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ON AN INCONSISTENCY IN TWO ONE-PARTICLE EXCHANGE MODELS
FOR π -N SCATTERING ^{*)}

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A B S T R A C T

One examines the consistency of the partial wave amplitude dispersion relations in the one-nucleon and rho exchange. For both models, even if the correct threshold behaviour is taken into account, one finds that an inconsistency arises whenever the forces are repulsive at high energy. For the rho exchange model, even if the force is attractive, there is also an inconsistency if the upper bound $g_{\rho\pi\pi} g_{\rho NN} \leq 4$ is violated. A plausible solution is presented to remedy the previous inconsistencies of the nucleon exchange model.

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I. INTRODUCTION

Recently, a large number of papers have been devoted to the applications of the one-particle exchange model. The semi-quantitative success, reached in predicting the general features of low-energy scattering as well as the value of parameters associated with the resonances, is a reason to try to investigate the internal consistency of this model. In 1961, Chew and Mandelstam ¹⁾ have shown that, for $\pi\pi$ scattering, a repulsive marginally singular force gives rise to an inconsistent partial wave dispersion relation or to ghosts in the N/D solution of it. This result was generalized by Halpern ²⁾ who showed that for any force whose asymptotic behaviour is given by $|s|^m$, $m > 0$ (s is the square of the energy), although the partial wave dispersion relation does not exist, one can still find solutions to the N/D equations, but these solutions contain an infinite number of ghosts.

In this paper, we want to analyze the consistency of the partial wave dispersion relations of two models for which the forces are $\pi\pi$ -singular, i.e., the one-nucleon and rho exchange for $\pi\pi$ -N scattering. Our results can be stated as follows:

- a) even if the correct threshold behaviour is taken into account the dispersion relations for the two models are inconsistent whenever the forces are repulsive at high energy;
- b) for the one-nucleon exchange model we show that one may remedy the inconsistency displayed in a), if we assume that the partial wave amplitude has at least a double pole at the origin;
- c) for the one-rho exchange model, even if the forces are attractive at high energy, there is no solution to the dispersion relation which does not oscillate at infinity, unless we have $g_{\rho\pi\pi} g_{\rho NN} \leq 4$.

In Section II, we recall the definition and the analytic properties in the W plane of the partial wave amplitude for π -N scattering and we derive the dispersion relation for the one-nucleon exchange model in a way suitable for our purpose.

In Section III, neglecting the consideration of the threshold behaviour, we derive the consistency condition which can be expressed as a sum rule (Eq. (23)).

In Section IV, we ensure the correct threshold behaviour and show that we still obtain an analogous sum rule (Eq. (31)).

In Section V, we assume that the partial wave amplitude has at least a double pole at the origin and get another sum rule (Eq. (35)) which can be fulfilled for any wave.

In Section VI, we briefly discuss the rho exchange model and find an analogous sum rule (Eq. (41)). It must be stressed that the consideration of Section V is here obviously ruled out.

II. DERIVATION OF THE π -N PARTIAL WAVE AMPLITUDE DISPERSION RELATIONS IN THE ONE-NUCLEON EXCHANGE MODEL

For the π -N scattering process, to each angular momentum correspond two partial wave amplitudes relative to $J = \ell \pm 1/2$. We shall denote them $f_{\ell}^{\pm}(W)$. If W is the total energy in the centre-of-mass system of the incident particles, these partial wave amplitudes are given in terms of the invariant amplitudes $A(s,t)$ and $B(s,t)$ by ³⁾

$$f_{\ell\pm}(w) = \frac{(w+M)^2 - \mu^2}{32\pi w^2} [A_\ell + (w-M)B_\ell] + \\ + \frac{(w-M)^2 - \mu^2}{32\pi w^2} [-A_{\ell\pm 1} + (w+M)B_{\ell\pm 1}] \quad (1)$$

where

$$[A_\ell + B_\ell] = \int_{-1}^{+1} d(\cos\theta) P_\ell(\cos\theta) [A(s,t); B(s,t)] \quad (2)$$

and M, μ the masses of the proton and the pion. They satisfy the MacDowell ⁴⁾ symmetry relations

$$f_{\ell\pm}(w) = -f_{\ell\pm 1}(-w) \quad (3)$$

and the unitarity conditions

$$\text{Im } f_{\ell\pm}(w) = q(w) |f_{\ell\pm}(w)|^2 + \frac{1 - \eta_{\ell\pm}^2(w)}{4 q(w)} \quad (4)$$

where

$$q^2(w) = \frac{[w^2 - (M+\mu)^2][w^2 - (M-\mu)^2]}{4 w^2} \quad (5)$$

and $\eta_{\ell\pm}$ is the absorption coefficient.

The positions of their singularities, as derived from the Mandelstam representation, are given in Fig. 1.

We introduce now the one-nucleon exchange model. We set

$$f_{\ell\pm}(w) = f_{\ell\pm}^U(w) + f_{\ell\pm}^P(w) \quad (6)$$

where $f_{\ell\pm}^U(w)$ contains all the singularities which are deduced from the two graphs of Fig. 2, their positions are given in Fig. 3; whereas $f_{\ell\pm}^P(w)$ contains only the "physical cuts" given in Fig. 4.

Writing now explicitly the isotopic spin index, it is easy to see that $f_{\ell\pm}^U(w)$ is given by

$$\begin{aligned} f_{\ell\pm}^{U,T}(w) = & -\frac{g^2 \alpha_T}{16\pi W^2} \left(\delta_{\ell 0} \frac{(W+M)^2 - \mu^2}{W+M} + \delta_{\ell \pm 1, 0} \frac{(W-M)^2 - \mu^2}{W-M} \right) + \\ & + \frac{g^2 \beta_T}{32\pi W^2 q^2} \left\{ [(W+M)^2 - \mu^2](W-M) Q_\ell(\eta) + [(W-M)^2 - \mu^2](W+M) Q_{\ell\pm 1}(\eta) \right\} \end{aligned} \quad (7)$$

with

$$\eta = 1 - \frac{W^2 - M^2 - 2\mu^2}{2q^2} \quad (8)$$

g is the π -N coupling constant and α_T, β_T are given by

T	$\frac{1}{2}$	$\frac{3}{2}$
α_T	3	0
β_T	-1	2

(9)

It satisfies the relations

$$f_{\ell_{\pm}}^{U,T}(w) = -f_{(\ell_{\pm}+1)_{\mp}}^{U,T}(-w) \quad (10)$$

and

$$|f_{\ell_{\pm}}^{U,T}(w)| = O\left(\frac{1}{|w|}\right) \quad (11)$$

when $|w| \rightarrow \infty$ along any direction in the complex w plane. Moreover, it is clear that $f_{\ell_{\pm}}^P(w)$ fulfils the properties

$$f_{\ell_{\pm}}^P(w) = -f_{(\ell_{\pm}+1)_{\mp}}^P(-w) \quad (12)$$

$$f_{\ell_{\pm}}^P(w) = f_{\ell_{\pm}}^{P*}(w^*) \quad (13)$$

Before going further, we would like to stress the following point: the definition (1) of $f_{\ell\pm}(W)$ shows us that this function may certainly possess a pole of the order two at $W = 0$. On the other hand, we see from the expression (7) that there exist some channels for which $f_{\ell\pm}^U(W)$ does not possess this pole. It seems reasonable in those cases to include this pole in the singularities of $f_{\ell\pm}^P(W)$. We shall first neglect this possibility, intending to go back to this question in Section V.

Let us now derive a dispersion relation for $f_{\ell\pm}(W)$ neglecting both its correct threshold behaviour and the possible occurrence of a pole of order two or higher at the origin. For this, we make the hypothesis

$$\text{Max} |f_{\ell\pm}^P(w)| \leq e^{\delta |w|^\varepsilon} \quad \varepsilon < 1, \delta > 0 \quad (14)$$

The relations (4), (11), (12), (13) and the Phragmén-Lindelöf theorems imply that

$$f_{\ell\pm}^P(w) \Rightarrow 0 \quad (15)$$

when $|W| \rightarrow \infty$ along any direction in the complex plane. A dispersion relation without subtraction may be written

$$f_{\ell\pm}(w) = f_{\ell\pm}^U(w) + \frac{1}{\pi} \int_{H+\mu}^{+\infty} \frac{\text{Im} f_{\ell\pm}(w')}{w' - w} dw' + \frac{1}{\pi} \int_{-\infty}^{-(H+\mu)} \frac{\text{Im} f_{\ell\pm}(w')}{w' - w} dw' \quad (16)$$

When we change W' in $-W'$ the last integral becomes

$$- \frac{1}{\pi} \int_{M+\mu}^{+\infty} \frac{\operatorname{Im} f_{\ell_{\pm}}(-W'+i\varepsilon)}{W'+W} dW'$$

and due to the relations (3) and (4) we have

$$\operatorname{Im} f_{\ell_{\pm}}(-W'+i\varepsilon) = \operatorname{Im} f_{(\ell_{\pm}+1)_{\mp}}(W'+i\varepsilon) \geq 0. \quad (17)$$

Thus, we obtain the well-known dispersion relation

$$\begin{aligned} f_{\ell_{\pm}}(W) = & f_{\ell_{\pm}}^U(W) + \frac{1}{\pi} \int_{M+\mu}^{+\infty} \frac{\operatorname{Im} f_{\ell_{\pm}}(W'+i\varepsilon)}{W'-W} dW' - \\ & - \frac{1}{\pi} \int_{M+\mu}^{+\infty} \frac{\operatorname{Im} f_{\ell_{\pm}}(-W'+i\varepsilon)}{W'+W} dW'. \end{aligned} \quad (18)$$

In particular, if W is real and greater than $M+\mu$

$$\begin{aligned} \operatorname{Re} f_{\ell_{\pm}}(W) = & f_{\ell_{\pm}}^U(W) + \frac{P}{\pi} \int_{M+\mu}^{+\infty} \frac{\operatorname{Im} f_{\ell_{\pm}}(W'+i\varepsilon)}{W'-W} dW' - \\ & - \frac{1}{\pi} \int_{M+\mu}^{+\infty} \frac{\operatorname{Im} f_{\ell_{\pm}}(-W'+i\varepsilon)}{W'+W} dW'. \end{aligned} \quad (18')$$

III. SEARCH FOR AN INCONSISTENCY IN THE DISPERSION RELATION (18')

The consistency criterion, we are going to use, has already been applied to different physical situations ⁵⁾. Let us write the dispersion relation (18') for W positive and very big.

Applying the Frye and Warnock formula ⁶⁾ to extract the dominant terms of the two integrals, we get

$$\begin{aligned} \operatorname{Re} f_{\ell_{\pm}}(W) &= \frac{a}{W} - \\ &- \frac{1}{\pi W} \left(\int_{\mu}^W [\operatorname{Im} f_{\ell_{\pm}}(W' + i\varepsilon) + \operatorname{Im} f_{\ell_{\pm}}(-W' + i\varepsilon)] dW' \right) + o\left(\frac{1}{W}\right) \end{aligned} \quad (19)$$

where a can be computed from the relation (7).

From the unitarity condition we know that $\operatorname{Re} f_{\ell_{\pm}}(W)$ is at most of $O(\frac{1}{W})$. This implies that the two integrals of the left-hand side of (19) must converge, i.e., that

$$W \operatorname{Im} f_{\ell_{\pm}}(\pm W) \rightarrow 0 \quad (20)$$

when $W \rightarrow +\infty$.

Using the unitarity condition we deduce

$$W |f_{\ell_{\pm}}(W)| > W |\operatorname{Re} f_{\ell_{\pm}}(W)| \rightarrow 0 \quad (21)$$

when $W \rightarrow +\infty$.

As, for W very big, we have $q(W) \sim W/2$ we see that

$$\operatorname{Re} f_{\ell\pm}(w) = o\left(\frac{1}{w}\right) \quad (22)$$

and we obtain the sum rule

$$\begin{aligned} \lim_{w \rightarrow +\infty} w f_{\ell\pm}^{\nu}(w) &= \\ &= \frac{1}{\pi} \int_{\mu+\mu}^{\infty} [\operatorname{Im} f_{\ell\pm}(w'+i\varepsilon) + \operatorname{Im} f_{\ell\pm}(-w'+i\varepsilon)] dw' > 0 \end{aligned} \quad (23)$$

Thus, whenever the forces are repulsive at high energy, i.e., when the left-hand side is negative, the dispersion relation (18) is meaningless.

The values of the right-hand side of (23) are given in the Table up to $\ell = 3$. We would like to show now that this situation remains unaltered when we take into account the correct threshold behaviour.

IV. STUDY OF THE CONSISTENCY OF THE PARTIAL WAVE DISPERSION RELATIONS OF π -N SCATTERING IN THE ONE-NUCLEON EXCHANGE APPROXIMATION WHEN WE TAKE INTO ACCOUNT THE CORRECT THRESHOLD BEHAVIOUR

We want now to write a dispersion relation for the functions

$$a_{\ell\pm}(w) = p_{\ell\pm}(w) f_{\ell\pm}(w) \quad (24)$$

To simplify the notation we shall work from now on only with one of the waves (say $f_{\ell+}(W)$) and denote it simply by $f_{\ell}(W)$.

The study of the kinematical factor has already been made by Martin and Uretsky ⁷⁾ who showed that the dispersion relation for $a_{\ell}(W)$ is inconsistent if $\rho_{\ell}(W) \rightarrow 0$ when $|W| \rightarrow \infty$. Two kinematical factors have been used in the literature which do not fulfil this condition. They are

a)

$$\rho_{\ell}(W) = \frac{W^2}{(W+\mu)^2 - \mu^2} \frac{W^{2\ell}}{q^{2\ell}} \quad (25)$$

which was introduced by Frazer and Fulco ³⁾.

b)

$$\rho_{\ell}(W) = \frac{W^{2\ell+1}}{(W+M+\mu)[W^2 - (M+\mu)^2]^{\ell}} \quad (26)$$

which was introduced by Martin and Uretsky ⁷⁾.

They require that $f_{\ell}(W)$ has zeros of orders and positions given in Figs. 5 and 6. Both are such that $\rho(W) \rightarrow c > 0$ when $W \rightarrow +\infty$. The second one, as it is implied by the Mandelstam representation ⁶⁾, allows the amplitude to be different from zero at the two crossed thresholds $W = \pm(M-\mu)$. For that reason, we shall concentrate ourselves in the following on this kinematical factor, but we would like to stress that the considerations of this paragraph are also valid when we consider the Frazer and Fulco kinematical factor, because $\rho(-W)$ and $\rho(+W)$, when W is real, are both greater than zero. Under the assumption (14), we see that $a_{\ell}(W) - a_{\ell}^U(W)$ must satisfy a dispersion relation without subtraction. We get

$$a_\ell(w) = a_\ell^U(w) + \frac{1}{\pi} \int_{M+\mu}^{\infty} \frac{\text{Im } a_\ell(w')}{w' - w} dw' + \frac{1}{\pi} \int_{-\infty}^{-(M+\mu)} \frac{\text{Im } a_\ell(w')}{w' - w} dw'. \quad (27)$$

The second integral is equal to

$$- \frac{1}{\pi} \int_{M+\mu}^{+\infty} \frac{\text{Im } a_\ell(-w' + i\varepsilon)}{w' + w} dw'$$

with

$$\text{Im } a_\ell(-w' + i\varepsilon) = \rho_\ell(-w') \text{Im } f_\ell(-w' + i\varepsilon) > 0 \quad (28)$$

as from (26) we have

$$\rho_\ell(-w) = \frac{w^{2\ell+1}}{(w - M - \mu)[w^2 - (M + \mu)^2]^\ell} > 0 \quad (29)$$

if $w > M + \mu$.

The relation (27) can be written

$$a_\ell(w) = a_\ell^U(w) + \frac{1}{\pi} \int_{M+\mu}^{+\infty} \frac{\rho_\ell(w') \text{Im } f_\ell(w')}{w' - w} dw' - \frac{1}{\pi} \int_{M+\mu}^{+\infty} \frac{\rho_\ell(-w') \text{Im } f_\ell(-w')}{w' + w} dw' \quad (30)$$

From this, we easily deduce, as in Section III, the sum rule

$$\begin{aligned} \lim_{W \rightarrow +\infty} W a_\ell^U(W) &= \lim_{W \rightarrow +\infty} W f_\ell^U(W) = \\ &= \frac{1}{\pi} \int_{-\infty}^{+\infty} [\rho_\ell(W') \operatorname{Im} f_\ell(W') + \rho_\ell(-W') \operatorname{Im} f_\ell(-W')] dW' \end{aligned} \quad (31)$$

where the right-hand side is greater than zero.

The conclusions drawn after the sum rule (23) are valid for the present situation and the Table still gives the partial waves for which the inconsistency exists.

V. A WAY OUT OF THE INCONSISTENCY

We want now to modify as little as possible the Martin and Uretsky kinematical factor in order to obtain "consistent" dispersion relations. We wish to prove the following:

the assumption that $f_\ell(W)$, for every ℓ , has at least a pole of order two at the origin is equivalent to the addition of a term c_ℓ/W in the unphysical function $a_\ell^U(W)$ and the previous sum rule allows us to determine c_ℓ in the channels for which the dispersion relations were inconsistent.

Indeed, the simplest way to satisfy our assumption is to introduce the amplitude

$$\begin{aligned} t_\ell(W) &= \frac{W^{2\ell}}{(W + 1 + \mu)[W^2 - (1 + \mu)^2]}^\ell W^2 f_\ell(W) = \\ &= W a_\ell(W) \end{aligned} \quad (32)$$

and suppose that $b_\ell(0)$ is finite and different from zero. As $b_\ell(w)$ behaves now as a constant at infinity, we may write a dispersion relation without subtraction for the function

$$\frac{b_\ell(w) - b_\ell^\nu(w)}{w}$$

which in addition to the physical cuts has a simple pole at $w = 0$ with a residue equal to

$$c_\ell = b_\ell(0) - b_\ell^\nu(0) \quad (33)$$

The dispersion relation is

$$\begin{aligned} \frac{b_\ell(w) - b_\ell^\nu(w)}{w} &= \frac{c_\ell}{w} + \\ &+ \frac{1}{\pi} \int_{\mu+\mu}^{+\infty} \frac{\text{Im} b_\ell(w')}{w'(w'-w)} dw' + \frac{1}{\pi} \int_{-\infty}^{-(M+\mu)} \frac{\text{Im} b_\ell(w')}{w'(w'-w)} dw'. \end{aligned} \quad (34)$$

In terms of $a_\ell(w)$ it can be written

$$\begin{aligned} a_\ell(w) &= a_\ell^\nu(w) + \frac{c_\ell}{w} + \\ &+ \frac{1}{\pi} \int_{\mu+\mu}^{+\infty} \frac{\text{Im} a_\ell(w')}{w'-w} dw' - \frac{1}{\pi} \int_{\mu+\mu}^{+\infty} \frac{\text{Im} a_\ell(-w')}{w'+w} dw'. \end{aligned} \quad (34')$$

The sum rule (31) becomes now

$$\begin{aligned} \lim_{w \rightarrow +\infty} w a_\ell^\nu(w) &= \lim_{w \rightarrow +\infty} w f_\ell^\nu(w) = \\ &= -c_\ell + \frac{1}{\pi} \int_{\mu+\mu}^{+\infty} [\text{Im} a_\ell(w') + \text{Im} a_\ell(-w')] dw'. \end{aligned} \quad (35)$$

It is obvious that whenever the left-hand side is negative c_ℓ is determined and greater than zero. We can also redefine $a_\ell^U(w)$ and $a_\ell^P(w)$ by the expressions

$$a_\ell^{U'}(w) = a_\ell^U(w) + \frac{c_\ell}{w} \quad (36)$$

$$a_\ell^{P'}(w) = a_\ell^P(w) - \frac{c_\ell}{w}$$

which make explicit the fact that the initial assumption is equivalent to the addition of an attractive force.

VI. A SHORT DISCUSSION OF THE CONSISTENCY OF THE ONE-RHO EXCHANGE MODEL FOR π -N SCATTERING

The discussion of Sections II-IV can also be done for the one-rho exchange model. We give here only the main points of it.

The unphysical function is given by the graph of Fig. 7, and its mathematical expression is ⁸⁾

$$f_{\ell\pm}^{U,T}(w) = - \frac{g_{\rho\pi\pi} g_{\rho NN}}{4\pi} \frac{\beta_T}{4w^2 q^2} \times \quad (37)$$

$$\left\{ [(w+M)^2 - \mu^2] (w-M) Q_\ell(\gamma) + [(w-M)^2 - \mu^2] (w+M) Q_{\ell\pm 1}(\gamma) \right\}$$

where
$$\eta = 1 + \frac{m_p^2}{2q^2}$$

and

$$\beta_{1/2, 3/2} = (1, -2)$$

Its asymptotic behaviour is given by

$$f_{\ell_{\pm}}^{V,T}(w) = - \frac{g_{\pi\pi\pi} g_{\pi NN}}{2\pi} \beta_T \frac{\log w}{w} + O\left(\frac{1}{w}\right). \quad (38)$$

With the unphysical function given by (37) the dispersion relation is given by (18').

From the fact that the right-hand side is at most of $O(1/W)$ if $W \rightarrow \infty$ and from (38) we see that unitarity must be saturated. If we assume that

$$\lim_{W \rightarrow +\infty} W \operatorname{Im} f_{\ell_{\pm}}(w) = c > 0 \quad (39)$$

we may extract the dominant terms of the two integrals. We obtain

$$\begin{aligned} \frac{P}{\pi} \int_{H+\mu}^{+\infty} \frac{\operatorname{Im} f_{\ell_{\pm}}(w')}{w' - w} dw' &= - \frac{c}{\pi} \frac{\log w}{w} + O\left(\frac{1}{w}\right) \\ - \frac{1}{\pi} \int_{H+\mu}^{+\infty} \frac{\operatorname{Im} f_{\ell_{\pm}}(-w')}{w' + w} dw' &= - \frac{c}{\pi} \frac{\log w}{w} + O\left(\frac{1}{w}\right). \end{aligned} \quad (40)$$

The coefficients of the logarithmic terms must cancel; this gives the relation

$$-\frac{g_{\rho\pi\pi} \cdot g_{\rho NN}}{2\pi} \beta_T = \frac{4c}{\pi} \quad (41)$$

If $T = 1/2$ this relation is impossible and the corresponding dispersion relations are inconsistent. If $T = 3/2$ this relation may be fulfilled only if the following upper bound

$$g_{\rho\pi\pi} \cdot g_{\rho NN} \leq 4 \quad (42)$$

is satisfied. This bound seems much too low to allow values of the coupling constants of physical relevance⁹⁾.

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T A B L E

		$T = 1/2$	$T = 3/2$
$\ell = 0$	$J = \frac{1}{2}$	$-g^2/16\pi$	$-g^2/4\pi$
$\ell = 1$	$J = \frac{1}{2}$	$-g^2/16\pi$	$-g^2/4\pi$
	$J = \frac{3}{2}$	$-g^2/16\pi$	$+g^2/8\pi$
$\ell = 2$	$J = \frac{3}{2}$	$-g^2/16\pi$	$+g^2/8\pi$
	$J = \frac{5}{2}$	$+g^2/24\pi$	$-g^2/12\pi$
$\ell = 3$	$J = \frac{5}{2}$	$+g^2/24\pi$	$-g^2/12\pi$
	$J = \frac{7}{2}$	$-g^2/32\pi$	$+g^2/16\pi$

R E F E R E N C E S

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FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7