\rho)} \quad (6.7)$$

or

$$e^{2i\delta(\rho)} \equiv 1 - \eta(\rho) = \frac{1 - a^2(\rho) - b^2(\rho)}{[1+b(\rho)]^2 + a^2(\rho)} \quad (6.8)$$

The function $b(\rho)$ represents in this model the damping coming from the many-particle channels. Let us note a rather essential difference between the formulae (6.2) and (6.8): if the absorption due to the many-body channels takes its maximum value, the corresponding value of (6.2) is zero, whereas (6.8) takes the value -1. This last feature seems to be characteristic for all pure K-matrix formalisms.

To proceed along the lines suggested by Dietz and Pilkuhn we took for $a(\rho)$ the one-particle exchange expression (5.1) and then tried to fit the function $b(\rho)$ in order to obtain the elastic scattering as similar as possible to that in our model with a given function $f(\rho)$. [One can easily see from the formulae (6.2) and (6.8) that one cannot obtain identical elastic scattering in both models for $b(\rho) \geq 0$. Indeed, for $a^2(\rho) > 1$ and $1 - 2f(\rho) \approx 0$, no value

of $b(\rho)$ can bring the ratio of (6.8) to (6.2) close to 1]. This function $b(\rho)$ was then used in the calculation of the inelastic scattering. The results are plotted in Fig. 4 together with the corresponding curves for our model. One can see that the inelastic two-body cross-section in the Dietz-Pilkun model is slightly less peaked in the forward direction than in our model.

7. CONCLUSIONS

The first part of the present paper discusses the consequences of the unitarity condition of the scattering matrix for elastic and inelastic two-particle processes. These consequences can be conveniently split into two parts:

- (a) the effects due to the existence of many-particle, weakly correlated channels, and
- (b) the effects due to the existence of several two-particle channels.

The many-particle channels influence the two-particle reactions via the overlap matrix (2.6), implying a damping of the two-particle part of the S matrix. Using a physically reasonable simplifying assumption on the overlap matrix, one can prove that this damping is of the same form as that derived from the distorted wave Born approximation.

Under the same assumptions on the overlap matrix, the constraints due to the existence of several two-particle channels imply that the two-particle part of the S matrix should be a

unitary matrix multiplied by a factor determined by the overlap matrix. This means that the different two-body reactions cannot be treated independently of each other, i.e. there exist correlations between them.

We discussed a simple method to correct two-particle amplitudes, supposed to be given a priori (e.g. Born amplitudes), for the two types of unitarity effects mentioned under (a) and (b). Our formulae describing the corrected Born amplitudes represent a generalization of two approaches in this problem which have been proposed before: (i) the distorted wave Born approximation technique and (ii) the K-matrix approach. Both DBWA and K-matrix treatments appear as limiting cases of our formulae. If the unadorned Born terms are small, our formulae are well approximated by the DBWA formulae for absorption. If, on the other hand, one neglects the existence of many-particle processes, our approach is identical to (ii).

In the second part of the paper we developed a simple model for very high energy interactions introducing four two-body channels (AB , A^*B , AB^* , A^*B^* ; A^* , B^* are isobars of A, B), as well as many particle channels globally described by an overlap function. The two-body channels obey crossing symmetry under the assumption of $C = 1$ exchange. The model is found to depend on two arbitrary functions of the impact parameter ρ : one K-matrix element $a(\rho)$ and the overlap function $f(\rho)$. The detailed study of the model shows that:

- (i) If $C = 1$ exchange dominates high-energy scattering, the process of one isobar excitation ($A + B \rightarrow A + B^*$ or $A + B \rightarrow A^* + B^*$) should go down with energy

- (ii) If the excitation of two isobars ($A + B \rightarrow A^* + B^*$) has a non-vanishing cross-section in the high-energy limit, the observed value of σ_{el}/σ_{tot} and shape of the diffraction peak imply that the isobar cross-section should be small compared to the "true" many-particle production processes.
- (iii) Under the assumption that the particles in many-body states produced by the collision are uncorrelated, the shape of the diffraction peak calculated in our model can be very well approximated by the experimental curve (5.14). This feature of the model is essentially independent of the form of the interaction responsible for isobar excitation in two-body processes, provided both elastic and two-body inelastic cross-sections are small compared to the total cross-section. We consider this stability as a possible explanation of the universal shape (5.14) found for all measured elastic processes.
- (iv) The inelastic two-body differential cross-section is strongly peaked in the forward direction, and shows very small second and further diffraction maxima.

In the last part of the paper we compared the corrections to the one-particle exchange model proposed by Sopkovich (DWBA),⁷⁾ by Dietz and Pilkuhn¹⁷⁾, and by Arnold¹⁶⁾, with those proposed in the present paper. The comparison was done in the frame of the simple model we discussed before. It shows that for the case of small two-body channel coupling there is no essential difference between the shape and magnitude of the inelastic scattering calculated in the DWBA and Dietz-Pilkuhn approaches and in our model. The Arnold model predicts angular distributions which are less peaked in the forward direction.

ACKNOWLEDGEMENTS

The authors would like to thank Drs. G. Belletini and H. Pilkuhn for interesting discussions. They greatly appreciate the help of Mrs. E. Bias in the numerical work required by this investigation.

One of the authors, (A.B.), is grateful to the Theoretical Division of CERN for the kind hospitality.

REFERENCES AND FOOTNOTES

- 1) L. Van Hove, *Revs. Modern Phys.* 36, 655 (1964).
- 2) A. Białas, Th.W. Ruijgrok and L. Van Hove, "A Two-Particle Model for High-Energy Collisions", CERN preprint (1964), to be published in *Nuovo Cimento*.
- 3) W.N. Cottingham and R.F. Peierls, "The Impact Parameter Expansion of High-Energy Scattering Amplitudes", Brookhaven National Laboratory preprint (1964).
- 4) O. Czyzewski, B. Escoubès, Y. Goldschmidt-Clermont, M. Guinea-Moorhead, D.R.O. Morrison and S. de Unamuno-Escoubès, "Antiproton-proton elastic scattering at 4 GeV/c and derivation of diffraction slope at infinite energy", CERN preprint (1965), to be published in *Physics Letters*.
- 5) The value of this ratio is not given explicitly in Ref. 2. Instead we have used the parameter a/σ_{tot} where a describes the width of the diffraction peak $d\sigma_{\text{el}}/dt = (\sigma_{\text{tot}}^2/16\pi) e^{at}$. From this formula one can easily find $\sigma_{\text{el}}/\sigma_{\text{tot}} \approx (1/16\pi)\sigma_{\text{tot}}/a$.
- 6) M. Baker and R. Blankenbecler, *Phys. Rev.* 128, 415 (1962).
- 7) N.J. Sopkovich, *Nuovo Cimento* 26, 186 (1962).
- 8) D.S. Chernavskii, *JETP* 45, 1558 (1963), English translation: *Soviet Physics JETP* 18, 1072 (1964).

- 9) A. Dar, M. Kugler, Y. Dothan and S. Nussinov,
Phys.Rev.Letters 12, 82 (1964).
- 10) A. Dar and W. Tobocman, Phys.Rev.Letters 12, 511 (1964).
- 11) L. Durand and Y.T. Chiu, Phys.Rev.Letters 12, 399 (1964)
and "Single Particle Exchange Models for the Reactions
 $\pi p \rightarrow \rho p$, $\bar{p} p \rightarrow \bar{Y} Y$, $n p \rightarrow p n$ ", preprint (1964).
- 12) M.H. Ross and G.L. Shaw, Phys.Rev.Letters 12, 627 (1964).
- 13) K. Gottfried and J.D. Jackson, Nuovo Cimento 34, 735 (1964).
- 14) D.B. Lichtenberg and P.K. Williams, "Multichannel Approach
to High-Energy Peripheral Collisions", preprint (1964).
- 15) V. Barger, Nuovo Cimento 35, 700 (1965).
- 16) R.C. Arnold, Phys.Rev. 136, B1388 (1964).
- 17) K. Dietz and H. Pilkuhn, "K-Matrix Model of Peripheral
Interactions", CERN preprint (1964).
- 18) R. Omnès, "Final State Interaction in the Presence of
Absorption", UCRL preprint (1964).
- 19) E.J. Squires, Nuovo Cimento 34, 1328 (1964).

FIGURE CAPTIONS

- Figure 1 : The diagrams contributing to the K matrix.
- Figure 2 : Comparison of our model with the DWBA approach.
Continuous curves: elastic and inelastic scattering in our model. Dashed-dotted curve: inelastic scattering in DWBA approach. Dashed curve: uncorrected one-pion exchange model.
- Figure 3 : Comparison of Arnold's model with ours.
Continuous curves: elastic and inelastic scattering in Arnold's model. Dotted curves: elastic and inelastic scattering in our model.
- Figure 4 : Comparison of the Dietz-Pilkuhn model with ours.
Continuous curves: elastic and inelastic scattering in the Dietz-Pilkuhn model. Dotted curves: elastic and inelastic scattering in our model.

FIG.1

FIG. 2

FIG. 3

FIG. 4