CM-P00057112

BARYON NUMBER 2, UNITARY SINGLET AS A TEST OF
ONE-PARTICLE EXCHANGE DYNAMICAL MODELS

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ABSTRACT

$B_8 - B_8$ scattering is studied making use of the fact that the forces contain the same $\bar{B}BP$ Hamiltonian as the $P_8 - B_8$ problem. This circumstance is exploited to find consistency conditions in low partial waves using experimental data. Results are consistent only if a new particle exists with quantum numbers $B = 2$, $T = Y = J = 0$, and F/D ratio in $\bar{B}BP$ coupling $\simeq 1/3$. It is argued that if this state does not exist the dynamical calculations, based on one-particle exchange and lowest lying threshold dominance, would have to be reconsidered. Besides this, such a particle would have interesting properties, in particular if its mass lies below $2m_\Lambda$, as could very well be the case: a possibility would then be open to study the weak decays of Σ^0 .

65/85/5-TH.518

/p/cm

25 January 1965

I. INTRODUCTION

Analyticity methods have already been applied to a variety of problems. The aim is to explain the known particles as dynamically generated in low-lying two body scattering amplitudes with the same quantum numbers, the forces arising mainly from the exchange of one-particle states.

The general features that seem to emerge from such calculations are the following:

a) From a quantitative point of view, the results obtained are rather poor. Rough agreement with experiment is sometimes reached, but only in such problems where one has adjustable parameters such as cut-offs; it is then very difficult to judge whether this agreement is really meaningful, or only due to the extra freedom available.

b) On the other hand, the qualitative predictions of these dynamical models are often encouraging: to take only two now classical examples, it is rather gratifying that the force due to V exchange in PP scattering¹⁾ is most attractive in the $J = 1$ octet direct channel²⁾, where vector mesons appear as resonances, and that B exchange in PB scattering can account for the existence of the $\frac{3}{2}^+$ decuplet Δ , with values of the F-D mixing parameter appearing reasonable in the light of other considerations³⁾.

These two aspects of dynamical calculations look, in a sense, contradictory, and the whole programme is thirsty for clear-cut evidence to prove its validity, and should be tested in all possible ways.

Since quantitative results seem quite difficult to improve, at least with present techniques, one is left with the possibility of checking qualitative predictions. The most reliable way of doing this appears to be: take a situation where there exist well-established particles, and where their explanation in terms of one-particle exchange forces is unambiguous; apply then the same arguments to all analogous channels and look for the predictions they give. In other words, one tries to derive, from the dynamical models under discussion, testable consequences of the existence of some already known particles.

It is the purpose of this paper to present an example of such a test by studying BB scattering.

This problem is well suited for our purpose because of the following reasons:

1) A $B = 2$ particle, the deuteron, is already well known and strong arguments have been found for its being a composite particle formed by two nucleons⁴⁾. On the other hand, there is no isospin 1, 1S_0 nucleon-nucleon bound state, but the corresponding scattering length is large, indicating that such a state "nearly exists". (In fact the associated pole lies in the second sheet of the Riemann surface.)

2) The two crossed channels of $B + B \rightarrow B + B$ are identical and their one-particle intermediate states can be constituted either by P mesons or by vector mesons (belonging to the octet V or the singlet V_1). We shall see below that there seems to be no alternative (within the framework of such models, where one-particle exchanges are supposed to be the dominant processes) to the choice of the P exchange force as the most important one in generating the $B = 2$ bound states.

3) From the unitary symmetry point of view, the deuteron ($Y = 2$, $T = 0$) can only be classified in the representation $\overline{10}$ of SU_3 ⁵⁾.

Furthermore, when actual masses are taken for the baryons, no such problems as the ones discussed in Ref. 6 appear, so that exact SU_3 considerations should be a particularly good approximation here.

The only difficulty comes from the fact that the $\bar{B}BP$ couplings involve two parameters, one of which is not accurately known (the so-called F-D mixing parameter). However, as we shall discuss below, there already exist arguments restricting the region in which this parameter can lie.

The application of the dynamical considerations sketched above to the $B + B$ problem yields two main results:

a) In order to explain the qualitative features of NN scattering correctly, one needs a very definite value for the F-D mixing parameter in $\bar{B}BP$ couplings.

b) A $B = 2$ unitary singlet is predicted.

This last particle provides a rather clear-cut test of dynamical models: as we will see below, it seems very difficult to believe in the validity of such models if the preceding prediction, obtained in a particularly clean case, is not borne out experimentally.

Apart from these considerations, a $B = 2$ unitary singlet would be very interesting if its mass lay below $2m_{\Lambda}$, as could very well be the case. It could then provide a chance to study the weak interactions of the neutral Σ particle.

II. B + B IN EXACT UNITARY SYMMETRY

Let us consider B + B scattering in the limit of exact unitary symmetry.

The one-particle exchanges we will consider [point 2) of the introduction], are those of the P octet, V octet and V_1 singlet.

The V_1 singlet will give the same force in all representations of the direct channel and, according to Sakurai's rules⁷⁾, this force will be repulsive. (The scattered particles have equal baryon number.)

The forces due to P exchange can be split into two factors, a space-time one (see below), and a unitary spin one. The latter can be expressed in terms of the π -N coupling constant g ($g^2/4\pi \simeq 14$) and an F-D mixing parameter f defined as in Ref. 3. The way we have calculated these coefficients is explained in the Appendix, the result being⁸⁾:

$$\begin{aligned}
 U_P(27) &= 4/3 \, g^2 (4f^2 - 2f + 1) \\
 U_P(10) &= - 8/3 \, g^2 (4f^2 - 5f + 1) \\
 U_P(\overline{10}) &= 8/3 \, g^2 (2f^2 - f - 1) \\
 U_P(8-8) &= - 2g^2 (4f^2 - 2f + 1) \\
 U_P(8'-8') &= 2/3 \, g^2 (-4f^2 - 10f + 5) \\
 U_P(8-8') &= 0 \\
 U_P(1) &= 4/3 \, g^2 (-4f^2 - 10f + 5) .
 \end{aligned} \tag{1}$$

As for the V octet exchange, some difficulties might arise from the fact that there are two types of $\bar{B}BV$ couplings⁹⁾ (the so-called Dirac and Pauli types.) The Pauli coupling is not well known experimentally, but it is believed to be less important than the Dirac one and, as we shall only need the rough features of the V exchange force, we will consider that vector mesons are coupled to baryons essentially through $\bar{B}\gamma_\mu B$. Such a coupling being pure F, the corresponding unitary spin crossing factors can be obtained from formulae (1) by setting $f = 1$:

$$\begin{aligned}
U_V(27) &= 4g_V^2 \\
U_V(10) &= 0 \\
U_V(\overline{10}) &= 0 \\
U_V(8-8) &= -6g_V^2 \\
U_V(8'-8') &= -6g_V^2 \\
U_V(8-8') &= 0 \\
U_V(1) &= -12g_V^2.
\end{aligned} \tag{2}$$

Formulae (2) clearly show that V exchange cannot be expected to give the main force for the generation of the deuteron (i.e., the $\overline{10}$ representation). Furthermore, P mesons are strongly favoured against V mesons, both by their lower masses and above all their considerably stronger coupling: $\bar{B}BP$ constants are of the order of the π -N one, while universality⁷⁾ predicts $g_{\rho NN}^2/4\pi \simeq 2$.

One is then left with the P exchange force to try and explain the deuteron dynamically. The corresponding spatial factor is easy to calculate from the formulae of Ref. 11. The only facts that we will use here are⁸⁾:

$$B_P^{(0,0)} = -3B_P^{(1,0)} \tag{3a}$$

$$B_P^{(1,0)} < 0 \text{ in the physical region,} \tag{3b}$$

where $B_P^{(J,L)}$ is the space-time factor for P exchange projected on the partial wave of the direct channel with orbital and total angular momenta L and J respectively.

The representations 1, 8 and 27 built with two baryon octets are symmetric under interchange of the two constituents, while 8', 10 and $\overline{10}$ are antisymmetric. The Pauli principle then allows $L = 0, J = 1$ (triplet state) only for 8', 10 and $\overline{10}$ ¹²⁾, and $L = J = 0$ (singlet) only for 1, 8 and 27. However, the fact that the orbital angular momentum is the same in both cases allows one to compare the corresponding forces.

Taking into account Eqs. (3), one can plot $-U_P(8'-8')$, $-U_P(10)$, $-U_P(\overline{10})$, $3U_P(1)$, $3U_P(8-8)$ and $3U_P(27)$ as functions of f (see Fig. 1).

Consider first the triplet states only. The force in $\overline{10}$ is the strongest one⁸⁾ for:

$$0 < f < 0.65. \quad (4)$$

This is encouraging since the values of f needed to explain the $\frac{3}{2}^+$ decuplet³⁾ fall in the region defined by inequalities (4).

But one should simultaneously study the singlet states. The first problem comes from the representation 27, in which the force due to P exchange is always stronger (or equal for $f = \frac{1}{4}$) than in $\overline{10}$ (see Fig. 1). Now 27 contains the $T = 1$ 1S_0 nucleon-nucleon amplitude. One then expects the force in 27 to be nearly equal but a little weaker than in $\overline{10}$. But V exchange can provide the small difference needed. In fact, it does not contribute in $\overline{10}$ ¹³⁾ [see formulae (2)], whereas it gives repulsion in 27, according to Sakurai's rules; so a very satisfactory explanation of the experimental situation is obtained for¹⁴⁾:

$$f \simeq \frac{1}{4}. \quad (5)$$

But Fig. 1 immediately shows that the P exchange force is considerably stronger in the representation 1 than in $\overline{10}$. Furthermore, formulae (2) indicate that V exchange is also attractive in the unitary singlet state¹⁶⁾.

The dynamical model we have been discussing then fulfils the requirements discussed in the Introduction: the already known experimental facts are correctly accounted for qualitatively, and a prediction comes out when the same considerations are applied to the unitary singlet 1S_0 state, which should show a bound state, its binding energy being larger than the deuteron one.

III. DISCUSSION

1) The above argument could contain a small loop-hole through which the dynamical model could escape being tested. This is due to the fact that, as seen in Fig. 1, the P exchange is very rapidly decreasing, as a function of the F-D mixing parameter f , in the neighbourhood of $f = 1/4$, in the unitary singlet channel.

Let us then see whether it is possible, by choosing f substantially above $1/4$, to avoid the unitary singlet without spoiling the qualitative explanation of known experimental facts.

One looks for a situation in which the force in $\overline{10}$ is stronger than in any other representation. The representations that can compete with $\overline{10}$ are 1, 27 and $8'$.

It is easily seen that no solution can be found without V exchange playing a crucial role.

From the formulae of Ref. 11, one can compute the corresponding space-time factor, with the results:

$$B_V^{(0,0)} \simeq B_V^{(1,0)} < 0 \quad \text{in the physical region.} \quad (6)$$

Taking into account formulae (2), one can then express that $\overline{10}$ is subject to the strongest force by writing the following inequalities between the parameter f and a positive quantity λ , defined as being the ratio between the V and P exchange forces (coupling constants included):

The force in 27 will be weaker than in $\overline{10}$ if:

$$4(4f^2 - 2f + 1) - 4\lambda < -\frac{8}{3}(2f^2 - f - 1). \quad (7)$$

The force in 1 will have the same property if:

$$4(-4f^2 - 10f + 5) + 12\lambda < -\frac{8}{3}(2f^2 - f - 1). \quad (8)$$

Finally the corresponding condition for $8'$ reads:

$$-\frac{2}{3}(-4f^2 - 10f + 5) + 6\lambda < -\frac{8}{3}(2f^2 - f - 1). \quad (9)$$

By combining inequality (7) with each of the other two, one finds that f must verify simultaneously:

$$5f^2 - 7f + 2 < 0 \quad (10)$$

$$10f^2 - 3f - 1 < 0 . \quad (11)$$

These inequalities are compatible for

$$\frac{2}{5} < f < \frac{1}{2} . \quad (12)$$

So, when qualitative considerations are used, as here, one cannot completely discard the possibility that V exchange is just such that only $\overline{10}$ contains particles.

However, because of the following reasons, the corresponding situation appears quite unsatisfactory compared to the one presented in section II.

a) First, the values of f defined by inequalities (12) seem too high in the light of other considerations. (See for instance, the discussion on this point in Ref. 17.) To give only one argument, it is currently believed that the $\Sigma\Sigma\pi$ coupling is somewhat weaker than the $\Sigma\Lambda\pi$ one¹⁰). Now, in exact unitary symmetry the corresponding constants are given by³):

$$\begin{aligned} g_{\Sigma\Sigma\pi} &= 2f g \\ g_{\Sigma\Lambda\pi} &= 2/\sqrt{3} (1-f) g . \end{aligned} \quad (13)$$

f should then verify:

$$f < \frac{\sqrt{3}-1}{2} \quad (14)$$

which is not the case if f lies in the region defined in Eq (12)¹⁹).

b) Secondly, the V exchange force would have to be comparable in magnitude with the P exchange one. Indeed, for $f = 1/2$, inequality (7) gives:

$$\lambda > 1/3 \quad (15)$$

which means that the V contribution is as important as the P contribution.

If this was the case, the nearby singularity arguments used in such dynamical models, as the one we are discussing, would look rather misleading, since they are even strongly reinforced here by a factor of 7 between the $\bar{B}BP$ and $\bar{B}BV$ coupling constants.

c) More generally, if it turned out that the unitary singlet predicted in section II did not exist, the dynamical explanation of the $\bar{10}$ particles would lose the major part of its interest due to its ad hoc nature: one would have to postulate that V exchange actually contributes just the amount needed both to bring the force in 27, and to keep those in 1 and 8', below the critical level defined by $\bar{10}$; in other words, one would have no more than a plausibility argument, which could only be supported by very precise calculations. Notice that if such crucial difficulties plague the particularly simple and well suited problem we are studying here (see the Introduction), then it is very difficult to believe dynamical models dealing with considerably more complicated cases, such as, for instance P+B scattering.

It then appears that the existence or absence of the $B = 2$ unitary singlet predicted in section II constitutes a rather clear-cut test of the validity of the models now in use.

2) Exact unitary symmetry should be a particularly good approximation in B+B scattering for the considerations we have developed. Indeed, the various channels with the same hypercharge have very similar thresholds:

$$\begin{aligned} Y = 2 & \quad NN (\simeq 1880 \text{ MeV}) \\ Y = 1 & \quad N\Lambda (\simeq 2055 \text{ MeV}), N\Sigma (\simeq 2135 \text{ MeV}) \\ Y = 0 & \quad \Lambda\Lambda (\simeq 2230 \text{ MeV}), N\Xi (\simeq 2260 \text{ MeV}), \Lambda\Sigma (\simeq 2310 \text{ MeV}), \Sigma\Sigma (\simeq 2390 \text{ MeV}) \\ Y = -1 & \quad \Lambda\Xi (\simeq 2435 \text{ MeV}), \Xi\Sigma (\simeq 2515 \text{ MeV}) \\ Y = -2 & \quad \Xi\Xi (\simeq 2640 \text{ MeV}). \end{aligned} \quad (16)$$

Take then the representation $\overline{10}$ for example. The mass formula reduces in this supermultiplet to the equal spacing rule. Now, this is compatible with the hypothesis that the forces in every member of $\overline{10}$ are roughly equal, the splitting being mainly due to the separation of thresholds. Neglecting the binding energies that should be of the same order as the deuteron one, a dynamical model treating the exchanged particles (whatever they may be) as degenerate according to SU_3 , will predict a rough equal spacing, which formulae (16) indicate to be of the order of 210 MeV^{20} , i.e:

$$\begin{aligned}
 m(2,0) &\simeq 1880 \text{ MeV (linked to NN)} \\
 m(1, \frac{1}{2}) &\simeq 2090 \text{ MeV (linked to NA and N}\Sigma) \\
 m(0,1) &\simeq 2300 \text{ MeV (linked to N}\Xi, \Lambda\Sigma \text{ and } \Sigma\Sigma) \\
 m(-1, \frac{3}{2}) &\simeq 2510 \text{ MeV (linked to } \Sigma\Xi).
 \end{aligned}
 \tag{17}$$

Of course, for the unitary singlet the situation is even better: all channels contributing to it are nearly degenerate. No problems such as the ones discussed in Ref. 6, due to a great separation of thresholds within the same hypercharge states, are then likely to appear here.

Moreover, it is easily seen that the $\overline{N}\eta N$ coupling constant vanishes for $f = \frac{1}{4}^3$. So, at least for the NN channels, the forces coming from P exchange will be equal in $T = 0$ and $T = 1$, even when SU_3 breakdown is taken into account. Furthermore, in the unitary singlet channel, one can take roughly into account the mass differences between the pseudoscalar mesons by multiplying the contributions from the η and K by a factor smaller than one; the force still remains substantially stronger in 1 than in $\overline{10}$.

IV. SOME PROPERTIES OF A $B = 2$ UNITARY SINGLET

If the particle predicted in section II actually exists, how could it be detected, and what properties are expected for it?

1) Two types of experiments could be performed in order to search for a $B = 2$ singlet.

a)

$$K^- + D \rightarrow (BB)_0 + K^0.$$

This reaction offers the advantage of a rather rich beam and easy target, but the cross-section might be rather small because of the high transfer of strangeness required. (Cf. the example of the Ω baryon.)

b)

$$\Xi^- + p \rightarrow (BB)_0 + \pi^0.$$

If the mass of the singlet is of the order of 2200 MeV, some 200 MeV kinetic energy is necessary for the Ξ^- . This second reaction could, perhaps, be performed in the near future.

2) By direct use of the tables of Ref. 21, one can see that the wave function of a unitary singlet is composed as follows:

$$\psi_0 = 1/\sqrt{8} (p\Xi^- + \Xi^-p - n\Xi^0 - \Xi^0n + \Sigma^+\Sigma^- + \Sigma^-\Sigma^+ - \Sigma^0\Sigma^0 - \Lambda\Lambda). \quad (18)$$

The qualitative arguments developed in section II only tell that the unitary singlet should have a larger binding energy than the deuteron, i.e., lie below the $Y = 0$ member of the decuplet $\overline{10}$: it is difficult to figure out the dependence of the binding energy on the strength of the forces.

If the particle under discussion is a resonance, with respect to some of the channels that appear in (18), this formula enables one to calculate the corresponding partial widths in terms of one parameter. (Phase space cannot be evaluated without precise knowledge of the mass.) It will be interesting to see whether these partial widths agree with

experiment, since it is rather embarrassing that those of the $\frac{3}{2}^+$ resonances do not: one could then find arguments for or against the perturbation point of view usually taken, with respect to unitary symmetry breakdown.

On the other hand, all $B = 2$ channels might be closed, since the force in the representation 1 is much stronger than in $\overline{10}$ (three times so for $f = \frac{1}{4}$, neglecting V exchange which can only enlarge this factor).

If this is the case, the decay will take place through weak interactions. One could then try to check the virtual structure displayed in (18) through the observed branching ratios.

Actually this is very interesting, especially for the $(\Sigma^0 \Sigma^0)_0$ state: if the mass of the particle is smaller than $2m_\Lambda$, the electromagnetic decay

$$(\text{BB})_0 \rightarrow \Lambda + \Lambda + \gamma + \gamma \quad (19)$$

is impossible, energetically. Then, if one sees

$$\begin{aligned} (\text{BB})_0 \rightarrow \Sigma^0 \Sigma^0 \rightarrow \Lambda + \gamma + p + \pi^- \\ \text{(virtual)} \end{aligned} \quad (20)$$

one knows that the particle has decayed through its $\Sigma^0 \Sigma^0$ virtual state (which has a probability of 12.5%), so that the p and π^- come from a Σ^0 . This could give a chance of observing the weak decay of Σ^0 , which would certainly be useful.

V. CONCLUSION

It has been seen that in a particularly clean case, $B+B$ scattering, a dynamical model taking into account only one-particle exchanges could not be consistent if there does not exist, besides the decuplet $\overline{10}$ containing the deuteron, a 1S_0 unitary singlet. At the same time, the value of the F-D mixing parameter in $\bar{B}BP$ couplings is very accurately determined by such a model.

The prediction of a $B = 2$ singlet can provide a stringent test of such analyticity arguments.

Moreover, if this particle exists, it could have interesting properties, depending on its mass, such as allowing the Σ^0 weak decays to be observed.

A P P E N D I X

To calculate the unitary spin crossing coefficients [formulae (1)], one can use the method of Clebsch-Gordan coefficients²².

One starts with the Hamiltonian:

$$H = \sum_{i,j,k} C(8,8,1; -k,k,0) (-1)^{T_{3i} + 1/2 Y_i} \bar{B}_i (i\gamma_5) B_j P_k \quad (A.1)$$

$$\times \left(g_1 C(8,8,8; -i,j,-k) + g_2 C(8,8,8'; -i,j,-k) \right)$$

where:

An index like i stands for the set (Y, T, T_3) .

An index like $-i$ denotes $(-Y, T, -T_3)$.

B_m and P_n are the fields of the baryons and the pseudoscalar mesons, respectively.

$C(R_1, R_2, R; m_1, m_2, m)$ is the Clebsch-Gordan coefficient combining the member m_1 of the representation R_1 and the member m_2 of R_2 to make m of R .

The factor $(-1)^{T_{3i} + 1/2 Y_i}$ comes from the definition of phases: if B_i belongs to the B octet, the corresponding member of the $\bar{B}$ octet is $(-1)^{T_{3i} + 1/2 Y_i} \bar{B}_i$.

g_1 and g_2 are related to the πN coupling constant g and the F-D mixing parameter f defined in Ref. 3 by:

$$g_1 = 4\sqrt{10/3} (f-1) g$$

$$g_2 = -4\sqrt{6} f g. \quad (A.2)$$

Considering the scattering amplitude

$$B_\alpha + B_\beta \rightarrow B_\gamma + B_\delta \quad (A.3)$$

in which the scattering angle is defined between the centre of mass momenta of B_α and B_γ , one can easily calculate the t and u P exchange graphs by using the Hamiltonian (A.1).

One then separates them into a space-time factor, which can be projected into the relevant partial waves of the direct channel with the aid of Ref. 11, and a unitary spin one.

The crossing coefficients listed in section II are then obtained from the formulae:

$$\begin{aligned}
 U_P^t(R-R') = \frac{1}{8} \sum_{\alpha, \beta, \gamma, \delta, \xi} (-1)^{[T_{3\gamma} + T_{3\beta} + \frac{1}{2}(Y_\gamma + Y_\beta)]} C(8, 8, R; \alpha, \beta, m) \times C(8, 8, R'; \gamma, \delta, m) \\
 \times \left(g_1 C(8, 8, 8; -\gamma, \alpha, -\xi) + g_2 C(8, 8, 8'; -\gamma, \alpha, -\xi) \right) \\
 \times \left(g_1 C(8, 8, 8; -\delta, \beta, \xi) + g_2 C(8, 8, 8'; -\delta, \beta, \xi) \right) \quad (A.4)
 \end{aligned}$$

$$\begin{aligned}
 U_P^u(R-R') = \frac{1}{8} \sum_{\alpha, \beta, \gamma, \delta, \xi} (-1)^{[T_{3\beta} + T_{3\delta} + \frac{1}{2}(Y_\beta + Y_\delta)]} C(8, 8, R; \alpha, \beta, m) \times C(8, 8, R'; \gamma, \delta, m) \\
 \times \left(g_1 C(8, 8, 8; -\delta, \alpha, -\xi) + g_2 C(8, 8, 8'; -\delta, \alpha, -\xi) \right) \\
 \times \left(g_1 C(8, 8, 8; -\gamma, \beta, \xi) + g_2 C(8, 8, 8'; -\gamma, \beta, \xi) \right) \quad (A.5)
 \end{aligned}$$

U_P^t and U_P^u correspond to the t and u channel exchanges respectively.

Of course, in actual calculations, one uses the fact that the U_P 's are independent of m, and the properties of the Clebsch-Gordan coefficients, to reduce the number of non-vanishing terms in the summation. For instance, by interchanging γ and δ in U_P^t , one easily finds that

$$U_P^u(R-R') = \epsilon(R') U_P^t(R-R') \quad (A.6)$$

where $\epsilon(R')$ is ± 1 depending on whether R' is symmetric or antisymmetric, with respect to its constituents (see table 1 of Ref. 21). In the same way the permutation

$$\alpha \leftrightarrow \beta$$

$$\gamma \leftrightarrow \delta$$

leaves all crossing coefficients unchanged, except those for 8-8', which are then seen to be zero.

REFERENCES AND FOOTNOTES

- 1) P stands for the octet of pseudoscalar mesons, B for that of the $\frac{1}{2}^+$ baryons, V and V_1 for the vector mesons (octet and singlet respectively), Δ for the decuplet of $\frac{3}{2}^+$ baryons.
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20.

- 14) If $f = 1/4$ it is not possible to have both 10 and 27, and only these, resonating in the $3/2^+$ P+B amplitude¹⁵⁾.
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- 19) Notice that the value of f for which the (1) and (10) curves cross in Fig. 1 is already larger than the maximum obtained in (14).
- 20) It is not clear whether the masses themselves or their squares should be used in the mass formula. However, there is no important difference between the two possibilities⁵⁾.
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FIG.1