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THE LOW ENERGY Λ -N AND Λ - Λ INTERACTION

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A B S T R A C T

The Λ -N and Λ - Λ potentials are derived from field theory by means of a "perturbation-dispersion" method. The two pion exchange potential is calculated from the Mandelstam representation for the Λ -N (and Λ - Λ) scattering amplitude in one channel formalism. In the unitarity condition the $\pi\Lambda$ scattering amplitude is introduced as an extension of the CGLN model for πN scattering, where contributions from Σ and Y_1^* are represented by appropriate pole terms. Since the TPE potential alone is not sufficient to reproduce the low energy π -N scattering data, no attempt is made to fit the $\pi\Lambda\Sigma$ coupling constant. Forces due to scalar, pseudoscalar, and vector boson exchange are discussed and corresponding potentials are deduced in one particle exchange approximation.

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I. INTRODUCTION

The determination of the baryon-pion coupling constants has played a central role in the understanding of the nature of the strong interactions. The experimental analysis of the pion-baryon scattering is at present possible only for the $\bar{\pi} N$ system and from these experiments the $\bar{\pi} NN$ coupling constant was determined with reasonable accuracy ¹⁾. The experimental investigations of the $N-N$ system led independently ²⁾ to the same value of $g_{\bar{\pi} NN}$. Unfortunately for hyperons we cannot study directly the $\bar{\pi} Y$ scattering, nor does the YN (or YY) scattering experiment exist in practice. The experimental information is very scarce and uncertain ³⁾ and it is impossible to draw any quantitative conclusion about the values of the $\bar{\pi} YY$ coupling constants directly from scattering ⁴⁾ experiments. The observation of the resonances in the $\bar{\pi} Y$ systems is here only a valuable hint that these coupling constants are large and comparable with $g_{\bar{\pi} NN}$. The other, most important sources of experimental information are the Λ -nuclei bound states. From the experimental evidence on the Λ -hypernuclei and double Λ -hypernuclei we have already learned a great deal about the $\Lambda-N$ forces in the 3S_1 and 1S_0 states ⁵⁾ and also about the $\Lambda-\Lambda$ interaction in the 1S_0 state ⁶⁾. Theoretical analysis of the hypernuclear binding energies gave us the values of the parameters specifying the phenomenological potentials which represent the forces acting in the $\Lambda-N$ and $\Lambda-\Lambda$ systems. The latter information is usually a starting point for a more sophisticated examination of the nature of the $\Lambda-N$ and $\Lambda-\Lambda$ forces, and in particular, for the determination of the relevant coupling constants. The importance of this problem has been stressed by several authors attempting to interpret these forces in a field theoretical framework.

At the present time the experimental knowledge of the structure of hypernuclei is not sufficient to allow an analysis of other than central spin-dependent potentials. For assumed shapes and ranges of these potentials, parameters characterising the strength of the forces have been specified from a detailed analysis of light hypernuclei. For large separations, according to the

field theoretical arguments, these potentials arise dominantly from a two pion exchange mechanism. For smaller separations, the situation is less clear and several phenomenological potentials, both with, and without hard core have been used ⁷⁾ to represent the effective interaction. The parameters of the low energy Λ -N scattering were subsequently deduced from potentials which reproduce correctly the binding energies of hypernuclei.

The field theoretical calculations ^{8),9)} were based mainly on the two channel approach in which Λ -N and Σ -N channels were treated on the same footing in view of the small Λ - Σ mass difference (of about one half of the pion mass). In order to determine the $g_{\pi\Lambda\Sigma}$ and $g_{\pi\Sigma\Sigma}$ coupling constants (and also the corresponding hard core radii) de Swart and Iddings ⁹⁾ derived a field theoretical potential which was inserted into the coupled Schroedinger equations. From the solutions of these equations they obtained the scattering parameters and by comparing them with those deduced from the analysis of hypernuclei they attempted to fit the coupling constants. For the Λ - Λ case, the Λ - Λ and the Σ - Σ channels were treated on the same footing and a similar analysis has been done by de Swart ¹⁰⁾. The discussion of the results thus obtained will be given a little later; here we wanted instead to outline the general idea of the procedure.

It follows from the above that the potential plays in this analysis an essential role, being an unavoidable step in the procedure. We would like to emphasize that in the NN problem de facto we do not need a potential, since the scattering amplitude obtained from field theory can be checked directly by comparison with the experimental data. On the other hand, the NN potential has attracted for years a considerable interest and there exists a large literature on this subject. In the last few years great progress has been achieved in that field, in particular, dispersion methods have thrown a new light on the very definition of the potential ¹¹⁾. It has appeared, in particular, that many of the older approaches were incorrect and many controversial problems have been resolved. In spite of the fact that the Λ -N (or Λ - Λ) problem shows a great

resemblance to the NN problem, the analysis of the former is not satisfactory; the field theoretical derivation of the potential clarified to a large extent for the NN case, has not been generalized properly for the Λ -N or Λ - Λ case. There are two reasons to believe that the existing analysis of the Λ -N and Λ - Λ potentials is incomplete. The first objection concerns the basic approximation used in the derivation of the potential, for they have been shown to be unjustified, or even incorrect, in the NN case. The second objection is connected with the experimental evidence of the $\bar{N}$ Y resonances which in most papers are not accounted for in the derivation of the potential. Having in mind the analogy with the NN problem it seems natural to suspect that these defects may strongly influence the Λ -N (or Λ - Λ) potential.

Let us now discuss the difficulties of the theory in some more detail. The derivation of the potential in almost all papers, Refs. ^{8),9),10)}, is based on the following assumptions :

- a) the potential is given in the lowest order of the relevant coupling constant;
- b) the static approximation is used for baryons;
- c) the Λ and Σ are treated on the same footing due to the small Λ Σ mass difference (neglected in the calculations);
- d) the two channel formalism is applied, i.e., the potentials are represented by the following 2x2 matrices

$$V_{NY} = \begin{bmatrix} \langle N\Lambda | V | N\Lambda \rangle & \langle N\Sigma | V | N\Lambda \rangle \\ \langle N\Sigma | V | N\Lambda \rangle & \langle N\Sigma | V | N\Sigma \rangle \end{bmatrix}$$

for the $\Lambda + N = \Lambda + N$ and $\Lambda + N = \Sigma + N$ case, and

$$V_{YY} = \begin{bmatrix} \langle \Lambda\Lambda | V | \Lambda\Lambda \rangle & \langle \Sigma\Sigma | V | \Lambda\Lambda \rangle \\ \langle \Sigma\Sigma | V | \Lambda\Lambda \rangle & \langle \Sigma\Sigma | V | \Sigma\Sigma \rangle \end{bmatrix}$$

for the $\Lambda + \Lambda = \Lambda + \Lambda$ and $\Lambda + \Lambda = \Sigma + \Sigma$ case.

The assumption a) is not justified. Notice that the $\langle N\Lambda | V | N\Lambda \rangle$ and $\langle \Lambda\Lambda | V | \Lambda\Lambda \rangle$ elements do not contain the OPE contribution responsible for the outer part of the potential in the NN case. The OPE term dominating at the distances $\gtrsim \mu^{-1}$ (μ is the pion mass) is the same in perturbation theory and in exact theory, since it is represented as a pole of the scattering amplitude. As is well known, this is the reason why the lowest order perturbation theory leads to the agreement with the experiment in the low energy region. The TPE potential needs, however, a more careful treatment and the contribution from baryon excited states, for example, changes the results significantly. The outer region of the Λ -N and Λ - Λ interaction is ascribed just to the two pion exchange, so even for the very low energies the scattering amplitude depends sensitively upon the approximation method used for the derivation of the potential.

In order to avoid the unwanted energy dependence of the potential, the potential assumption b) was introduced. This leads to a new difficulty as to how to interpret the diagram with no meson in the intermediate state (time ordered Feynman diagram shown in Fig. 1). This diagram is of course divergent for $m_\Lambda = m_\Sigma$, but Lichtenberg and Ross⁸⁾ and also some other authors argued that in the many channel formalism this term is included as it appears to be a repetition of the second order off-diagonal potential. Hence, the many channel formalism is here necessary in order to avoid the difficulties with the "improper diagram". This procedure possesses, however, a severe disadvantage, since, as has been pointed out for the N-N problem in Ref. ¹⁵⁾, there are also some other interpretations of the improper diagram which lead to different potentials. Thus, apart from the unjustified expansion in the strong coupling constant the many channel formalism is ambiguous, and, on the other hand, in the one channel formalism we are left with the divergent improper diagram.

These difficulties are, however, superficial even in the framework of perturbation theory and they appear as consequences of the illegitimate approximations b) and c). For the NN case, Charap and Fubini¹²⁾ were able to show that the static approximation is not quite meaningful. Actually, the TPE

contribution to the NN potential can be written as

$$V(x) = \int_{4\mu^2}^{\infty} \eta(t) \frac{e^{-x\sqrt{t}}}{x} dt.$$

In the static approximation $m_N \rightarrow \infty$ but this limit is not defined for the $\eta(t)$ function at the lower end of the spectrum (i.e., for $m_N \rightarrow \infty$, $t \rightarrow 4\mu^2$). The recoil effect appears to be important in the asymptotic region where one would expect that it should be negligible. The static approximation is thus not well defined at the outer end of the potential. Fortunately, this approximation is not necessary as was shown also in Refs. ^{12),13)}. Remembering that the static approximation together with the assumption c) leads to the difficulty with the improper diagram, let us now examine the validity of neglecting of the $\Lambda \Sigma$ mass difference. Let us consider the simpler case of $\Lambda - \Lambda$ scattering at low energies (the $\Lambda - N$ case is very similar). The lowest order diagrams in perturbation theory are the fourth order diagrams, Fig. 3a and Fig. 4a. For the parallel diagram, the lowest mass in the intermediate state is $2m_\Sigma$ and for the crossed diagram, the corresponding mass is $2m_\Sigma + \mu$. Now, if we express the corresponding matrix elements as spectral integrals, the cut in the k^2 variable (k is the c.m. momentum) starts from $(m_\Sigma^2 - m_\Lambda^2)$ and from $1/4(2m_\Sigma + \mu)^2 - m_\Lambda^2$ for the parallel and crossed diagram, respectively. Hence, for the momenta $k \ll (m_\Lambda/\mu + 1/4\mu^2)^{1/2} \simeq 400$ MeV, our integrals are never singular, they depend weakly on energy, so in consequence, the potential will also be energy independent. This is obviously not so if we put $m_\Lambda = m_\Sigma$ [assumption c)]. The cut in the k^2 variable for the parallel diagram starts then from zero and the corresponding integral will be strongly singular, i.e., the potential will be strongly energy dependent. This singular part is intimately connected with the improper diagram of the static theory, but in fact, it gives a finite contribution if the illegitimate assumptions b) and c) are removed. Of course, in the NN case an energy dependent contribution does appear, but as shown by Charap and Tausner ¹³⁾, it is cancelled out with the energy dependent

contribution from the iterated second order diagram ¹⁴⁾.

In the above discussion we did not go beyond the perturbation theory. Very recently, Nogami and Bloore ¹⁵⁾ derived the Λ -N and Λ - Λ potentials accounting for the Y_1^* resonance in the $\bar{\pi}\Lambda$ system and the $\bar{33}$ resonance in the $\bar{\pi}N$ system together. In that paper, the Chew-Low static model for $\bar{\pi}N$ interaction is extended for the $\bar{\pi}\Lambda$ interaction and the Λ -N and Λ - Λ potentials are calculated by a method proposed by Miyazawa ¹⁶⁾. Nogami and Bloore use one channel formalism and in the improper diagram they take into account the recoil in an approximate way. The Λ - Σ mass difference is not neglected, so the improper diagram should give a finite contribution and in fact it does. It is also claimed that the static approximation does not spoil the results significantly, except for the asymptotic region, where as we have mentioned before, the potential is not defined. Our objections quoted as a), c), d) are removed and in view of this fact their method is a big step forward. However, according to the Miyazawa method, all baryons are static, i.e., only P waves are allowed to transmit the interaction. Apart from that, the off-mass-shell $\bar{\pi}$ -N and $\bar{\pi}$ - Λ scattering amplitudes are used, whereas it is the spirit of the S-matrix methods to introduce matrix elements on their physical shell ¹⁷⁾.

In this paper we derive the Λ -N and Λ - Λ potentials by means of the Mandelstam representation for the relevant elastic scattering amplitudes. The Mandelstam representation combined with unitarity condition gives the possibility of calculating the many pion contribution to the elastic Λ -N (or Λ - Λ) amplitude provided the baryon-antibaryon annihilation amplitude is given. The latter is not known exactly at present, but we can find out (in an approximate way) the two pion contribution to the $\bar{N}\bar{N} \rightarrow \bar{\pi}\pi$ (or $\bar{\Lambda}\bar{\Lambda} \rightarrow \bar{\pi}\pi$) annihilation amplitude, or being more precise, the analytic continuation of the low energy $\bar{\pi}$ -N (or $\bar{\pi}$ - Λ) scattering amplitude. For the NN case the potential was already derived by Cottingham and Vinh Mau ¹⁸⁾ who proceeded according to that prescription. The N-N scattering amplitude was there obtained by the method of Amati, Leader and Vitale ¹⁹⁾ who made use of the $\bar{\pi}$ -N scattering amplitude in the Bowcock, Cottingham and Lurić ²⁰⁾ approximation. The $\bar{\pi}$ -N scattering amplitude includes the one nucleon pole term, the contribution from the $\bar{33}$ resonance approximated by a second pole

term, and a contribution from $\bar{\pi} - \pi$ interaction. The N-N potential thus obtained was then compared with the phenomenological potentials of Breit²¹⁾ and Hamada²²⁾ and it has appeared that the field theoretical OPE and TPE potentials together reproduce properly the essential features of the phenomenological N-N interaction. Moreover, this approach enables one to avoid the illegitimate approximations mentioned above and formulates a constructive definition of a potential in terms of the $\bar{\pi} - N$ and $\bar{\pi} - \Lambda$ scattering parameters.

Encouraged by these facts, we follow the CVM method for the derivation of the two pion contribution to the $\Lambda - N$ and $\Lambda - \Lambda$ potentials in a one channel formalism. In section II we extend the ALV method for constructing the $\Lambda - N$ and $\Lambda - \Lambda$ scattering amplitudes in terms of the Mandelstam representation. As in ALV we express the T-matrix in terms of the perturbative invariants which appear to be more convenient than the invariants used by other authors. In order to combine the Mandelstam representation with the unitarity condition we construct the $\Lambda \bar{\Lambda} \rightarrow \pi \pi$ annihilation amplitude as an analytic continuation of the low energy $\bar{\pi} - \Lambda$ scattering amplitude. The latter is obtained in an approximate way. We keep the $\sum$ pole term and the contribution from the Y_1^* resonance (1385 MeV) is represented by a second pole²³⁾. This is justified by the very narrow width of this resonance ($\Gamma/2 = 25$ MeV). In section III we derive the TPE contribution to the $\Lambda - N$ and $\Lambda - \Lambda$ potentials. We use the non-relativistic approximation and the potentials are expressed as a continuous superposition of Yukawa potentials. The explicit forms of the corresponding spectral functions are given in section IV. The TPE potential contains as parameters $g_{\pi \Sigma \Lambda}$, $g_{\pi NN}$, and the masses and widths of the resonances which, with the exception of $g_{\pi \Lambda \Sigma}$, are well known from experiment. However, the TPE potential alone is not sufficient to explain the strong spin dependence of the low energy $\Lambda - N$ scattering and we were not able to fit the $g_{\pi \Lambda \Sigma}$ coupling constant with a spin independent hard core. The results and discussion are given in section V and in the Appendix we give for completeness the explicit expressions for the $\Lambda - N$ potentials arising from the one boson exchange ($\eta, \omega, K, K^*, \dots$).

II. THE SCATTERING MATRIX

We begin with the Λ -N scattering. In order to introduce the Mandelstam representation we define the usual invariant variables (Fig. 2)

$$\begin{aligned} w &= -(p_1 + p_2)^2, \\ t &= -(p_1 - p'_1)^2, \\ \bar{t} &= -(p_1 - p'_2)^2, \end{aligned} \quad (1)$$

where p_1, p_2 and p'_1, p'_2 are the initial and final four momenta, respectively, and $w + t + \bar{t} = 2m_\Lambda^2 + 2m_N^2$. In the Λ -N c.m. system (w channel) w is the total energy and $t, \bar{t}$ are the two momentum transfers

$$\begin{aligned} w &= (E_\Lambda + E_N)^2, \\ t &= -2\tilde{k}^2(1 - \cos\theta), \\ \bar{t} &= -2\tilde{k}^2(1 + \cos\theta) + (E_\Lambda - E_N)^2, \\ E_\Lambda &= (\tilde{k}^2 + m_\Lambda^2)^{1/2}; \quad E_N = (\tilde{k}^2 + m_N^2)^{1/2}, \end{aligned} \quad (2)$$

where $|\tilde{k}|$ is the c.m. momentum and θ is the corresponding scattering angle.

The causal scattering matrix M is an operator in a 4×4 spin space

$$S = 1 + i \delta(p_1 + p_2 - p'_1 - p'_2) \frac{m_\Lambda m_N}{(2\pi)^2 E_\Lambda E_N} \bar{u}(p'_1) \bar{u}(p'_2) M u(p_1) u(p_2)$$

where by $u(p)$ we denote a four component positive energy spinor representing the spin of the nucleon and/or the Λ particle. The scattering matrix M can be expressed in terms of γ matrices by introducing the so-called spinor invariants.

The number of the spinor invariants is the same as the number of independent matrix elements of M in the spin space in an arbitrary representation. It is easy to see that for identical particles (e.g., $N-N$, or $\Lambda - \Lambda$ scattering) we have five matrix elements, since in the JLSM representation we have four elements with the total spin $S = 1$ (triplet-triplet transitions) and one element with $S = 0$ (singlet-singlet transition). For different particles (e.g., $\Lambda - N$ scattering) there is also a singlet-triplet transition possible, which is forbidden for identical particles by the Pauli principle. Consequently, for the $\Lambda - \Lambda$ case we have five, and for $\Lambda - N$ we have six spinor invariants. There are several sets of invariants known for the NN case and the choice among them is to some extent arbitrary. The most suitable for our purposes are the perturbative invariants introduced in ALV. This set of invariants appears in a natural way in the calculation and is generalized in a straightforward way to the $\Lambda - N$ case. In the NN problem the perturbative invariants are constructed as scalar products of the γ matrices with the following four vectors

$$\Pi_1 = \frac{1}{2}(p_1 + p_1') ; \quad \Pi_2 = \frac{1}{2}(p_2 + p_2') \quad (4)$$

The perturbative invariants defined in ALV are

$$\begin{aligned} P_1 &= 1^{(1)} 1^{(2)} ; \quad P_2 = i \left(1^{(1)} \gamma^{(2)} \Pi_1 + \gamma^{(1)} \Pi_2 1^{(2)} \right) ; \\ P_3 &= i \gamma^{(1)} \Pi_2 i \gamma^{(2)} \Pi_1 ; \quad P_4 = \gamma^{(1)} \gamma^{(2)} ; \quad P_5 = \gamma_5^{(1)} \gamma_5^{(2)} \end{aligned} \quad (5)$$

In the $\Lambda - N$ case P_2 splits into two independent invariants $i \gamma^{(1)} \cdot \Pi_2 1^{(2)}$ and $i \gamma^{(2)} \cdot \Pi_1 1^{(1)}$, so after some re-ordering we are left with the following set of invariants

$$\begin{aligned}
P_1 &= 1^{(1)} 1^{(2)}; \quad P_2 = i \gamma^{(2)} \Pi_1 1^{(1)}; \quad P_3 = i \gamma^{(1)} \Pi_2 i \gamma^{(2)} \Pi_1 \\
P_4 &= i \gamma^{(1)} \Pi_2 1^{(2)}; \quad P_5 = \gamma_5^{(1)} \gamma_5^{(2)}; \quad P_6 = \gamma^{(1)} \gamma^{(2)}
\end{aligned} \quad (6)$$

Now, the spin dependence of the scattering matrix can be exhibited by expressing M in terms of P_i

$$M = \sum_{i=1}^6 p_i(w, t, \bar{t}) P_i \quad (7)$$

where the scalar functions $p_i(w, t, \bar{t})$ depend only on the variables $w, t, \bar{t}$.

For the NN case the crossing properties of the functions p_i and their Mandelstam representation were extensively considered in ALV. We shall not repeat here their arguments based on perturbation theory since the reasoning would be quite analogous. We write the conjectured Mandelstam representation in the special form

$$\begin{aligned}
p_i(w, t, \bar{t}) &= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dt'}{t'-t} \left[\rho_i(w, t') - (-1)^i \rho_i(\bar{t}, t') \right] + \\
&+ \sum(\text{poles}) + \frac{1}{\pi^2} \int_{-\infty}^{\infty} \frac{dt'}{t'-t} \int_{-\infty}^{\infty} \frac{dx}{(m_A + m_N)^2} \chi_i(x, t') \left[\frac{1}{x-w} + \frac{1}{x-\bar{t}} \right] dx \quad (8)
\end{aligned}$$

In contrast to the NN case, the OPE term is not present here due to the isospin conservation. The contribution from the two pion exchange is represented explicitly by the first integral in the expression (8). The functions $\rho_i(w, t')$ are analytic functions of w except for a cut along the real axis from $w = (m_A + m_N)^2$ to ∞ , and are real for $w < (m_A + m_N)^2$. The $[w \leftrightarrow \bar{t}]$ term corresponds to diagrams with crossed meson lines and the crossing properties of the perturbative invariants make necessary the factor $(-1)^i$. The two pion exchange term contains

the lowest singularities and corresponds to the outer part of the Λ -N forces.

The second term in formula (8) represents one particle exchange, and for example, η or ω exchange can be approximated by an appropriate pole in the t-channel. From the third channel ($\bar{t}$ -channel) we have also poles connected with the K meson (or K^*) exchange, all of which we have written symbolically as $\sum$ (poles). The masses of those particles are larger than 2μ and consequently the second term represents shorter range forces in the Λ -N system²⁴⁾.

The two and many particle exchange, except for the TPE, is represented by the third term in the formula (8). The functions χ_i are real Mandelstam spectral functions. The third term contributes also to the short range forces in the Λ -N system.

The separation of the TPE contribution can be made unambiguously by considering the elastic unitarity relation in the t-channel. In this channel t is the total energy of the nucleon-antinucleon (and also $\Lambda - \bar{\Lambda}$) system, and w and $\bar{t}$ are the two momentum transfers. The functions ρ_i are defined in such a way that the unitarity relation in the t-channel yields

$$\sum_{i=1}^6 [\rho_i(w, t') - (-1)^i \rho_i(\bar{t}, t')] \rho_i = \quad (9)$$

$$= \frac{1}{(2\pi)^2 2t} \sum \langle N \bar{N} | \tau_1^+ | 2\pi \rangle \langle 2\pi | \tau_2 | \Lambda \bar{\Lambda} \rangle,$$

where the summation is taken only over all two pion intermediate states that conserve the four momenta. The contribution of remaining possible intermediate states is related to the second and third term of expression (8). The amplitude $\langle 2\pi | \tau_2 | \Lambda \bar{\Lambda} \rangle$ describes the $\Lambda + \bar{\Lambda} = 2\pi$ annihilation process and is an analytic continuation of the pion - lambda scattering amplitude. This amplitude can be written in the form

$$\langle \pi_\beta \pi_\alpha | \tau_2 | \Lambda \bar{\Lambda} \rangle = [-A_1 + i\sigma^{(2)} \cdot \frac{1}{2}(q_2 - q_1) B_\Lambda] \delta_{\alpha\beta}, \quad (10)$$

where α and β are the isotopic spin indices of the two pions and q_1 and q_2 are their four momenta. A_Λ and B_Λ are two scalar functions of the variables

$$\begin{aligned} t &= -(p_2 - p_2')^2 = 4E_\Lambda^2 = 4\omega^2, \\ S_2 &= -(q_1 - p_2)^2 = -(q^2 + p_\Lambda^2 + 2q p_\Lambda), \\ \bar{S}_2 &= -(q_2 - p_2)^2 = -(q^2 + p_\Lambda^2 - 2q p_\Lambda), \\ t + S_2 + \bar{S}_2 &= 2\mu^2 + 2m_\Lambda^2, \quad q \equiv \frac{1}{2}(q_2 - q_1), \end{aligned} \quad (11)$$

where q and ω are the c.m. momentum and energy of the pions, and p_Λ is the lambda-antilambda c.m. momentum.

The second amplitude occurring in the unitarity relation (9) describes the nucleon-antinucleon creation process, $2\pi \rightarrow N + \bar{N}$. This amplitude $\langle \bar{N} N | \tau_1^+ | 2\pi \rangle = \langle 2\pi | \tau_1 | \bar{N} N \rangle^*$ is an analytic continuation of the pion-nucleon scattering amplitude and can be expressed as

$$\begin{aligned} \langle \pi_\alpha \pi_\beta | \tau_1 | N \bar{N} \rangle &= [-A_N^{(+)} + i\gamma^{(1)} \cdot \frac{1}{2}(q_2 - q_1) B_N^+] \delta_{\alpha\beta} + \\ &+ [-A_N^{(-)} + i\gamma^{(1)} \cdot \frac{1}{2}(q_2 - q_1) B_N^{(-)}] \frac{1}{2} [\tau_\beta, \tau_\alpha]. \end{aligned} \quad (12)$$

The scalar functions $A_N^{(\pm)}$, $B_N^{(\pm)}$ depend on the variables $t, s_1, \bar{s}_1$ defined as

$$\begin{aligned} t &= -(p_1 - p_1')^2 = 4E_N^2 = 4\omega^2, \\ S_1 &= -(q_2 - p_1')^2 = -(q^2 + p_N^2 + 2q p_N), \\ \bar{S}_1 &= -(q_1 - p_1')^2 = -(q^2 + p_N^2 - 2q p_N), \\ t + S_1 + \bar{S}_1 &= 2\mu^2 + 2m_N^2, \end{aligned} \quad (13)$$

where $\underline{p}_N$ is the nucleon-antinucleon momentum in the c.m. system.

We see that the t variable in expressions (11) and (13) coincides with the t of the formula (1). Inserting the expressions for $\langle 2\pi | \bar{N} | A \bar{A} \rangle$ and $\langle \bar{N} | \bar{N} | 2\pi \rangle$ into the unitarity relation (9) we obtain

$$\begin{aligned} \sum_{i=1}^6 [\vartheta_i(w, t') - (-1)^i \vartheta_i(\bar{w}, t')] \mathcal{P}_i = \\ = \frac{3}{16\pi^2 t} \int d^3 q_1 d^3 q_2 \delta(q_1 + q_2 - p_1 - p_1') \left[-A_N^{(+)*} + i\gamma^{(1)} \frac{1}{2}(q_2 - q_1) B_N^{(+)*} \right] \times \\ \times \left[-A_\Lambda + i\gamma^{(2)} \frac{1}{2}(q_2 - q_1) B_\Lambda \right]. \end{aligned} \quad (14)$$

In order to perform the integration in the formula (14) it is convenient to go to the two pion c.m. system. Because of the presence of the δ functions, the magnitude of the q vector defined by formula (11) is fixed and equal to $(\frac{1}{4}t - \mu^2)^{\frac{1}{2}}$, so the integration is extended only over the solid angle of $\underline{q}$

$$\begin{aligned} \sum_{i=1}^6 [\vartheta_i(w, t') - (-1)^i \vartheta_i(\bar{w}, t')] \mathcal{P}_i = \\ = \frac{3}{(8\pi)^2} \frac{|q|}{t^{\frac{1}{2}}} \int d\Omega_q \left[-A_N^{(+)*} + i\gamma^{(1)} \frac{1}{2} q B_N^{(+)*} \right] \left[-A_\Lambda + i\gamma^{(2)} \frac{1}{2} q B_\Lambda \right]. \end{aligned} \quad (15)$$

It is evident from the formula (15) that the functions ϑ_i can be calculated provided we introduce some knowledge of the functions $A_N^{(+)}$, $B_N^{(+)}$, A_Λ , B_Λ in the appropriate integration region. Since the $\bar{K} - N$ scattering problem is not solved completely, and even less is known about $\bar{K} - \Lambda$ scattering, the calculation can be done only in an approximate way. Our approach is here quite similar to that adopted in ALV and this is essentially the Cini-Fubini²⁹⁾ philosophy of making use of the weak dependence of every quantity if the corresponding singularities are distant from the considered region. We are looking here for the

low energy Λ -N scattering amplitude with the variables near their threshold values. In this region $\sqrt{s} \simeq (m_\Lambda + m_N)^2$, $\bar{t} \simeq (m_\Lambda - m_N)^2$, and t small the main contribution to the $\mathcal{G}_i$ functions comes from the $A_N^{(+)}$, $B_N^{(+)}$, and A_Λ , B_Λ functions near the physical region of the low energy $\bar{\pi}$ -N and $\bar{\pi}$ - Λ scattering, respectively. In other words, these are the lowest singularities in the s_1 and s_2 planes that determine the $\mathcal{G}_i$ functions in the interesting region. The low energy $\bar{\pi}$ -N scattering amplitude can be extrapolated out of the physical region by using the expressions for $A_N^{(+)}$ and $B_N^{(+)}$ constructed by Bowcock, Cottingham and Lurié²⁰⁾. These authors approximate the $\bar{\pi}$ -N scattering amplitude by taking into account the lowest singularities, i.e., the single nucleon pole term, and the contribution from the Δ resonance represented by a second pole term. The $\bar{\pi}$ - $\bar{\pi}$ interaction could also be included by adding terms which have cuts in the t variable. The same reasoning is applied here for the construction of the $\bar{\pi}$ - Λ scattering amplitude. The lowest singularities are taken into account by including the pole term²⁵⁾ and the contribution from the Y_1^* resonance is itself approximated by a second pole term. The explicit forms of the functions $A_N^{(+)}$, $B_N^{(+)}$, A_Λ , B_Λ constructed according to the above prescription are (the $\bar{\pi}$ - $\bar{\pi}$ interaction neglected)²⁶⁾

$$\left. \begin{aligned} A_N^{(+)} &= \left(\frac{1}{s_N - s_L} + \frac{1}{s_N - \bar{s}_1} \right) G_A^N, \\ B_N^{(+)} &= \left(\frac{1}{m_N^2 - s_L} - \frac{1}{m_N^2 - \bar{s}_1} \right) g_N^2 - \left(\frac{1}{s_N - s_1} - \frac{1}{s_N - \bar{s}_1} \right) G_B^N \end{aligned} \right\} (16a)$$

$$\left. \begin{aligned} A_\Lambda &= \left(\frac{1}{m_\Sigma^2 - s_2} + \frac{1}{m_\Sigma^2 - \bar{s}_2} \right) (m_\Lambda - m_\Sigma) g_\Lambda^2 + \\ &\quad + \left(\frac{1}{s_Y - s_2} + \frac{1}{s_Y - \bar{s}_2} \right) G_A^\Lambda, \\ B_\Lambda &= \left(\frac{1}{m_\Sigma^2 - s_2} - \frac{1}{m_\Sigma^2 - \bar{s}_2} \right) g_\Lambda^2 - \left(\frac{1}{s_Y - s_2} - \frac{1}{s_Y - \bar{s}_2} \right) G_B^\Lambda \end{aligned} \right\} (16b)$$

with

$$G_A^N = \left[\frac{1}{3} \frac{\sqrt{s_N} - m_N}{E_N^* - m_N} + \frac{\sqrt{s_N} + m_N}{E_N^* + m_N} \left(1 + \frac{t}{2q_N^2} \right) \right] \frac{4\sqrt{s_N} q_N^2 g_N^2}{3 m_N^2} \quad (17a)$$

$$G_B^N = \left[\frac{1}{3} \frac{1}{E_N^* - m_N} - \frac{1}{E_N^* + m_N} \left(1 + \frac{t}{2q_N^2} \right) \right] \frac{4\sqrt{s_N} q_N^2 g_N^2}{3 m_N^2}$$

$$G_A^\Lambda = 4\pi \left[\frac{1}{3} \frac{\sqrt{s_Y} - m_\Lambda}{E_Y^* - m_\Lambda} + \frac{\sqrt{s_Y} + m_\Lambda}{E_Y^* + m_\Lambda} \left(1 + \frac{t}{2q_Y^2} \right) \right] \frac{3\Gamma\sqrt{s_Y}}{q_Y},$$

$$G_B^\Lambda = 4\pi \left[\frac{1}{3} \frac{1}{E_Y^* - m_\Lambda} - \frac{1}{E_Y^* + m_\Lambda} \left(1 + \frac{t}{2q_Y^2} \right) \right] \frac{3\Gamma\sqrt{s_Y}}{q_Y}, \quad (17b)$$

where s_N and s_Y are total energies of the 33 and Y_1^* resonance respectively, Γ is the width of the Y_1^* resonance, g_N is the πNN coupling constant ($g_N^2 = 4\pi \cdot 14.4$), g_Λ is the $\pi \Lambda \Sigma$ coupling constant, E_N^* and E_Y^* are the nucleon and lambda energies at the position of the appropriate resonance, and

$$q_N = (E_N^{*2} - m_N^2)^{\frac{1}{2}}, \quad q_Y = (E_Y^{*2} - m_\Lambda^2)^{\frac{1}{2}}.$$

The normalization is here such that the phase shift of the resonant wave approaches $\frac{1}{2}\pi$ at the position of the resonance. The resonant partial wave amplitude is taken to be of the Breit-Wigner form

$$\frac{1}{q} e^{i\delta} \sin \delta = \frac{\Gamma/2 q_r}{\omega_r - \omega - i\Gamma/2}$$

with $\Gamma \rightarrow 0$. For the 33 resonance we make use of the Chew-Low effective range formula and our expressions for G_A^N , G_B^N coincide with those given in CVM.

We suppose here that the Y_1^* resonance occurs in the $P_{3/2}$ wave as is strongly suggested by experiment. Expressions (17) are written under the

assumption that both resonances are in the $P_{3/2}$ state (but we have also examined the possibility of some other waves ascribed to the Y_1^* resonance ²⁷⁾).

In order to calculate the functions $\mathcal{S}_i$ we develop the right-hand side of equation (15) into spinor invariants. This can be easily done by introducing four orthogonal four-vectors ²⁸⁾

$$\begin{aligned} M &= (\Pi_2 \cdot \Pi_2)^{-1/2} \Pi_2 + (\Pi_1 \cdot \Pi_1)^{-1/2} \Pi_1; \quad \Delta = p_1 - p'_1 = p'_2 - p_2; \\ K &= (\Pi_2 \cdot \Pi_2)^{-1/2} \Pi_2 - (\Pi_1 \cdot \Pi_1)^{-1/2} \Pi_1; \quad L_\lambda = \epsilon_{\lambda\mu\nu\zeta} M_\mu K_\nu \Delta_\zeta \end{aligned} \quad (18)$$

In the c.m. system of the t channel we have

$$\begin{aligned} M &= \left[\frac{p_N}{|p_N|} - \frac{p_A}{|p_A|}, 0 \right]; \quad K = \left[-\frac{p_N}{|p_N|} - \frac{p_A}{|p_A|}, 0 \right]; \\ L &= \left[i\sqrt{t} \, \underline{M} \times \underline{K}, 0 \right]; \quad \Delta = \left[0; \sqrt{t} \right], \end{aligned} \quad (19)$$

and the scalar product $i\gamma \cdot 1/2(q_2 - q_1)$ can be decomposed into the scalar products of the gammas and the vectors (18)

$$i\gamma \cdot \frac{1}{2}(q_2 - q_1) = (\underline{q} \cdot \underline{M}) i\gamma M + (\underline{q} \cdot \underline{K}) i\gamma K + (\underline{q} \cdot \underline{L}) i\gamma L. \quad (20)$$

Expressing now $i\gamma \cdot M$, $i\gamma \cdot K$, and $i\gamma \cdot L$ in terms of $i\gamma^{(2)} \cdot \Pi_1$ and $i\gamma^{(1)} \cdot \Pi_2$ the perturbative invariants come out in a straightforward way in the integral of the formula (15). After some manipulations, Eq. (15) can be rewritten in the

following form

$$\begin{aligned}
 & \sum_{i=1}^6 [\rho_i(w, t') - (-1)^i \rho_i(\bar{t}, t')] \mathcal{P}_i = \\
 & = \frac{3}{(8\pi)^2} \frac{q}{t^2} \left[\left(a + \frac{m_N}{p_N} b^- + \frac{m_\Lambda}{p_\Lambda} c^+ + \frac{m_\Lambda m_N}{p_\Lambda p_N} e \right) \mathcal{P}_1 + \right. \\
 & \quad - \frac{1}{p_N} \left(c^- + \frac{m_N}{p_N} d^- \right) \mathcal{P}_2 + \frac{1}{p_\Lambda p_N} e \mathcal{P}_3 + \\
 & \quad \left. - \frac{1}{p_\Lambda} \left(b^+ + \frac{m_\Lambda}{p_\Lambda} d^+ \right) \mathcal{P}_4 - f \mathcal{P}_5 \right], \quad (21)
 \end{aligned}$$

where $p_N = |\underline{p}_N|$, $p_\Lambda = |\underline{p}_\Lambda|$, and

$$a = \int d\Omega_q A_N^{(+)*} A_\Lambda;$$

$$b^\pm = \int d\Omega_q B_N^{(+)*} A_\Lambda \left(\frac{\underline{q} \cdot \underline{M}}{M^2} \pm \frac{\underline{q} \cdot \underline{K}}{K^2} \right);$$

$$c^\pm = \int d\Omega_q B_\Lambda A_N^{(+)*} \left(\frac{\underline{q} \cdot \underline{M}}{M^2} \pm \frac{\underline{q} \cdot \underline{K}}{K^2} \right);$$

$$d^\pm = \int d\Omega_q B_N^{(+)*} B_\Lambda \left[\left(\frac{\underline{q} \cdot \underline{M}}{M^2} \pm \frac{\underline{q} \cdot \underline{K}}{K^2} \right)^2 + t(K^2 + M^2) \left(\frac{\underline{q} \cdot \underline{L}}{L^2} \right)^2 \right]; \quad (22)$$

$$e = \int d\Omega_q B_N^{(+)*} B_\Lambda \left[\left(\frac{\underline{q} \cdot \underline{M}}{M^2} \right)^2 - \left(\frac{\underline{q} \cdot \underline{K}}{K^2} \right)^2 + t(K^2 - M^2) \left(\frac{\underline{q} \cdot \underline{L}}{L^2} \right)^2 \right];$$

$$f = \int d\Omega_q B_N^{(+)*} B_\Lambda \left(\frac{\underline{q} \cdot \underline{L}}{L^2} \right)^2.$$

Inserting the expressions (16) for $A_N^{(+)}$, $B_N^{(+)}$, A_Λ , B_Λ , into (22) the integration over $d\Omega_q$ can be performed analytically and this will be done in the next sections.

For completeness we will now discuss briefly $\Lambda - \Lambda$ scattering. For indistinguishable particles one has to take into account the Pauli principle which is achieved by subtracting in expression (3) an antisymmetrized term. The definition of the field theoretical scattering matrix describing $\Lambda - \Lambda$ scattering is

$$S = 1 + i \int d^4x \delta(p_1 + p_2 - p'_1 - p'_2) \left(\frac{m_\Lambda}{2\pi E_\Lambda} \right)^2 \bar{u}(p_1) \bar{u}(p_2) M u(p'_1) u(p'_2)$$

- antisymmetrized term,

(23)

where the antisymmetrized term is obtained from the direct term by interchanging all quantum numbers of the Λ particles in the initial state. In the decomposition of M into spinor invariants the ALV set (5) is used and we get exactly the same expressions as for the NN case. The Mandelstam representation is assumed in the same form as given by formula (8) except for the non-zero strangeness one particle pole terms. The unitarity relation corresponding to the formula (21) coincides with the expression (3,35) given in ALV and will not be written down explicitly.

III. DERIVATION OF THE POTENTIAL

The concept of a potential $V(r)$ is connected intimately with the Schroedinger equation, i.e., we wish to replace the field theory in the low energy region by a Schroedinger equation with a suitably chosen "equivalent" potential. The definition of the potential follows from the requirement that after inserting $V(r)$ into the Schroedinger equation we obtain the same scattering matrix

$$S = 1 - 2\pi i \delta(E_i - E_f) T \quad (24)$$

as that obtained from the field theory. This equivalency requirement leads to the relation

$$\langle k_f | T | k_i \rangle = - \frac{1}{4\pi^2} \bar{u}(p_1) \bar{u}(p_2) M u(p_1) u(p_2) \quad (25)$$

where M is the scattering matrix defined in the preceding section.

For local, and for some trivially non-local potentials (e.g., the spin orbit force), the scattering matrix is energy independent. In general, however, M will be energy dependent and from (25) the potential $V(\underline{r})$ associated with T should be also energy dependent, i.e., non-local in space and time. In order to get rid of this unwanted energy dependence in older approaches, the static approximation was introduced. This approximation is not justified here and fortunately it is not necessary, because as in the NN case, in the low energy region (sufficiently below the $N\pi$ channel) M depends weakly on energy. This can be demonstrated by taking into account the two pion contribution to M (for the time being we are interested in the outer part of the potential only). In our approximation we have four diagrams contributing to $\varphi_i(w, t)$ functions (Fig. 3) and the energy dependence of $\varphi_i(w, t)$ resulting from formula (21) can be written in the following form

$$\varphi_i(w, t) = \frac{1}{\pi} \int \frac{\sigma_i(w', t)}{w' - w} dw' \quad (26)$$

where the variable w is near its threshold value $w \simeq (m_A + m_N)^2$. For the first diagram, Fig. 3a, the cut in the w plane runs from $(m_N + m_\Sigma)^2$ to ∞ , so the integral (27) is never singular and in the low energy region depends weakly on

energy. The integrals associated with the remaining diagrams in Fig. 3, or associated with the crossed diagrams (Fig. 4) have cuts which are very distant from the low energy physical region and the energy dependence of such terms would be very small. According to the Cini-Tubini²⁹⁾ philosophy, we replace w in these integrals by its threshold value $(m_\Lambda + m_N)^2$, i.e., we put

$$g_L(w, t) \simeq g_L((m_\Lambda + m_N)^2, t), \quad (27a)$$

and similarly for the crossed diagrams $\bar{t}$ is replaced by the threshold value $(m_\Lambda - m_N)^2$

$$g_L(\bar{t}, t) \simeq g_L((m_\Lambda - m_N)^2, t). \quad (27b)$$

In this approximation the first term in the Mandelstam representation, and thus the resulting TPE potential, will be energy independent, but we do not exclude the velocity dependent forces. To make that evident we have to employ the non-relativistic approximation and replace the spinor invariants by a set of six suitably chosen Pauli operators. It is convenient to introduce the following set of Pauli spin operators³⁰⁾

$$\begin{aligned} \tilde{\Omega}_C &= 1, \\ \tilde{\Omega}_S &= \underline{\sigma}_N \cdot \underline{\sigma}_\Lambda, \\ \tilde{\Omega}_T &= \underline{\Delta}^2 \underline{\sigma}_N \cdot \underline{\sigma}_\Lambda - 3 \underline{\sigma}_N \cdot \underline{\Delta} \underline{\sigma}_\Lambda \cdot \underline{\Delta}, \\ \tilde{\Omega}_{SO N} &= -\frac{1}{2} i \underline{\sigma}_N \cdot (\underline{k}_i \times \underline{k}_f), \\ \tilde{\Omega}_{SO \Lambda} &= -\frac{1}{2} i \underline{\sigma}_\Lambda \cdot (\underline{k}_i \times \underline{k}_f), \\ \tilde{\Omega}_{SO 2} &= \underline{\sigma}_N \cdot (\underline{k}_i \times \underline{k}_f) \underline{\sigma}_\Lambda \cdot (\underline{k}_i \times \underline{k}_f), \\ \underline{\Delta} &\equiv \underline{k}_f - \underline{k}_i, \end{aligned} \quad (28)$$

TABLE I

	$\tilde{\Omega}_c$	$\tilde{\Omega}_s$	$\tilde{\Omega}_T$	$\tilde{\Omega}_{s0\Lambda}$	$\tilde{\Omega}_{s0N}$	$\tilde{\Omega}_{s02}$
P_1	1	0	0	$-\frac{1}{2m_\Lambda^2}$	$-\frac{1}{2m_N^2}$	$-\frac{1}{16m_N^2m_\Lambda^2}$
P_2	$-m_N$	0	0	$-\frac{1}{m_\Lambda}\left(1+\frac{m_N}{2m_\Lambda}\right)$	$\frac{1}{2m_N}$	$-\frac{1}{8m_\Lambda m_N^2}\left(1+\frac{m_N}{2m_\Lambda}\right)$
P_3	$m_\Lambda m_N$	0	0	$1+\frac{m_N}{2m_\Lambda}$	$1+\frac{m_\Lambda}{2m_N}$	$-\frac{1}{4m_\Lambda m_N}\left(1+\frac{m_N}{2m_\Lambda}\right)\left(1+\frac{m_\Lambda}{2m_N}\right)$
P_4	$-m_N$	0	0	$-\frac{1}{2m_\Lambda}$	$-\frac{1}{m_N}\left(1+\frac{m_N}{2m_\Lambda}\right)$	$-\frac{1}{8m_\Lambda m_\Lambda^2}\left(1+\frac{m_\Lambda}{2m_N}\right)$
P_5	0	$\frac{t}{12m_N m_\Lambda}$	$\frac{1}{12m_N m_\Lambda}$	0	0	0
P_6	1	$\frac{t}{6m_N m_\Lambda}$	$-\frac{1}{12m_N m_\Lambda}$	$\frac{1}{m_\Lambda m_N}\left(1+\frac{m_N}{2m_\Lambda}\right)$	$\frac{1}{m_\Lambda m_N}\left(1+\frac{m_\Lambda}{2m_N}\right)$	$-\frac{1}{16m_N^2 m_\Lambda^2}$

where $\vec{\sigma}_N$ and $\vec{\sigma}_\Lambda$ are the Pauli spin operators for the nucleon and the Λ -particle respectively ³¹⁾.

Now, we can go from the set $\mathcal{P}_i$ to the above defined set $\tilde{\Omega}_\alpha$

$$\langle \mathcal{P}_i \rangle = \sum_\alpha \chi_{i\alpha} \langle \tilde{\Omega}_\alpha \rangle, \quad (29)$$

where the coefficients $\chi_{i\alpha}$ can be obtained by a direct calculation. In the non-relativistic (adiabatic) approximation we expand $\chi_{i\alpha}$ into a power series and neglect all terms of order k^2/m_Λ^2 and k^2/m_N^2 compared to unity. In this approximation the transformation matrix $\chi_{i\alpha}$ is independent of the energy; the explicit expression for $\chi_{i\alpha}$ is given in the Table 1.

In order to make use of the relation (25) we replace now the spinor invariants $\mathcal{P}_i$ by the set $\tilde{\Omega}_\alpha$ and in the spectral representation (8) we neglect all distant cuts corresponding to the short range forces. The field theoretical scattering matrix M in this approximation is

$$\begin{aligned} \bar{u}(p'_1) \bar{u}(p'_2) M u(p_1) u(p_2) \simeq & \sum_{i,\alpha} \chi_{i\alpha} \langle \tilde{\Omega}_\alpha \rangle \times \\ & \times \left\{ \frac{1}{\pi} \int_{-4\mu^2}^{\infty} \frac{dt'}{t'-t} \left[\rho_i((m_\Lambda+m_N)^2, t') - (-1)^i \rho_i((m_\Lambda-m_N)^2, t') \right] + \right. \\ & \left. + \frac{1}{\pi^2} \int_{-4\mu^2}^{\infty} \frac{dt'}{t'-t} \int_0^{\infty} dk'^2 \frac{\xi_\alpha(k'^2, t')}{m_\Sigma - m_\Lambda + (k'^2/2m') - (k^2/2m)} \right\} \end{aligned} \quad (30)$$

where $m^{-1} = m_\Lambda^{-1} + m_N^{-1}$; $m^{1-1} = m_\Sigma^{-1} + m_N^{-1}$, and we have dropped the pole terms.

Due to the required relation (25) the T matrix and consequently the Fourier transform of the potential $\tilde{V}(\underline{\Delta})$ associated with T will be also expressed in terms of the operators $\tilde{\Omega}_\alpha$

$$\tilde{V}(\underline{\Delta}) = \sum_\alpha \tilde{V}_\alpha(|\underline{\Delta}|) \tilde{\Omega}_\alpha \quad (31)$$

Transforming $\tilde{V}(\underline{\Delta})$ from the momentum space to the $\underline{x}$ space we obtain the potential $V(\underline{r})$ associated with the T matrix

$$V(\underline{r}) = \sum_{\alpha} V_{\alpha}(r) \Omega_{\alpha}, \quad (32)$$

with

$$\begin{aligned} \Omega_C &= 1, \\ \Omega_S &= \underline{\sigma}_N \cdot \underline{\sigma}_\Lambda, \\ \Omega_T &= (3/\gamma^2) \underline{\sigma}_N \cdot \underline{\gamma} \underline{\sigma}_\Lambda \cdot \underline{\gamma} - \underline{\sigma}_N \cdot \underline{\sigma}_\Lambda, \\ \Omega_{S0N} &= \frac{1}{2} \underline{\sigma}_N \cdot \underline{L}, \\ \Omega_{S0\Lambda} &= \frac{1}{2} \underline{\sigma}_\Lambda \cdot \underline{L}, \\ \Omega_{S02} &= \frac{1}{2} [\underline{\sigma}_N \cdot \underline{L} \underline{\sigma}_\Lambda \cdot \underline{L} + \underline{\sigma}_\Lambda \cdot \underline{L} \underline{\sigma}_N \cdot \underline{L}], \\ \underline{r} &= \underline{r}_\Lambda - \underline{r}_N; \quad \underline{L} = \underline{r} \times \underline{\Delta} \end{aligned} \quad (33)$$

The functions $V_{\alpha}(r)$ in expression (32) are scalar functions of the relative lambda-nucleon separation only.

Suppose now for a moment that we are given a potential (32) as a continuous superposition of Yukawa potentials, i.e.,

$$\tilde{V}(\underline{\Delta}) = - \frac{1}{(2\pi)^2} \sum_{\alpha} \tilde{\Omega}_{\alpha} \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\eta_{\alpha}(t')}{t' - t} dt'. \quad (34)$$

For this potential the Mandelstam representation can be written as ³²⁾

$$\begin{aligned} \langle k_f | T | k_i \rangle = \\ = -\frac{1}{(2\pi)^2} \sum_{\alpha} \tilde{\Omega}_{\alpha} \left\{ \frac{1}{\pi} \int_{9\mu^2}^{\infty} \frac{\eta_{\alpha}(t')}{t'-t} dt' + \frac{1}{\pi^2} \int_{16\mu^2}^{\infty} \frac{dt'}{t'-t} \int_0^{\infty} dk'^2 \frac{\xi_{\alpha}(k'^2, t')}{k'^2 - k^2} \right\}, \end{aligned} \quad (35)$$

where $\eta_{\alpha}(t)$ and $\xi_{\alpha}(k'^2, t)$ are real spectral functions.

Comparing (30) and (35) we see that apart from the double spectral function, Eq. (25) is in fact satisfied with an energy independent potential of the form (34). The double spectral integrals of formulae (30) and (35) have a distant cut in the t variable and the influence of these terms on the outer part of the potential is very small. It should be also noticed that the double spectral integrals of formulae (30) and (35) are different, since the cuts in the t variable start from $9\mu^2$ and $16\mu^2$, respectively. In the expression for M we have 3π exchange contribution (in general any number of pions), but in T only an even number of pions can be exchanged as a repetition of the TPE potential. Since it is the $\pi \Sigma \Sigma$ coupling that is responsible for the 3π contribution we can interpret the above mentioned difference as an effect of the closed ΣN channel. However, as far as long range forces are concerned, all distant cuts in the t variable are not important and the coupling to the ΣN channel is very small.

The density $\eta_{\alpha}(t)$ of the field theoretical potential obtained from equation (25) can be expressed in terms of the g_i functions

$$\eta_{\alpha}(t) \approx \sum_i \chi_{i\alpha} [g_i((m_{\Lambda} + m_N)^2, t) - (-1)^i g_i((m_{\Lambda} - m_N)^2, t)]. \quad (36)$$

This formula together with expressions (21) and (22) determines our potential (34). The explicit expressions for $\eta_{\alpha}(t)$ will be given in the next section.

The $\Lambda - \Lambda$ potential [denoted hereafter by $U(r)$] is obtained in an analogous way, but corresponding to the five operators P_i of ALV the T matrix and the $\Lambda - \Lambda$ potential are expressed in terms of five Pauli operators. Then one has

$$\tilde{U}(\underline{\Delta}) = - \frac{1}{(2\pi)^2} \sum_{\alpha} \tilde{O}_{\alpha} \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\lambda_{\alpha}(t')}{t' - t} dt', \quad (37)$$

with

$$\tilde{O}_c = 1,$$

$$\tilde{O}_s = \underline{\sigma}_1 \cdot \underline{\sigma}_2,$$

$$\tilde{O}_T = \underline{\Delta}^2 \underline{\sigma}_1 \cdot \underline{\sigma}_2 - 3 \underline{\sigma}_1 \cdot \underline{\Delta} \underline{\sigma}_2 \cdot \underline{\Delta}, \quad (38)$$

$$\tilde{O}_{s0} = -\frac{1}{2} i (\underline{\sigma}_1 + \underline{\sigma}_2) \cdot (\underline{k}_i \times \underline{k}_f),$$

$$\tilde{O}_{s02} = \underline{\sigma}_1 \cdot (\underline{k}_i \times \underline{k}_f) \underline{\sigma}_2 \cdot (\underline{k}_i \times \underline{k}_f),$$

where $\underline{\sigma}_1$ and $\underline{\sigma}_2$ are the Pauli matrices and $\lambda_{\alpha}(t)$ is the spectral density function of the potential.

IV. THE SPECTRAL DENSITIES OF THE $\Lambda - N$ AND $\Lambda - \Lambda$ POTENTIAL

In configuration space the $\Lambda - N$ potential can be written as ³³⁾

$$V(\underline{r}) = \sum_{\alpha} V_{\alpha}(r) \Omega_{\alpha}, \quad (39)$$

with

$$\begin{aligned}
V_C(r) &= -\frac{1}{(2\pi)^2} \int_4^\infty \eta_C(t) \frac{e^{-r\sqrt{t}}}{r} dt, \\
V_S(r) &= -\frac{1}{(2\pi)^2} \int_4^\infty \eta_S(t) \frac{e^{-r\sqrt{t}}}{r} dt, \\
V_T(r) &= -\frac{1}{(2\pi)^2} \int_4^\infty \eta_T(t) \frac{e^{-r\sqrt{t}}}{r} t \left(1 + \frac{3}{r\sqrt{t}} + \frac{3}{r^2 t}\right) dt, \\
V_{SO,N}(r) &= +\frac{1}{(2\pi)^2} \int_4^\infty \eta_{SO,N}(t) \frac{e^{-r\sqrt{t}}}{r^2 \sqrt{t}} \left(1 + \frac{1}{r\sqrt{t}}\right) dt, \\
V_{SO,\Lambda}(r) &= +\frac{1}{(2\pi)^2} \int_4^\infty \eta_{SO,\Lambda}(t) \frac{e^{-r\sqrt{t}}}{r^2 \sqrt{t}} \left(1 + \frac{1}{r\sqrt{t}}\right) dt, \quad (40) \\
V_{SO,2}(r) &= -\frac{1}{(2\pi)^2} \int_4^\infty \eta_{SO,2}(t) \frac{e^{-r\sqrt{t}}}{r^3} t \left(1 + \frac{2}{r\sqrt{t}}\right) dt.
\end{aligned}$$

Taking the inverse Fourier transform of (37) we get the $\Lambda - \Lambda$ potential in the following form

$$u(r) = \sum_{\beta} u_{\beta}(r) O_{\beta}, \quad (41)$$

with

$$O_c = 1,$$

$$O_s = \underline{\sigma}_1 \cdot \underline{\sigma}_2,$$

$$O_T = \left(\frac{3}{4}\right) \underline{\sigma}_1 \cdot \underline{\sigma}_2 \underline{L} \cdot \underline{L} - \underline{\sigma}_1 \cdot \underline{\sigma}_2, \quad (42)$$

$$O_{so} = \frac{1}{2} (\underline{\sigma}_1 + \underline{\sigma}_2) \cdot \underline{L},$$

$$O_{so2} = \frac{1}{2} [\underline{\sigma}_1 \cdot \underline{L} \underline{\sigma}_2 \cdot \underline{L} + \underline{\sigma}_2 \cdot \underline{L} \underline{\sigma}_1 \cdot \underline{L}],$$

$$U_c(r) = -\frac{1}{(2\pi)^2} \int_0^\infty \lambda_c(t) \frac{e^{-r\sqrt{t}}}{r} dt,$$

$$U_s(r) = -\frac{1}{(2\pi)^2} \int_0^\infty \lambda_s(t) \frac{e^{-r\sqrt{t}}}{r} dt,$$

$$U_T(r) = -\frac{1}{(2\pi)^2} \int_0^\infty \lambda_T(t) \frac{e^{-r\sqrt{t}}}{r} t \left(1 + \frac{3}{r\sqrt{t}} + \frac{3}{r^2 t}\right) dt, \quad (43)$$

$$U_{so}(r) = +\frac{1}{(2\pi)^2} \int_0^\infty \lambda_{so}(t) \frac{e^{-r\sqrt{t}}}{r^2} \sqrt{t} \left(1 + \frac{1}{r\sqrt{t}}\right) dt,$$

$$U_{so2}(r) = -\frac{1}{(2\pi)^2} \int_0^\infty \lambda_{so2}(t) \frac{e^{-r\sqrt{t}}}{r^3} t \left(1 + \frac{2}{r\sqrt{t}}\right) dt.$$

In this paper we will confine our attention to the central, spin-spin, and tensor forces only. Although the velocity dependent forces appear in a natural way in our approach and there is no difficulty in calculating the corresponding spectral densities, we feel that the very nature of these forces is more complicated. We are not able to discuss this problem now, but, at least at sufficiently low energies, $k \ll 300$ MeV, these forces are not important. Notice that in the

adiabatic approximation used throughout this paper (by adiabatic approximation we mean the expansion in (k^2/m^2) powers) the velocity dependent forces are of the same order as the non-adiabatic corrections to the C, S, and T forces. These energy dependent corrections are however entirely neglected in this calculation. Moreover, our knowledge of the Λ -N and Λ - Λ forces is obtained from hyper-nuclei and concerns presumably the S state interaction where the velocity dependent forces of spin-orbit type do not contribute at all. Therefore, at present, there is no way to compare the velocity dependent potentials with experiment.

Having in mind the following relations (see Table I)

$$\eta_S(t) = -2t\eta_T(t); \quad \lambda_S(t) = -2t\lambda_T(t) \quad (44)$$

we see that the central, spin-spin, and tensor potentials are determined by four spectral densities $\eta_C, \eta_T, \lambda_C, \lambda_T$. Inserting (21) into (36) we obtain

$$\begin{aligned} \eta_C(t) = & \frac{3}{(2\pi)^2} \frac{g}{16iF} \left[\left[a + \frac{m_N}{p_N} b^- + \frac{m_\Lambda}{p_\Lambda} c^+ + \frac{2m_\Lambda m_N}{p_\Lambda p_N} e + \right. \right. \\ & + \frac{m_\Lambda}{p_\Lambda} b^+ + \frac{m_N}{p_N} c^- + \frac{m_\Lambda^2}{p_\Lambda^2} d^+ + \frac{m_N^2}{p_N^2} d^- - f \Big] + \\ & + X \left[a + \frac{m_N}{p_N} b^- - \frac{m_\Lambda}{p_\Lambda} c^+ + \frac{2m_\Lambda m_N}{p_\Lambda p_N} e - \frac{m_\Lambda}{p_\Lambda} b^+ - \right. \\ & \left. \left. - \frac{m_\Lambda^2}{p_\Lambda^2} d^+ - \frac{m_N}{p_N} c^- - \frac{m_N^2}{p_N^2} d^- + f \right] \right] \quad (45) \end{aligned}$$

$$\eta_T(t) = \frac{3}{(2\pi)^2} \frac{q}{16\sqrt{t}} \cdot \frac{1}{12 m_A m_N} \{ \Pi[f] - X[f] \}, \quad (46)$$

where by $\Pi[]$ and $X[]$ we denote the contribution from the parallel (Fig. 3) and crossed (Fig. 4) diagrams, respectively. In performing the integrations written down explicitly in expressions (22) we take the adiabatic limit, i.e., we put in the parallel diagrams $w \simeq (m_A + m_N)^2$, $\bar{t} \simeq (m_A - m_N)^2$, t - very small; and for the crossed diagrams $w \simeq (m_A - m_N)^2$, $\bar{t} \simeq (m_A + m_N)^2$, t - very small.

The final results are

$$\begin{aligned} \eta_C(t) = & \frac{3}{16\pi} \frac{q/\sqrt{t}}{m_N m_A} \left\{ g_A^2 G_A^N \delta X(y_2, x_1) + g_A^2 g_N^2 \delta m_N x_1 Y(y_2, x_1) \right. \\ & + g_A^2 (G_A^N m_A y_2 - G_B^N \delta m_N x_1) Y(y_2, x_1) - g_A^2 g_N^2 Z(y_2, x_1) + \\ & + g_A^2 G_B^N Z(y_2, x_1) + G_A^A G_A^N X(y_2, x_2) + g_N^2 G_A^A m_N x_2 Y(y_2, x_2) \\ & - (G_B^N G_A^A m_N x_2 + G_A^N G_B^A m_A y_2) Y(y_2, x_2) - G_B^A G_B^N Z(y_2, x_2) \\ & \left. + g_N^2 G_B^A Z(y_2, x_2) \right\}, \quad (47) \end{aligned}$$

$$\begin{aligned} \eta_T(t) = & \frac{1}{32\pi} \frac{q^2/\sqrt{t}}{m_A m_N} \left\{ g_A^2 [G_B^N T(y_2, x_1) - g_N^2 T(y_2, x_1)] \right. \\ & \left. + G_B^A [g_N^2 T(y_2, x_2) - G_B^N T(y_2, x_2)] \right\}, \end{aligned}$$

where

$$X(y, x) = \frac{2m_\Lambda m_N}{q} \frac{m_\Lambda y \phi(x) - m_N x \Psi(y)}{m_\Lambda^2 y^2 - m_N^2 x^2},$$

$$Y(y, x) = -\frac{1}{q} \frac{m_\Lambda y \Psi(y) - m_N x \phi(x)}{m_\Lambda^2 y^2 - m_N^2 x^2},$$

$$Z(y, x) = -1 + \frac{1}{m_\Lambda^2 y^2 - m_N^2 x^2} \left[y \Psi(y) \frac{m_\Lambda^2 y^2}{2q m_N} - x \phi(x) \frac{m_N^2 x^2}{2q m_\Lambda} \right],$$

$$T(y, x) = \frac{m_\Lambda y \Psi(y) - m_N x \phi(x)}{m_\Lambda^2 y^2 - m_N^2 x^2} \left(1 + \frac{m_\Lambda^2 y^2 + m_N^2 x^2}{8q^2 m_\Lambda^2 m_N^2} \right);$$

$$X_1 = \frac{1}{2}t + m_\Sigma^2 - m_\Lambda^2 - 1, \quad q = \left(\frac{1}{4}t - 1\right)^{\frac{1}{2}},$$

$$X_2 = \frac{1}{2}t + s_y - m_\Lambda^2 - 1, \quad \delta = m_\Lambda - m_\Sigma$$

$$y_1 = \frac{1}{2}t - 1, \quad \phi(x) = \tan^{-1}(2q m_\Lambda / x)$$

$$y_2 = \frac{1}{2}t + s_N - m_N^2 - 1, \quad \Psi(y) = \tan^{-1}(2q m_N / y);$$

$$\begin{aligned} \lambda_c(t) = & \frac{3}{16\pi} \frac{q/\sqrt{t}}{m^2} \left\{ (q^2 - G_B)^2 + \frac{1}{2} (G_B - G_A)^2 \frac{X_2^2}{X_2^2 + 4m^2 q^2} + \right. \\ & \left. + \frac{1}{2} q^4 \left(1 + \frac{2\delta m}{X_1}\right)^2 \frac{X_2^2}{X_1^2 + 4m^2 q^2} + \right. \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} (G_B - G_A) \frac{\phi_2 X_2}{g m} \left[\frac{2 g^2 X_2^2}{X_2^2 - X_1^2} + \frac{2 \delta m}{X_1} \frac{2 g^2 X_1^2}{X_2^2 - X_1^2} + \frac{3}{2} G_B - \frac{1}{2} G_A \right] \\
& - \frac{1}{2} g^2 \left(1 + \frac{2 \delta m}{X_1} \right) \left[G_B \frac{2 X_1^2}{X_2^2 - X_1^2} - G_A \frac{2 X_2^2}{X_2^2 - X_1^2} + \frac{3}{2} g^2 - \frac{1}{2} g^2 \frac{2 \delta m}{X_1} \right] \frac{\phi_2 X_2}{g m} \Bigg\};
\end{aligned}$$

$$\begin{aligned}
\lambda_T(t) = & \frac{1}{16\pi} \frac{g/\sqrt{t}}{8m^4} \left\{ g^4 \left(1 + \frac{2gm}{X_2} \phi_2 - \frac{X_2}{2gm} \phi_2 \right) + \right. \\
& + g^2 G_B 8gm \left(1 + \frac{X_1^2 + X_2^2}{8g^2 m^2} \right) \frac{X_2 \phi_2 - X_1 \phi_1}{X_2^2 - X_1^2} + \\
& \left. + G_B^2 \left(1 - \frac{2gm}{X_2} \phi_2 - \frac{X_2}{2gm} \phi_2 \right) \right\};
\end{aligned}
\tag{48}$$

where the following abbreviations were introduced

$$g \equiv g_\Lambda; \quad m \equiv m_\Lambda; \quad G_A \equiv G_A^\Lambda \frac{2m}{X_2}; \quad G_B \equiv G_B^\Lambda$$

$$\phi_1 = \phi(X_1); \quad \phi_2 = \phi(X_2).$$

V. DISCUSSION OF THE RESULTS

In the preceding section we have derived the $\bar{\Lambda}$ -N and $\bar{\Lambda}$ - Λ TPE potentials as continuous superposition of Yukawa potentials. The central, spin-spin, and tensor potentials were evaluated numerically using the CERN IBM 709 computer and the r dependence of these potentials is given in Figs. 5, 6, 7, and 8 (we take $g_N^2 = 4\pi \cdot 14.4$ and $g_\Lambda^2 = g_N^2$). The central potentials are only slightly different from those obtained by Nogami and Bloore¹⁵⁾ which are given as dashed lines in Figs. 5 and 6. The spin-spin and tensor potentials are not compared, since they are smaller by about a factor of 10 in comparison with the results of Ref. 15). There is only an agreement in sign; the spin-spin potential is negative, while the tensor is positive. Since the pseudoscalar coupling contains γ_5 matrices the spin dependent forces come out in our calculation as a relativistic effect, from the mixing of small and large spinor components. Consequently, the central potential is roughly m^2/μ^2 times larger than the spin dependent potentials. Although, in $\gamma_c(t)$ and $\lambda_c(t)$, in the leading terms large cancellations take place, the central potential is still larger by about a factor of 100 in comparison with the spin dependent potentials³⁴⁾. As has been pointed out in CVM, the cancellations of the leading terms refer to the S wave $\bar{\pi}$ -N and $\bar{\pi}$ - Λ contribution and are implicitly contained in the model assumption. The model adopted here is essentially the CGLN approach extended to $\bar{\pi}$ - Λ scattering. In this model, the $\bar{\pi}$ - Λ scattering length for the S wave is given as the difference of two large expressions which are approximately equal in magnitude³⁵⁾. Due to the small contribution from the S wave, the central potential is very close to that calculated in Ref. 15), where the S wave was completely neglected. Some improvements of the central potential are certainly possible, especially if a more exact $\bar{\pi}$ - Λ scattering amplitude is used. Unfortunately, the parameters of the low energy $\bar{\pi}$ - Λ scattering are not known experimentally and one is not able to introduce corrections to our model³⁶⁾. We believe, however, that apart from such corrections the TPE potential is rather well established.

At small distances ($r \lesssim 0.4 \text{ fm}^{-1}$) our potential is not adequate and a phenomenological hard core has to be introduced, with the hard core radii and the g_{Λ}^2 coupling constant as adjustable parameters. For spin independent hard cores (i.e., with singlet and triplet hard core radii equal to each other) the S wave Λ -N scattering will be presumably the same in the triplet and singlet state, owing to the predominance of the central potential. This is in contradiction to the phenomenological analysis of light hypernuclei, which suggests that the S wave scattering length is about 7 times larger in the singlet state. Thus, the TPE potential alone is not sufficient to explain the strong spin dependence of the Λ -N forces. On the other hand with a spin dependent hard core one is not able to fit g_{Λ}^2 , because the scattering lengths depend very sensitively on the hard core radius, being much less affected by the variation of g_{Λ}^2 .

In the NN problem the situation is quite different because higher phase shifts are known experimentally. For higher waves, the centrifugal barrier shields the inner part of the potential and the results are not very sensitive to the hard core radius. Unfortunately almost all experimental information about the Λ -N interaction concerns the S state and therefore the details of the inner part of the potential are also very important.

Further improvements of the theory could be achieved by considering heavy boson exchange potentials³⁷⁾. Since the many meson exchange processes are probably dominated by resonances, one can approximate the corresponding forces by one particle exchange potentials. The longest range potentials are due to the exchange of pseudoscalar mesons (η, K), vector mesons (ω, K^*), and perhaps also a scalar particle. The derivation of these potentials is elementary³⁸⁾, but since K and K^* give rise to the exchange forces in the Λ -N system, for completeness we give in the Appendix explicit expressions for such potentials in a system of four arbitrary baryons. Although the number of relevant coupling constants could be reduced by assuming some higher symmetry, e.g., the SU_3 symmetry³⁹⁾, the number of parameters is still so large that some additional experimental information is needed. In addition to the Λ -N scattering lengths,

the other quantities known from experiment are the average Λ -N cross-section and the binding energy of the Λ particle in infinite nuclear matter ⁴⁰⁾. It seems that some other piece of information can be obtained also from the phenomenological analysis of the double Λ hyperfragment ⁶⁾ and from the $p + \bar{p} \rightarrow \Lambda + \bar{\Lambda}$ annihilation process. For higher energies, however, the whole task becomes much more complicated since one has to do with a many channel problem. At the present moment we are not able to discuss this problem in its full complexity, and we have confined ourselves only to the derivation of the forces, but further analysis is in progress.

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A P P E N D I X

For a process $1+2 \rightarrow 3+4$ the coupling and mass assignment is given in Fig. 9. The meson of mass μ_1 is supposed to be a scalar (μ_S), pseudoscalar (μ_P), or vector particle (μ_V). The resulting one-boson exchange potentials are given below, where indices C, S, T, SO2 have the same meaning as before and by SOS and SOA we denote the symmetric and antisymmetric spin-orbit force $1/2(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{L}$. Pauli matrices $\vec{\sigma}_1$ are understood to be saturated by spinors χ_1 and χ_3 and matrices $\vec{\sigma}_2$ by χ_2 and χ_4 , respectively. Recoil corrections of order $O(\mu^2/m^2)$, as well as other possible velocity dependent potentials, are neglected. We give only expressions for direct potentials; exchange potentials are obtained simply by the $3 \leftrightarrow 4$ interchange.

Scalar case

$$V_C(r) = - \frac{g_1^{(S)} g_2^{(S)}}{4\pi} \mu \frac{e^{-\mu r}}{\mu r},$$

$$V_S(r) = V_T(r) = 0,$$

$$V_{SOS}(r) = - \frac{g_1^{(S)} g_2^{(S)}}{4\pi} \frac{\mu^3}{4} \left(\frac{1}{m_1 m_3} + \frac{1}{m_2 m_4} \right) \left(1 + \frac{1}{\mu r} \right) \frac{e^{-\mu r}}{\mu^2 r^2},$$

$$V_{SOA}(r) = - \frac{g_1^{(S)} g_2^{(S)}}{4\pi} \frac{\mu^3}{4} \left(\frac{1}{m_1 m_3} - \frac{1}{m_2 m_4} \right) \left(1 + \frac{1}{\mu r} \right) \frac{e^{-\mu r}}{\mu^2 r^2},$$

$$V_{SO2}(r) = + \frac{g_1^{(S)} g_2^{(S)}}{4\pi} \frac{\mu^5}{16 m_1 m_2 m_3 m_4} \left(1 + \frac{2}{\mu r} \right) \frac{e^{-\mu r}}{\mu^3 r^3},$$

with

$$\mu^2 = \mu_s^2 - (m_1 - m_3)^2.$$

Pseudoscalar case

$$V_C(r) = V_{SOS}(r) = V_{SOA}(r) = V_{SO2}(r) = 0,$$

$$V_S(r) = + \frac{g_1^{(P)} g_2^{(P)}}{4\pi} \cdot \frac{\mu^3}{48} \left(\frac{1}{m_1} + \frac{1}{m_3} \right) \left(\frac{1}{m_2} + \frac{1}{m_4} \right) \frac{e^{-\mu r}}{\mu r},$$

$$V_T(r) = + \frac{g_1^{(P)} g_2^{(P)}}{4\pi} \cdot \frac{\mu^3}{48} \left(\frac{1}{m_1} + \frac{1}{m_3} \right) \left(\frac{1}{m_2} + \frac{1}{m_4} \right) \times \\ \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2} \right) \frac{e^{-\mu r}}{\mu r},$$

with

$$\mu^2 = \mu_P^2 - (m_1 - m_3)^2.$$

Vector case

$$V_C(r) = + \frac{g_1^{(V)} g_2^{(V)}}{4\pi} \mu \frac{e^{-\mu r}}{\mu r},$$

$$V_S(r) = + \frac{g_1^{(V)} g_2^{(V)}}{4\pi} \cdot \mu^3 \frac{1}{24} \left(\frac{1}{m_1} + \frac{1}{m_3} \right) \left(\frac{1}{m_2} + \frac{1}{m_4} \right) \frac{e^{-\mu r}}{\mu r},$$

$$V_T(r) = - \frac{g_1^{(V)} g_2^{(V)}}{4\pi} \cdot \mu^3 \frac{1}{48} \left(\frac{1}{m_1} + \frac{1}{m_3} \right) \left(\frac{1}{m_2} + \frac{1}{m_4} \right) \times \\ \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2} \right) \frac{e^{-\mu r}}{\mu r},$$

$$V_{SO S}(r) = - \frac{g_1^{(v)} g_2^{(v)}}{4\pi} \mu^3 \frac{1}{4} \left(\frac{1}{m_2 m_4} + \frac{1}{m_1 m_3} + \frac{2}{m_2 m_3} + \frac{2}{m_1 m_4} \right) \left(1 + \frac{1}{\mu r} \right) \times \frac{e^{-\mu r}}{\mu^2 r^2}$$

$$V_{SO A}(r) = - \frac{g_1^{(v)} g_2^{(v)}}{4\pi} \mu^3 \frac{1}{4} \left(\frac{1}{m_2 m_4} + \frac{1}{m_1 m_3} + \frac{2}{m_2 m_3} - \frac{2}{m_1 m_4} \right) \left(1 + \frac{1}{\mu r} \right) \times \frac{e^{-\mu r}}{\mu^2 r^2}$$

$$V_{SO 2}(r) = - \frac{g_1^{(v)} g_2^{(v)}}{4\pi} \frac{\mu^5}{16 m_1 m_2 m_3 m_4} \left(1 + \frac{2}{\mu r} \right) \frac{e^{-\mu r}}{\mu^3 r^3}$$

with

$$\mu^2 = \mu_V^2 - (m_1 - m_3)^2$$

In the vector case we have neglected also corrections of order $(m_1 - m_3) \cdot (m_2 - m_4) / \mu^2$ in comparison with unity.

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the form $\vec{\sigma}_A \cdot \vec{p} \vec{\sigma}_N \cdot \vec{p}$; $\vec{\sigma}_N \times \vec{\sigma}_A \cdot \vec{p}$; etc.
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- 34) We have learned from Dr. Nogami that the spin dependent potentials calculated recently by a relativistic Miyazawa method are very close to ours (Y. Nogami, B. Ram and I.J. Zucker. The Long Range Part of Λ -N and $\Lambda\Lambda$ Potential - to be published). Although that method seems to be different, both results are now in good agreement. We are grateful to Dr. Nogami for informing us of these results.

- 35) This is easily seen by expanding (16b) in $1/m_\Lambda$ powers. The S wave scattering length a is given by

$$4\pi a \approx -\frac{g_\Lambda^2}{m_\Lambda} \frac{(-2m_\Lambda \delta/x_1) X_1^2 - 4m_\Lambda^2 \omega^2}{X_1^2 - 4m_\Lambda^2 \omega^2} + \frac{1}{m_\Lambda} \frac{G_4^\Lambda (2m_\Lambda/x_2) X_2^2 - G_8 4m_\Lambda^2 \omega^2}{X_2^2 - 4m_\Lambda^2 \omega^2}$$

$$\text{where } \omega^2 = q^2 + \mu^2; x_1 = \frac{m_\Sigma^2}{2} - m_\Lambda^2; x_2 = s_Y - m_\Lambda^2.$$

Now notice that both terms are large, but for $t \approx 4\mu^2$ we have

$$(-2m_\Lambda \delta/x_1) \approx 1, g_\Lambda^2 \approx G_B^\Lambda \approx G_R^\Lambda (2m_\Lambda/x_1)$$

(provided that $g_\Lambda^2 \approx g_N^2 \approx 15$) and thus $4\pi a$ is very small.

- 36) The $\bar{\Lambda}$ -N (and probably $\bar{\Lambda}$ - Λ) S waves are known to be influenced by the $\bar{\Lambda}$ - $\bar{\Lambda}$ interaction which was ignored here thus far. We think, however, that this effect could be taken approximately into account by adding to the TPE potential the contribution from the $\bar{\Lambda}$ - $\bar{\Lambda}$ resonances exchange potential, i.e., the η and the ω exchange potential (see Appendix).
- 37) The Λ -N forces in one-boson exchange approximation were investigated recently by B.W. Downs and R.J.N. Phillips - to be published. In this paper, however, the TPE potential is completely ignored.
- 38) For direct potentials it follows simply from (40) with

$$\eta_\alpha = \sum_i g_i(t) \chi_{i\alpha}$$

where

$$g_i = \pi \delta(t - \mu_i^2) S_{ii}$$

for scalar bosons;

$$g_i = -\pi \delta(t - \mu_i^2) S_{ii}$$

for pseudoscalar bosons; and

$$g_i = -\pi \delta(t - \mu_V^2) \delta_{i6}$$

for vector boson; the coefficients $\chi_{i\alpha}$ are given in Table I.

- 39) M. Gell-Mann, Phys. Rev. 125, 1064 (1962); Y. Ne'eman, Nucl. Phys. 26, 222 (1961).
- 40) Cf. Ref. ³⁷⁾ and references cited therein.

FIGURE CAPTIONS

Figure 1 - The "improper diagram".

Figure 2 - The Λ -N scattering.

Figure 3 - The parallel diagrams with and without contribution from the 33 and Y_1^* resonances.

Figure 4 - The crossed diagrams with and without contribution from the 33 and Y_1^* resonances.

Figure 5 - The Λ -N central potential $V_C(r)$ as a function of relative separation $x = \mu r$ (in μ units). Our results are represented by a solid curve, the dashed curve is taken from Ref. ¹⁵⁾. For $x > 0.85$ the potentials have been increased by a factor of 10.

Figure 6 - The Λ - Λ central potential $U_C(r)$ as a function of relative separation $x = \mu r$ (in μ units). Our results are represented by the solid curve, the dashed curve is taken from Ref. ¹⁵⁾. For $x > 0.85$ the potentials have been increased by a factor of 10.

Figure 7 - Spin-spin potentials plotted against the relative separation $x = \mu r$ (in μ units). The solid curve represents the Λ - Λ potential $U_S(r)$, the dashed curve represents the Λ -N potential $V_S(r)$. These potentials have been increased by factors of 10^2 and 10^3 as indicated at the bottom of the figure.

Figure 8 - Tensor potentials plotted against the relative separation $x = \mu r$ (in μ units). The solid curve represents the Λ - Λ potential $U_T(r)$, the dashed curve represents the Λ -N potential $V_T(r)$. These potentials have been increased by factors of 10^2 and 10^3 as indicated at the top of the figure.

Figure 9 - One-boson exchange diagram.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

Λ -N CENTRAL POTENTIAL

FIG. 5

Λ Λ CENTRAL POTENTIAL

FIG. 6

FIG. 7

FIG. 8

FIG. 9