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FIRST ORDER VIOLATION OF SU_3 INVARIANCE IN THE VECTOR
AMPLITUDE OF LEPTONIC DECAYS OF STRANGE PARTICLES

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Cabibbo's theory of leptonic decays of strange particles based on unitary symmetry ¹⁾ seems to lead to results which compare well with experimental data. However, one expects modifications of this theory due to the symmetry violating interaction which is responsible, among other things, for the mass splittings within the unitary octet.

Assuming that the violation of SU_3 invariance can be described by a local Lagrangian $\mathcal{L}_1(x)$ which transforms like an $I=0$ $S=0$ member of an SU_3 multiplet (but not necessarily like the eight component of an octet), we shall prove that the first order correction to the vector decay amplitude in the limit of zero momentum transfer can be accounted for by using unrenormalized coupling constants but with wave functions corresponding to the physical masses.

As we shall see, this wave function correction is in fact only of second order for the relevant matrix element. So in first order there is no modification of the matrix element for the time component of the vector current.

Our proof is valid for bosons as well as for baryons and can be easily generalized to other groups. We shall give a detailed proof only for the baryon case. We write down the hadron vector current density in the form used by Cabibbo :

$$J_\mu(x) = \cos \theta j_\mu^{(0)}(x) + \sin \theta j_\mu^{(1)}(x). \quad (1)$$

The S matrix element describing the baryon leptonic decay $B \rightarrow B' + \ell + \bar{\nu}$ through the vector interaction is written in the following way :

$$S_{fi}^{(0)} = \frac{G}{\sqrt{2}} \int d^4x M_\mu^{(0)}(x) e^{iqx} \bar{u}_\ell(p_\ell) \gamma^\mu (1 + \gamma_5) u_\nu(p_\nu) \quad (2)$$

where $q = -(p_\ell + p_\nu)$ is the momentum transfer and

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$$M_{\mu}^{(0)}(x) = -i \langle B' | J_{\mu}(x) | B \rangle. \quad (3)$$

$|B\rangle$ and $|B'\rangle$ stand for ingoing (outgoing) physical states of the baryons B and B' in a field theory with exact unitary symmetry. In the limit of zero momentum transfer : $M_{\mu}^{(0)}(x)$ is given in terms of the generalized SU_3 Clebsch-Gordan coefficients by :

$$M_{\mu}^{(0)}(x) = -i \bar{\varphi}_{B'}(x) \left[\cos \theta (f_{BB'}^1 + f_{BB'}^2) + \sin \theta (f_{BB'}^4 + f_{BB'}^5) \right] \gamma_{\mu} \varphi_B(x)$$

To avoid divergences in our proof it is convenient to describe the state $|B\rangle$ and $|B'\rangle$ by normalizable wave packets $\varphi_B(x)$ and $\varphi_{B'}(x)$ solutions of the Dirac equation with the degenerate mass M_0 .

The matrix element corresponding to the first order correction is given by ²⁾ :

$$S_{fi}^{(1)} = \frac{G}{\sqrt{2}} \int d^4x e^{iqx} M_{\mu}^{(1)}(x) \bar{u}_e \gamma^{\mu} (1 + \gamma_5) v_{\nu} \quad (5)$$

with

$$M_{\mu}^{(1)}(x) = - \langle B' | \int T(I_{\mu}(x) \mathcal{L}_1(x')) | B \rangle d^4x' \quad (6)$$

Let us take, for a moment, infinitely narrow wave packets to describe the particles B and B' and use translational invariance :

$$\begin{aligned}
M_{\mu}^{(1)}(x) &= -\langle p' | \int T(J_{\mu}(x) \mathcal{L}_1(x')) | p \rangle d^4x' \\
&= -e^{-i(p'-p) \cdot x} \langle p' | \int T(J_{\mu}(0) \mathcal{L}_1(y) d^4y | p \rangle = e^{-i(p'-p) \cdot x} M_{\mu}^{(1)}(p, p') \quad (7)
\end{aligned}$$

where we have set $y = x' - x$.

Using relativistic invariance $M_{\mu}^{(1)}(p, p')$ can be written in the following way :

$$M_{\mu}^{(1)}(p, p') = \bar{u}(p') (a^{(1)} \gamma_{\mu} + b^{(1)} \sigma^{\mu\nu} q_{\nu} + c^{(1)} q^{\mu}) u(p) \quad (8)$$

In the zero momentum transfer limit assuming the particle B at rest we have :

$$M_{\mu}^{(1)}(p, p') = \delta_{\mu,0} \bar{u}(p') a^{(1)}(0) u(p)$$

The form factor $c^{(1)}$ can be left out if we limit ourselves to electronic decays of baryons. The other neglected terms, first order correction to the "so-called" forbidden matrix element already of the order of $\delta_{M/M}$ are in fact second order corrections. It follows then that we can restrict ourselves to the time component $M_0^{(1)}(x)$ of $M_{\mu}^{(1)}(x)$.

In the limit $q \rightarrow 0$, $S_{if}^{(1)}$ is proportional to :

$$\int M_0^{(1)}(x) d^4x = -\langle B' | T(J_0(x) \mathcal{L}_1(x')) | B \rangle d^4x d^4x' \quad (9)$$

Let us perform now first the integration over d^3x . In the T product will appear the quantity $\int J_0(\vec{x}, t) d^3x$ which is nothing but a linear combination of the infinitesimal operators of the group, namely using the Gell-Mann notation :

$$\int J_0(\vec{x}, t) d^3x = (F_1 + i F_2) \cos \theta + (F_4 + i F_5) \sin \theta \quad (10)$$

These operators acting on one baryon states of an octet will give another baryon state of the same octet with the same energy momentum. Therefore, if we insert between $J_0(x)$ and $\mathcal{L}_1(x)$ a complete set of intermediate states, in the limit $q \rightarrow 0$ the only states which contribute are the one baryon states of the octet containing B and B'.

More specifically, let us compute $M_\mu^{(1)}(x)$ keeping only the one baryon intermediate states :

$$M_0^{(1)}(x) = - \int d^4x \left\{ \sum_n \theta(x_0 - x'_0) \langle B' | J_0(x) | n \rangle \langle n | \mathcal{L}_1(x') | B \rangle + \theta(x'_0 - x_0) \langle B' | \mathcal{L}_1(x') | n \rangle \langle n | J_0(x) | B \rangle \right\}. \quad (11)$$

In the limit $q \rightarrow 0$ the only contribution to $S_{fi}^{(1)}$ comes from one baryon states with an energy momentum n close to that of B and B'. So we can write :

$$\langle n | \mathcal{L}_1(x') | B \rangle \simeq \bar{\varphi}_B(x') \delta M_B \varphi_B(x') \quad (12)$$

where δM_B is the mass shift of particle B induced by the symmetry breaking interaction $\mathcal{L}_1(x)$ in first order. Of course equation (12) is valid provided there is no degeneracy between the particles which we consider and some member of another multiplet.

In the same manner we have from equations (3) and (4) :

$$\langle B' | J_0(x) | n \rangle \simeq \bar{\varphi}_{B'}(x) \left[\cos \theta \left(f_{B'n}^{(1)} + f_{B'n}^{(2)} \right) + \sin \theta \left(f_{B'n}^{(4)} + f_{B'n}^{(5)} \right) \right] \gamma_0 \varphi_B(x) \quad (13)$$

We can then rewrite $M^{(1)}(x)$ in the following form :

$$M_0^{(1)}(x) = -i \left[\cos \theta \left(f_{B'B}^{(1)} + f_{B'B}^{(2)} \right) + \sin \theta \left(f_{B'B}^{(4)} + f_{B'B}^{(5)} \right) \right] \left[\bar{\varphi}_{B'}(x) \gamma_0 \delta \varphi_B(x) + \delta \bar{\varphi}_{B'}(x) \gamma_0 \varphi_B(x) \right] \quad (14)$$

where $\delta \varphi_B$ and $\delta \bar{\varphi}_{B'}$ are given by the following expressions :

$$\delta \varphi_B(x) = -i \int d^4x' \theta(x_0 - x'_0) \left(\sum_n \varphi_n(x) \otimes \bar{\varphi}_n(x') \right) \delta M_B \varphi_B(x') \quad (15)$$

$$\delta \bar{\varphi}_{B'}(x) = -i \int d^4x' \theta(x'_0 - x_0) \left(\sum_n \varphi_n(x') \otimes \bar{\varphi}_n(x) \right) \delta M_{B'} \bar{\varphi}_{B'}(x') \quad (16)$$

We shall now compare these expressions with the result of a first order perturbation performed on the Dirac equation

$$\left(\gamma^\mu \frac{\partial}{\partial x^\mu} + M_0 \right) \varphi'(x) = \delta M \varphi'(x), \quad (17)$$

the mass displacement term $\delta M \varphi'(x)$ being considered as the perturbation.

The perturbed wave function $\delta\varphi(x) = \varphi'(x) - \varphi(x)$ is given in terms of the Feynman propagator $S_F(x-x')$ by :

$$\delta\varphi(x) = \varphi'(x) - \varphi(x) = \frac{i\delta M}{2} \int S_F(x-x') \varphi'(x') d^4x' \quad (18)$$

where $\varphi(x)$ is a solution of the Dirac equation with the degenerate mass M_0 . Similarly, we have :

$$\delta\bar{\varphi}(x) = \bar{\varphi}'(x) - \bar{\varphi}(x) = \frac{i\delta M}{2} \int \bar{\varphi}(x') S_F(x'-x) d^4x'. \quad (19)$$

Let us write now $S_F(x-x')$ as a vacuum expectation value of the time ordered product of the free fields $\psi(x)$ and $\bar{\psi}(x')$:

$$S_F(x-x') = -2 \langle \phi_0 | T(\psi(x) \bar{\psi}(x')) | \phi_0 \rangle \quad (20)$$

Inserting between the two field operators a complete set of orthogonal states we get the following expressions :

$$S_F(x-x') = -2 \left\{ \sum_{n \in E_+} \theta(x_0 - x'_0) \varphi_n(x) \bar{\varphi}_n(x') - \sum_{n \in E_-} \theta(x'_0 - x_0) \chi_n(x) \bar{\chi}_n(x') \right\} \quad (21)$$

where $\chi_n(x)$ is a negative energy wave packet of the Dirac equation. Since the states $|B\rangle$ and $|B'\rangle$ we are considering correspond to a very small kinetic energy, it is legitimate in the computation of the perturbed wave function to neglect the contribution of the negative energy

states. It follows then that we can identify $\varphi_B(x) + \delta\varphi_B(x)$, $\bar{\varphi}_{B'}(x) + \delta\bar{\varphi}_{B'}(x)$, with the perturbed wave functions of the ingoing and outgoing baryons under the influence of the mass splitting δM_B and $\delta M_{B'}$. In other words, the effect of the symmetry breaking on the matrix element of the time component of the current can be accounted for by taking Dirac wave functions with the physical masses. More precisely, the matrix element of the time component of the current is proportional to the overlap between the function $\varphi_B(\vec{x}, t)$ and $\varphi_{B'}(\vec{x}, t)$ with the perturbed mass. Using plane waves for φ_B and $\varphi_{B'}$ and taking the particle B at rest, the lack of overlap is given by :

$$1 - \sqrt{\frac{M_B + E'}{2E'}}$$

which is of the order of $(\delta M/M_0)^2$, in agreement with the known result of perturbation theory where the lack of overlap between the unperturbed wave function and the first order perturbed wave function is of second order. Therefore to the first order in the symmetry breaking there is no modification of the matrix element of the time component of the current.

In principle, our considerations apply as well to mesons as to baryons, except that in that case $\delta M/M_0$ is of order 1 and therefore the use of first order perturbation is problematic unless one makes some kind of dynamical assumption to explain the validity of the Gell-Mann Okubo mass formula.

A remark should be made at this point concerning our assumption that the symmetry breaking interaction can be described by a local Lagrangian $\mathcal{L}_1(x)$. This limitation implies that our proof would not be valid for instance in a theory where the symmetry breaking would be due to a second order effect as in certain subnuclear particle models where the symmetry breaking term in the Lagrangian transforms like the $I=0$ $S=0$ component of a unitary triplet ³⁾.

In conclusion we would like to stress that the proof we have given holds independently of the representation to which the symmetry breaking interaction belongs provided it carries the proper quantum numbers. The result which we get would therefore remain true if there were some admixture of representation 27, for instance, in the symmetry breaking interaction. In the same way, our result remains valid if B and B' are not members of an octet provided they belong to the same representation of the group.

After this work was completed, we received a preprint of M. Ademollo and R. Gatto dealing with a similar problem from a somewhat different point of view.

R E F E R E N C E S

- 1) N. Cabibbo, Phys.Rev. Letters 10, 531 (1963);
M. Gell-Mann, Phys.Rev. 125, 1067 (1962).
- 2) Similar considerations were applied by R.E. Behrends and A. Sirlin
[Phys.Rev. Letters 4, 186 (1960)] to the renormalization of the
vector current in β decay due to electromagnetic mass differences.
- 3) J. Schwinger, Phys.Rev. Letters 12, 237 (1964); "Field theory of
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