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CM-P00057063

WEAK INTERACTIONS AND UNITARY SYMMETRY

(Review talk at Royal Society Meeting,
London, 5 November 1964)

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I will briefly discuss the recent advances in the understanding of weak interactions which have been made possible by the discovery of approximate SU_3 symmetry of strong interactions ¹⁾.

I will only consider here leptonic weak interactions since this is the field in which the SU_3 approach has been more successful. In the V-A theory of Feynman and Gell-Mann ²⁾, Marshak and Sudarshan ³⁾, the weak interaction of leptons among themselves and with hadrons is described by a Lagrangian

$$\mathcal{L} = \frac{g}{\sqrt{2}} (J_\mu l_\mu + J_\mu^* l_\mu^* + l_\mu l_\mu^*) \quad (1)$$

where l_μ and J_μ are "weak" currents of leptons and of hadrons respectively. If weak interactions are mediated by vector mesons, Eq. (1) could still be adequate to give a phenomenological description of those low energy weak processes which do not involve real vector mesons. The current l_μ is known to be of the form:

$$l_\lambda = (\bar{e} \gamma_\lambda (1 + \gamma_5) \nu_e) + (\bar{\nu}_\mu \gamma_\lambda (1 + \gamma_5) \mu) \quad (2)$$

It is impossible to give a similar explicit description of the current of strongly interacting particles, J_λ ; given the difficulties of field theory of strong interactions, such a description would in fact be useless. To characterize J_λ we must recur to properties which are meaningful in spite of strong interactions, or, more explicitly, to properties which are meaningful thanks to the exact or approximate symmetries of strong interactions.

2.

The main properties of the $\Delta S = 0$ part of J_λ can be assumed to be:

- a) the vector part of $J_\lambda (\Delta S = 0)$ is a conserved current and in fact is proportional to the current which carries the I^+ component of isotopic spin [CVC hypothesis ²⁾

$$V_\lambda (\Delta S = 0) \propto j_\lambda^{(+)} \quad (3)$$

The proportionality factor in Eq. (3) can be measured by comparing the vector coupling constant G_V in beta decay (as measured, e.g., in $^{14}\text{O} \rightarrow ^{14}\text{N}$ and other pure Fermi decays) and the coupling constant in muon decay, equal to G by Eq. (1); in terms of these we can write explicitly

$$V_\lambda (\Delta S = 0) = \frac{G_V}{G} j_\lambda^{(+)} \quad (4)$$

- b) Goldberger-Treiman relations for the axial current ⁴⁾. These relations are best understood by assuming an intimate relation among the axial current and the source of the pion field, e.g., in the form proposed by Gell-Mann and Lévy ⁵⁾:

$$j_\lambda A_\lambda (\Delta S = 0) \propto \phi_{\pi^-} \quad (5)$$

Let us note that Eqs. (4) and (5) indicate a particular kindness of nature. In the case of the vector part $V_\lambda (\Delta S = 0)$ Eq. (4) gives a complete characterization from the point of view of weak interactions: $j_\lambda^{(+)}$ is a current uniquely defined in the world of strong interactions. Particular problems on its structure (form factors, et.) should concern strong interaction physicists. The characterization of $A_\lambda (\Delta S = 0)$ afforded by Eq. (5) is also nearly complete.

SU_3 symmetry connects strange particles with non-strange particles. Its validity in strong interactions suggests that properties of $\Delta S \neq 0$ weak currents should be similar to those of $\Delta S = 0$ ones. We note that the I spin current $j_\lambda^{(+)}$ is a member of the octet of conserved vector currents j_λ^i , required by SU_3 invariance, and that the pion is also a member of an octet of PS mesons. This suggests a straightforward generalization of Eqs. (5) and (6):

- i) The axial and vector parts of J_λ are members of octets. The vector part of J_λ is a member of the octet j_λ^i to which the I spin current and the electromagnetic current also belongs. The axial part of J_λ should belong to an octet of axial currents g_λ^i , whose divergence is proportional to the (renormalized) fields of the PS octet:

$$\partial_\lambda g_\lambda^i = \alpha \phi^i \quad (6)$$

A first consequence of assumption i) is the absence of $\Delta S = -\Delta Q$ currents and a selection rule $\Delta I = \frac{1}{2}$ for the $\Delta S = \Delta Q = 1$ currents. In any octet we have in fact only two members with $\Delta Q = 1$ (as required for the interaction with a charged lepton pair); these are the homologous of π^+ and K^+ .

4.

We will add two further hypotheses:

- ii) V_λ and A_λ are "parallel" members of the respective octets: i.e., we will write

$$V_\lambda = a \, j_\lambda^{(0)} + b \, j_\lambda^{(1)} \quad (7a)$$

$$A_\lambda = a \, g_\lambda^{(0)} + b \, g_\lambda^{(1)} \quad (7b)$$

This assumption is suggested by the wish to have all "bare" weak interactions of the form $\gamma_\lambda(1+\gamma_5)$. Parallelism as defined in Eqs. (7) [note that Eq. (7b) defines the arbitrary parameter in Eq. (6)] is preserved by SU_3 conserving strong interactions, while the form $\gamma_\lambda(1+\gamma_5)$ is not, due to the presence of F and D type octets, as we will see in the following.

- iii) Universality. This is imposed by putting

$$a^2 + b^2 = 1 \quad (8)$$

or, as function of an angle θ to be determined experimentally,

$$\begin{aligned} a &= \cos \theta \\ b &= \sin \theta \end{aligned} \quad (9)$$

This formulation of universality is suggested by the following consideration: in the limit of exact SU_3 the definition of strangeness becomes completely arbitrary; any two orthogonal combinations of π^+ and K^+ , e.g., could be chosen as having $S = 0$ and 1 respectively. In the SU_3 limit, in fact, unitary transformations of the form $\exp[i\alpha F_7]$ become physically possible. With such a transformation the ratio b/a can be made to assume any value; for example

$$a \pi^+ + b K^+ = e^{2i\theta F_7} \frac{1}{\sqrt{a^2 + b^2}} (a \pi^+ + b K^+) e^{2i\theta F_7} \quad (10)$$

The value of the ratio, b/a or equivalently, the direction of J_λ in the $\Delta Q = 1$ subspace of the octet space, is devoid of any meaning in the SU_3 limit, where in fact J_λ would define the only privileged direction in the $\Delta Q = 1$ plane. Equation (8) affirms that, whatever the direction of J_λ (which is undefined in the SU_3 limit), its length (an SU_3 invariant) should be unitary. The angle θ introduced in Eq. (9) could be considered as the angle among the direction in the $\Delta Q = 1$ plane defined by weak interactions and the one chosen in the same plane by SU_3 breaking.

Direct consequences of ii) and iii):

- 1) θ should result the same when measured in axial and vector interactions.

Two simple determinations of θ are obtained by comparing rates for $K^+ \rightarrow \mu^+ + \nu$ and $\pi^+ \rightarrow \mu^+ + \nu$ for axial interactions and the rates for $K^+ \rightarrow \pi^0 + e^+ + \nu$ and $\pi^+ \rightarrow \pi^0 + e^+ + \nu$, for vector interactions. The results are ⁹⁾:

$$|\theta| = 0.264 \quad (\text{axial}) \quad (11a)$$

$$|\theta| = 0.24 \quad (\text{vector}) \quad (11b)$$

The agreement is indeed remarkable. Some difference is to be expected due to SU_3 breaking interactions which can affect in different ways the axial and vector parts of J_λ . In a recent paper, Ademollo and Gatto⁶⁾ have proved that, under the usual assumption of octet transformations of the SU_3 breaking interactions, these interactions fail in the lowest order to renormalize the vector part of J_λ . No such theorem exists for the axial currents. According to this theorem the value of θ as determined from vector transitions [e.g., that given in Eq. (11b)] should be (with good approximation) equal to the "true" value.

2) The $^{14}O-\mu^+$ lifetime puzzle.

The vector coupling constant in beta decay is in this theory expected to be

$$G_V = G \cos \theta \quad (12)$$

so that [see Eq. (11b)]

$$\delta G^2 = \frac{G^2 - G_V^2}{G^2} = \sin^2 \theta = 0.056 \quad (13)$$

The same quantity can be measured experimentally by comparing G , as determined by the muon lifetime, with G_V as determined from pure Fermi beta decays, like $^{14}O \rightarrow ^{14}N$ after applying the necessary radiative corrections to both cases. The experimental result⁷⁾

$$\delta G^2 = 0.044 \pm 0.003 \quad (14)$$

is in satisfactory agreement with the theoretical expectation.

Fit to leptonic decays of baryons

The theory described above ⁸⁾ can be applied to the description of leptonic decays among members of the baryon octet, of the general form

$$B_1 \rightarrow B_2 + e^- + \bar{\nu} \quad (15)$$

Once θ is known, the vector part in such a decay is uniquely determined, at least in the SU_3 limit (in the limit of zero momentum transfer these predictions are strengthened by the theorem given in Ref. ⁵⁾).

The main axial contribution is a $\gamma_\lambda \gamma_5$ term, whose coefficient in each individual reaction of type 15 can be expressed as a combination of two unknown members, F and D, with known coefficients $f(B_1 \rightarrow B_2)$ and $d(B_1 \rightarrow B_2)$. These two sets of coefficients correspond to the two ways in which we can build an octet current out of two (baryon) octets.

$$G_A(B_1 \rightarrow B_2) = G \left[f(B_1 \rightarrow B_2) F + d(B_1 \rightarrow B_2) D \right] \quad (16)$$

In this way we obtain:

$$G_A(n \rightarrow p) = G (F + D) \cos \theta \quad (17a)$$

$$G_A(\Sigma^- \rightarrow n) = G (F - D) \sin \theta \quad (17b)$$

$$G_A(\Lambda^0 \rightarrow p) = \sqrt{\frac{3}{2}} G \left(F + \frac{1}{3} D \right) \sin \theta \quad (17c)$$

$$G_A(\Sigma^- \rightarrow \Lambda) = \sqrt{\frac{2}{3}} G D \cos \theta \quad (17d)$$

etc.

The corresponding G_V are obtained from Eqs. (17) by putting $F = 1$, $D = 0$.

The unknowns F and D can be obtained by either substituting in Eq. (17) the value of G_A , where this is known, as in the case of neutron beta decay, or by fitting the total decay rate which is with a good approximation (few %) given by :

$$\Gamma(B_1 \rightarrow B_2 \ell \bar{\nu}) = \frac{(M_1 - M_2)^5}{64 \pi^3} \left(1 - 3 \frac{M_1 - M_2}{M_1 + M_2} \right) \left[3 G_A^2(B_1 \rightarrow B_2) + G_V^2(B_1 \rightarrow B_2) \right] \quad (18)$$

Recent analyses ^{10),11)} which make use of the latest experimental data show that a satisfactory fit is obtained putting ¹¹⁾:

$$\begin{aligned} F &= 0.437 \\ D &= 0.742 \end{aligned} \quad (19)$$

Use of Goldberger-Treiman relations

These lead to the prediction that the D/F ratio in the axial weak currents should be equal to that obtained from PS meson-baryon coupling. There seems to be reasonable agreement among the ratio determined from Eq. (19) and the D/F ratio for PS-B coupling as evaluated in dynamical models of the decuplet resonances ¹¹⁾.

SU₆ symmetry

In the relation of Professor Matthews you heard of the recent attempts of Gürsey, Radicati, Sakita, Pais and others to consider a group SU₆ which contains both SU₃ and spin rotations.

According to this model the D/F ratio in both PS-B coupling and axial weak interactions is expected to be $3/2$, in good agreement with Eq. (19).

Final remarks

The discovery of SU_3 symmetry has allowed a simple understanding of lepton-hadron interactions. The vector and axial weak currents of hadrons are identified with currents which have special meanings for strong interactions. This is now true for both $\Delta S = 0$ and $\Delta S = 1$ parts. The origin of the $\Delta S = \Delta Q$ and $\Delta I = \frac{1}{2}$ selection rules is also clarified. Finally lepton-hadron interactions are seen to be described in terms of two universal members: the Fermi coupling constant, G , and an angle, θ , which describes the relation among symmetry breaking and weak interactions.

Many other problems, and in particular that concerning the absence of neutral currents, are left unsolved.

R E F E R E N C E S

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