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DO THE $\frac{3}{2}^+$ BARYONS BELONG TO THE REPRESENTATION 10 OF SU_3 ?

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A B S T R A C T

The classification of the $\frac{3}{2}^+$ baryons in the representation 10 of SU_3 is re-examined in the light of recent evidence for the possible existence of a $\frac{3}{2}^+$ 27-plet. It is found that the mass formula ("equal spacing rule") alone is not sufficient to decide whether these baryons belong to a pure unitary decuplet or result from strong 10-27 mixing. Some consequences of this latter scheme are derived, and it is shown in particular that it does not solve the problems related to the widths of the $\frac{3}{2}^+$ resonances. An experimental test is proposed that could confirm or infirm the 10 assignment independently from the mass formula; the experiment, though difficult, does not look impossible, and seems worth performing, due to its crucial importance for unitary symmetry.

INTRODUCTION

Isospin symmetry of strong interactions has been proved and tested in several independent ways, through branching ratios and selection rules. Furthermore, assignment of strongly interacting particles to given isospin multiplets is made easier by the clustering of masses: particles with identical space-time properties and similar dynamical behaviour group themselves in such a way that there is no ambiguity in classifying them in well separated multiplets.

The situation is far from being as satisfactory for higher symmetry groups such as SU_3 . The now rather general belief that unitary symmetry is actually relevant to strong interactions is essentially based on two facts:

a) In general, the number of known particles with the same spatial properties and their isospin and hypercharge quantum numbers seem to be the ones needed to fill SU_3 representations.

b) The Gell-Mann-Okubo mass formula has had some rather impressive achievements, among which the prediction of the recently discovered Ω baryon¹⁾.

Now it is well known, on the other hand, that these two points are not always fulfilled in quite a satisfactory way.

Furthermore, the breakdown of unitary symmetry, which is in particular responsible for the mass splittings described by the Gell-Mann-Okubo mass formula, is not yet well understood.

We wish to discuss here two related questions:

a) To what extent is the evidence for particles to belong to definite SU_3 representations actually compelling?

b) Is it possible to find other tests of such assignments than the mass formula?

Though similar considerations could be extended to other cases, we will be concerned here with the $\frac{3}{2}^+$ baryons, because they constitute essentially the show-case of unitary symmetry.

The prediction of the Ω followed from the assignment of the already known N^* (1238 MeV), Y_1^* (1385 MeV), Ξ^* (1530 MeV), which all have $J^P = \frac{3}{2}^+$, to the representation 10 of SU_3 ; the mass experimentally found for the Ω verifies with great accuracy the "equal spacing rule" which the mass formula reduces to in a decuplet.

Now this assignment is plagued with two main difficulties.

First, it is somewhat surprising, in a sense, that the mass formula works so well, especially in the case of the decuplet. Indeed, the most obvious way of understanding such a formula is to derive it from perturbation theory. But then, if energies are already correct in first order, zeroth order "wave functions" should also be a good approximation. Now it is well known that the widths of the $\frac{3}{2}^+$ resonances predicted in this way are rather far from their experimental values ²⁾.

Secondly, experiments in the energy region where the " $\frac{3}{2}^+$ decuplet" lies are not such as to discard the existence of other particles definitely. Actually, evidence has recently been presented for an isospin 2 $\pi\Xi$ resonance around 1415 MeV ⁴⁾. Of course, it has to await confirmation before it can be welcome into the society of well-established particles, but it is made interesting by the theoretical problems it raises, since these problems would remain, as we shall see, even if this particular Y_2^* eventually "disappeared".

From the unitary symmetry point of view, this Y_2^* can only be classified in the representation 27. As for its spatial properties, they have not been determined experimentally; however, a reasonable guess seems to be $J^P = \frac{3}{2}^+$ ⁵⁾.

In Section I we consider the possibility of strong 10-27 mixing, with the condition that the experimentally well-verified equal spacing rule still holds for the $N^*(1238 \text{ MeV})$, $Y_1^*(1385 \text{ MeV})$, $\Xi^*(1530 \text{ MeV})$ and $\Omega(1675 \text{ MeV})$. Some implications of such a mixing are discussed in Section II; the most striking is that, in this scheme, the $Y_2^*(1415 \text{ MeV})$, if it exists, should be accompanied by a $T = \frac{3}{2} \pi\Xi$ resonance around 1550 MeV and a $Y = -2$, $T = 1$ bound state near 1690 MeV. Finally, Section III deals with the second question mentioned above: is it possible to test the 10 assignment independently from the mass formula? It turns out that such a test actually exists, and seems to be possible to be carried out experimentally in the near future, if not at present.

SECTION I: 10-27 mixing with equal spacing

The equal spacing rule between the N^* (1238 MeV), Y_1^* (1385 MeV), Ξ^* (1530 MeV), and Ω (1675 MeV) is amazingly well verified experimentally.

However, as mentioned in the introduction, one has no independent evidence for the 10 assignment so far.

Furthermore, if the Y_2^* exists and is indeed a $\frac{3}{2}^+$ state, the fact that its mass ($\approx 1415 \text{ MeV}$ ⁴⁾) falls very near the mean mass of the now accepted decuplet makes strong mixing of representations nearly unavoidable a priori⁹⁾: of course, the Y_2^* itself does not mix with any member of the decuplet, but one expects at least some of its partners susceptible of mixing to lie in the same energy region¹¹⁾.

The first question is then: is it possible to have a strong 10-27 mixing without spoiling the above-mentioned equal spacing rule? The answer is yes, as we shall see¹³⁾.

Let us then look in more detail at the strong mixing case. We consider a decuplet, for which the mass formula writes:

$$m(Y, T) = m_{10} + a_{10} Y \quad (1)$$

and which is supposed to mix strongly with a 27 representation where the corresponding equation is:

$$m(Y, T) = m_{27} + a_{27} Y + b_{27} \left[T(T+1) - \frac{1}{4} Y^2 \right] \quad (2)$$

The mixing energy parameter will be called f ¹⁴⁾, and the coefficients that appear in the mass-matrices can be read off reference 15).

The N^* , Y_1^* and Ξ^* belonging to the decuplet mix with the analogous members of the 27-plet, and the relevant 2×2 matrices $M(Y, T)$, together with the corresponding eigenvalue equations for the physical masses, write:

$$M(1, \frac{3}{2}) = \begin{pmatrix} m_{10} + a_{10} & -\frac{3}{4} f \\ -\frac{3}{4} f & m_{27} + a_{27} + \frac{7}{2} b_{27} \end{pmatrix} \quad (3a)$$

$$m_1^2 - m_1 \left(m_{10} + a_{10} + m_{27} + a_{27} + \frac{7}{2} b_{27} \right) + (m_{10} + a_{10})(m_{27} + a_{27} + \frac{7}{2} b_{27}) = \frac{9}{16} f^2 \quad (3b)$$

$$M(-1, \frac{1}{2}) = \begin{pmatrix} m_{10} - a_{10} & -\frac{3\sqrt{5}}{20} f \\ -\frac{3\sqrt{5}}{20} f & m_{27} - a_{27} + \frac{1}{2} b_{27} \end{pmatrix} \quad (4a)$$

$$m_{-1}^2 - m_{-1} \left(m_{10} - a_{10} + m_{27} - a_{27} + \frac{1}{2} b_{27} \right) + (m_{10} - a_{10})(m_{27} - a_{27} + \frac{1}{2} b_{27}) = \frac{9}{80} f^2 \quad (4b)$$

$$M(0,1) = \begin{pmatrix} m_{10} & -\frac{\sqrt{30}}{10} f \\ -\frac{\sqrt{30}}{10} f & m_{27} + 2 b_{27} \end{pmatrix} \quad (5a)$$

$$m_0^2 - m_0(m_{10} + m_{27} + 2 b_{27}) + m_{10}(m_{27} + 2 b_{27}) = \frac{3}{10} f^2 \quad (5b)$$

Of course, the other states of 27 remain pure, as does the $Y = -2, T = 0$ member of $10(\Omega)$. Since the mass of this last particle is given by:

$$m_{\Omega} = m(-2, 0) = m_{10} - 2 a_{10} \quad (6)$$

equal spacing can be imposed by requiring that $m_{10} - 2a_{10} - \Delta$ be a solution of (4b), $m_{10} - 2a_{10} - 2\Delta$ of (5b), and $m_{10} - 2a_{10} - 3\Delta$ of (3b) (experimentally, $\Delta \simeq 146$ MeV).

The corresponding conditions are:

$$(a_{10} + \Delta)(-m_{10} + 2a_{10} + \Delta + m_{27} - a_{27} + \frac{1}{2} b_{27}) = \frac{9}{80} f^2 \quad (7a)$$

$$(a_{10} + \Delta)(-m_{10} + 2a_{10} + 2\Delta + m_{27} + 2b_{27}) = \frac{3}{20} f^2 \quad (7b)$$

$$(a_{10} + \Delta)(-m_{10} + 2a_{10} + 3\Delta + m_{27} + a_{27} + \frac{7}{2} b_{27}) = \frac{3}{16} f^2 \quad (7c)$$

Of course, the trivial solution (no mixing):

$$\begin{cases} a_{10} = -\Delta \\ f = 0 \end{cases} \quad (8)$$

is possible. If one discards it for the moment, only two among equations (7) happen to be independent; so one can keep only one of them and:

$$\Delta = a_{10} + \frac{1}{2}(m_{27} - m_{10}) - 2(a_{27} + b_{27}), \quad (9)$$

which is a consequence of the other two.

Notice that, if different coefficients appeared in the right-hand side of equations (7), this system would have no solution. It is because of the degeneracy just mentioned that it is possible to maintain equal spacing between the $N^*(1238 \text{ MeV})$, $Y_1^*(1385 \text{ MeV})$, $\Xi^*(1530 \text{ MeV})$, and

$\Omega(1675 \text{ MeV})$, even if 10-27 mixing is in operation. If, just for comparison, one investigates the possibility of 8-10 mixing, demanding equal spacing for the relevant physical particles, one obtains only the trivial solution. (This can be readily understood if one notices that the base and vertex of the "10 triangle" do not mix with members of 8.)

The equal spacing Δ' between the second set of particles that undergo mixing is given by:

$$\Delta' = \frac{1}{2} (m_{10} - m_{27}) - 2a_{10} + a_{27} + \frac{1}{2} b_{27} \quad (10)$$

We will then be faced with the following alternative: either we consider that strong mixing is actually in operation, and then we lose the attractive, because natural, explanation of the equal spacing rule as coming from the Gell-Mann-Okubo formula in a pure decuplet; or we think that this last explanation should be kept, and then we do not understand why the ω and ϕ should mix while the $\frac{3}{2}^+$ decuplet and 27-plet should not. Neither of these situations is very satisfactory, especially if one remembers that the latter is plagued by the above-mentioned difficulty about the widths of the $\frac{3}{2}^+$ resonances (actually, this was the main reason why $\frac{3}{2}^+$ baryons were first thought to constitute a 27-plet¹⁰), since $Y_1^* \rightarrow \pi + \Sigma$ is forbidden in such a scheme).

Anyway, the mass formula alone is not sufficient to decide whether the $\frac{3}{2}^+$ baryons are to be classified in a pure decuplet or whether they arise from strong 10-27 mixing.

SECTION II: Some implications of 10-27 mixing with equal spacing

Let us now discuss some of the consequences of the scheme developed in Section I, to see whether we can find arguments in favour of it, or against.

Six parameters enter in such a scheme. As we have already imposed the equal spacing conditions, we have only three known independent quantities (the masses of the Ω and Y_2^* and the equal spacing Δ , say), and four equations:

$$m_{\Omega} = m_{10} - 2a_{10} \quad (11a)$$

$$m_{Y_2^*} = m_{27} + 6b_{27} \quad (11b)$$

$$\Delta = a_{10} + \frac{1}{2}(m_{27} - m_{10}) - 2(a_{27} + b_{27}) \quad (11c)$$

$$f^2 = \frac{80}{9}(a_{10} + \Delta)(-m_{10} + 2a_{10} + \Delta + m_{27} - a_{27} + \frac{1}{2}b_{27}) \quad (11d)$$

It is however remarkable that, even with two equations less than parameters, the positions of two of the conjectured members of the 27-plet can be immediately predicted:

$$m(-1, \frac{3}{2}) = \frac{1}{4}(3m_{Y_2^*} + m_{\Omega} + 2\Delta) \simeq 1553 \text{ Mev.} \quad (12a)$$

$$m(-2,1) = \Delta + \frac{1}{2}(m_{Y_2^*} + m_{\Omega}) \simeq 1691 \text{ MeV} \quad (12b)$$

(These two masses and the Y_2^* one are equally spaced.)

Concerning the other particles, one can easily see that solutions might exist that put all of them at higher energies ($\geq 2 \text{ GeV}$), but still such that m_{10} and m_{27} are close enough to justify mixing. Anyhow, one needs at least two more particles to be known before the rest can be located, if one believes in the existence of the $\frac{3}{2}^+$ 27-plet.

Let us now turn to the widths of the physical N^* (1238 MeV), Y_1^* (1385 MeV), and Ξ^* (1530 MeV). Because of the difficulty mentioned above, several groups have proposed solutions taking into account first order SU_3 violations in the coupling constants ^{16,17)}, but the importance of the breaking terms needed to fit the experimental values seems greater than what appears reasonable from a perturbation theory point of view ¹⁷⁾.

One could hope to obtain better agreement in the 10-27 mixing case. Using equations (3), (4) and (5), one immediately calculates $\alpha_{10}(Y,T)$ and $\alpha_{27}(Y,T)$, the components of the eigenvector of $M(Y,T)$ corresponding to the well-known particle with these quantum numbers. Let:

$$R = \frac{a_{10} + \Delta}{f} \quad (13)$$

Then, by taking into account equation (6) and the equal spacing rule, one gets:

$$\left\{ \alpha_{10}\left(1, \frac{3}{2}\right) = \left[1 + 16 R^2\right]^{-\frac{1}{2}} \right. \quad (14a)$$

$$\left\{ \alpha_{27}\left(1, \frac{3}{2}\right) = 4R \left[1 + 16 R^2\right]^{-\frac{1}{2}} \right. \quad (14b)$$

$$\left\{ \alpha_{10}(0, 1) = \left[1 + \frac{40}{3} R^2\right]^{-\frac{1}{2}} \right. \quad (15a)$$

$$\left\{ \alpha_{27}(0, 1) = \frac{20}{\sqrt{30}} R \left[1 + \frac{40}{3} R^2\right]^{-\frac{1}{2}} \right. \quad (15b)$$

$$\left\{ \alpha_{10}\left(-1, \frac{1}{2}\right) = \left[1 + \frac{80}{9} R^2\right]^{-\frac{1}{2}} \right. \quad (16a)$$

$$\left\{ \alpha_{27}\left(-1, \frac{1}{2}\right) = \frac{20}{3\sqrt{5}} R \left[1 + \frac{80}{9} R^2\right]^{-\frac{1}{2}} \right. \quad (16b)$$

The reduced widths of the corresponding particles are then given, in terms of two amplitudes A_{10} and A_{27} , by (cf. ref. 15):

$$g_{\chi_1^* \pi \Sigma} = \frac{1}{\sqrt{6}} \alpha_{10}(0, 1) A_{10} \quad (17a)$$

$$g_{Y_1^* \pi \Lambda} = \frac{1}{2} \alpha_{10}(0,1) A_{10} + \frac{\sqrt{30}}{10} \alpha_{27}(0,1) A_{27} \quad (17b)$$

$$g_{N^* \pi N} = \frac{1}{\sqrt{2}} \alpha_{10}\left(1, \frac{3}{2}\right) A_{10} + \frac{1}{\sqrt{2}} \alpha_{27}\left(1, \frac{3}{2}\right) A_{27} \quad (17c)$$

$$g_{\Xi^* \pi \Xi} = \frac{1}{2} \alpha_{10}\left(-1, \frac{1}{2}\right) A_{10} + \frac{\sqrt{5}}{10} \alpha_{27}\left(-1, \frac{1}{2}\right) A_{27} \quad (17d)$$

One could think that, with such a number of parameters, a fit of the experimental widths should be easy. Actually, as was the case for the mass formula above, one is faced with rather strong conditions. Let us give two examples to illustrate this.

Suppose first that we want $g_{Y_1^* \pi \Sigma} \approx 0$. Neglect of A_{10} leads to the relations:

$$g_{Y_1^* \pi \Lambda}^2 \left[1 + \frac{40}{3} R^2\right] = \frac{1}{2} g_{N^* \pi N}^2 \left[1 + 16 R^2\right] = 9 g_{\Xi^* \pi \Xi}^2 \left[1 + \frac{80}{9} R^2\right] \quad (18)$$

It is then easy to see that the reduced widths deduced from experimental data ⁽¹⁸⁾ do not satisfy such relations. Indeed, R^2 being positive, the first and third members of 18 are compatible only if:

$$\left[g_{Y_1^* \pi \Lambda}^2 - 9 g_{\Xi^* \pi \Xi}^2\right] \left[2 g_{\Xi^* \pi \Xi}^2 - \frac{1}{3} g_{Y_1^* \pi \Lambda}^2\right] > 0, \quad (19)$$

while the second and third ones imply that:

$$\left[\frac{1}{2} g_{N^*\pi N}^2 - g_{\Xi^*\pi \Xi}^2 \right] \left[10 g_{\Xi^*\pi \Xi}^2 - g_{N^*\pi N}^2 \right] > 0 \quad (20)$$

Neither of these inequalities is actually satisfied ¹⁸⁾.

The other example concerns the fact that, whatever the values of A_{10} and A_{27} , equations (17) require $g_{Y_1^*\pi\Lambda}$ and $g_{N^*\pi N}$ to verify:

$$2 g_{Y_1^*\pi\Lambda}^2 \left[1 + \frac{40}{3} R^2 \right] = g_{N^*\pi N}^2 \left[1 + 16 R^2 \right] > \quad (21)$$

so that a necessary condition is:

$$\left[2 g_{Y_1^*\pi\Lambda}^2 - g_{N^*\pi N}^2 \right] \left[g_{N^*\pi N}^2 - \frac{5}{3} g_{Y_1^*\pi\Lambda}^2 \right] > 0 \quad (22)$$

The width of the Y_1^* being rather well known ²⁰⁾, the N^* one is limited by the above inequality to:

$$112 \text{ Mev} < \Gamma_{N^*\pi N} < 137 \text{ Mev} \quad (23)$$

which, this time, seems compatible with present experimental data ²⁰⁾.

The results of this section can then be summarized as follows:

1: If one allows the 27-plet, to which Y_2^* must belong, to mix with the decuplet, and imposes the experimentally verified equal spacing, then two particles can be predicted: a $Y = -1$, $T = \frac{3}{2}$ one around 1550 MeV, and a $Y = -2$, $T = 1$ one around 1690 MeV. While the latter is certainly not excluded by present experiments, the former should already have been seen, lying in a region which has been fairly well explored (see, for instance, ref. 12). Of course, as mentioned in the introduction, it is not possible to say that experimental data definitely forbid the existence of such a particle, but they do give strong arguments against it.

2: 10-27 mixing with equal spacing does not radically improve the theoretical situation insofar as the widths of the $\frac{3}{2}^+$ resonances are concerned. But perhaps such problems are rather profound; the perturbation theory point of view mentioned in the introduction may be completely misleading, and may have to be replaced, say, by some kind of "octet enhancement" arguments ²¹⁾.

The 10-27 mixing scheme does not seem then to compensate for the loss of the "natural" explanation of the equal spacing rule by any attractive feature. If a $\frac{3}{2}^+$ 27-plet actually exists, it would perhaps be preferable that it did not appreciably mix with the already known decuplet. An argument to "explain" this absence of mixing could be derived from higher symmetry groups: in SU_6 ²²⁾, for instance, ω and ϕ appear in the same representation (35), while a $\frac{3}{2}^+$ 27-plet would have to be classified in a supermultiplet different from 56, which the decuplet is supposed to belong to.

SECTION III: An independent test for the 10 assignment

The preceding discussions show the relevance of the second question asked in the introduction, i.e., the importance of testing the 10 assignment for $\frac{3}{2}^+$ baryons independently from the mass formula.

Needless to say that, if the $Y_2^*(1415 \text{ MeV})$ were confirmed, some evidence for or against this assignment could come from a search of the two particles predicted in the 10-27 mixing scheme (Section II).

However, it turns out that a much neater, though more elusive, experimental test actually exists for the $\frac{3}{2}^+$ baryons.

It is well known, from U-spin considerations ²³⁾, that the decays:

$$Y_1^{*-} \rightarrow \Sigma^- + \gamma \quad (24a)$$

$$\Xi_1^{*-} \rightarrow \Xi^- + \gamma \quad (24b)$$

are forbidden by unitary symmetry, if the Y_1^* and Ξ_1^* belong to a pure decuplet, while their analogues for Y_1^{*+} and Ξ^{*0} are allowed.

Notice that the statement that the decay modes, Eq. (24), are forbidden is not sufficient for the test to be applicable. Indeed, the way unitary symmetry is broken remains obscure, and it is very probable that some such decays will be found. Furthermore, it is easy to see that (24a) would not be very probable either if Y_1^* belonged to a 27-plet or resulted from 10-27 mixing.

The important quantities are the ratios:

$$R_{Y_1^*} = \Gamma[Y_1^{*-} \rightarrow \Sigma^- + \gamma] / \Gamma[Y_1^{*+} \rightarrow \Sigma^+ + \gamma] \quad (25a)$$

$$R_{\Xi^*} = \Gamma[\Xi^{*-} \rightarrow \Xi^- + \gamma] / \Gamma[\Xi^{*0} \rightarrow \Xi^0 + \gamma] \quad (25b)$$

If Y_1^* and Ξ^* are actually decuplet states, these ratios should be found much smaller than 1 (if SU_3 breaking interactions are roughly 10 times weaker in intensity than SU_3 conserving ones, one expects $R_{Y_1^*}$ and R_{Ξ^*} to be of the order of $1/10$ if the decuplet assignment is correct).

Let us now look at how the prediction would be changed if Y_1^* and Ξ^* resulted from 10-27 mixing. The formulae of reference 24) show that the 27 parts of Y_1^* and Ξ^* are the following mixtures of U-spin:

$$|27; Y_1^{*-}\rangle = \sqrt{\frac{5}{6}} |U=\frac{3}{2}; U_3=\frac{1}{2}\rangle + \frac{1}{\sqrt{6}} |U=\frac{1}{2}; U_3=\frac{1}{2}\rangle \quad (26a)$$

$$|27; Y_1^{*+}\rangle = -\sqrt{\frac{5}{6}} |U=\frac{3}{2}; U_3=-\frac{1}{2}\rangle + \frac{1}{\sqrt{6}} |U=\frac{1}{2}; U_3=-\frac{1}{2}\rangle \quad (26b)$$

$$|27; \Xi^{*-}\rangle = \frac{\sqrt{5}}{3} |U=\frac{3}{2}; U_3=-\frac{1}{2}\rangle + \frac{2}{3} |U=\frac{1}{2}; U_3=-\frac{1}{2}\rangle \quad (26c)$$

$$|27; \Xi^{*0}\rangle = \sqrt{\frac{5}{6}} |U=2; U_3=-1\rangle + \frac{1}{\sqrt{6}} |U=1; U_3=-1\rangle \quad (26d)$$

To calculate $R_{Y_1^*}$ and R_{Ξ^*} (formulae 25), we prefer to work with isospin eigenstates. In such an approach, the photon is considered as belonging to an incomplete octet 8_Y , with zeros everywhere except at the centre, where:

$$\begin{aligned} |8_Y; Y=0, T=1, T_3=0\rangle &= \frac{\sqrt{3}}{2} |\gamma\rangle \\ |8_Y; Y=0, T=0, T_3=0\rangle &= \frac{1}{2} |\gamma\rangle \end{aligned} \quad (27)$$

and isospin is conserved.

When considering such decays as:

$$\left. \begin{matrix} 10 \\ 27 \end{matrix} \right\} \longrightarrow 8_B + 8_Y \quad , \quad (28)$$

where 8_B is the baryon octet, one can use the Clebsch-Gordan coefficients tabulated in reference 25) to find the already mentioned results for 10 and:

$$R_{Y_1^*}(27) = \Gamma[\gamma_1^{*-}(27) \rightarrow \Sigma^- + \gamma] / \Gamma[\gamma_1^{*+}(27) \rightarrow \Sigma^+ + \gamma] \quad (29a)$$

$$= 1$$

$$R_{\Xi^*}(27) = \Gamma[\Xi^{*-}(27) \rightarrow \Xi^- + \gamma] / \Gamma[\Xi^{*0}(27) \rightarrow \Xi^0 + \gamma] \quad (29b)$$

$$= 4$$

The difference between pure 27 and pure 10 is then very neat (R_{Ξ^*} has even been inverted).

Though it is difficult to make definite predictions, in the 10-27 mixing case, without knowing more about the amount of mixing, values of $R_{Y_1^*}$ and R_{Ξ^*} small against 1 are characteristic of the 10 assignment. In other words, if $R_{Y_1^*}$ or R_{Ξ^*} turned out to be comparable with 1 or larger, this assignment would have to be abandoned.

From the experimental point of view, the detection of such electromagnetic decays as the ones discussed here is certainly difficult, especially for the Ξ^{*c} , but a rough value for $R_{Y_1^*}$ is not perhaps impossible to obtain (notice that neither absolute rates nor very accurate measurements are needed).

CONCLUSION

We have evoked here some problems related to the classification of $\frac{3}{2}^+$ baryons according to SU_3 representations.

While only ten equally spaced $\frac{3}{2}^+$ baryons were known, the choice of the representation 10 was unique.

Once other particles are found in the same energy region (preliminary evidence for a Y_2^* has recently been announced ⁴), one is led to question the preceding assignment.

One then finds that equal spacing is not characteristic of the decuplet, but could also hold if a strong 10-27 mixing was in operation.

When one adds this to the fact that the $\frac{3}{2}^+$ resonance widths are not correctly predicted when these particles are classified in a pure 10, one sees that such a scheme should be supported by other independent pieces of evidence.

Now, if the $Y_2^*(1415 \text{ MeV})$ is actually confirmed and has $J^P = \frac{3}{2}^+$, two possibilities are open: either strong 10-27 mixing takes place, and then one can predict two particles (a $T = \frac{3}{2} \Xi^*$ around 1550 MeV and a $Y = -2, T = 1 \frac{3}{2}^+$ baryon near 1690 MeV); or the decuplet and 27-plet do not mix appreciably, which could be understood in a higher symmetry group such as SU_6 . It should be emphasized, though, that the 10-27 mixing case is not much more successful than pure 10 concerning the widths.

Anyhow, the electromagnetic decays of Y_1^* and Ξ^* can provide a clear-cut test of the 10 assignment which is independent of the mass formula. These electromagnetic decays, though difficult to observe, are not perhaps outside present experimental reach, and their importance seems to deserve consideration.

ACKNOWLEDGEMENTS

We would like to thank Professors J. Prentki and Ph. Meyer for very useful discussions. One of us (H.R.R.) acknowledges a profitable stay at CERN, during which time part of this work was performed.

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- 2) If one starts from the $N^* \pi N$ width as given, one finds a discrepancy of a factor of 2 in the $\Xi^* \pi \Xi$ one, and the branching ratio $Y_1^* \rightarrow \pi + \Sigma / Y_1^* \rightarrow \pi + \Lambda$ is calculated to be around 15%, while experimentally it is much smaller and even compatible with zero ($4 \pm 4\%$)³⁾.
- 3) See, for instance, M.H. Alston et al., 1962 International Conference on High Energy Physics at CERN, p.311.
- 4) Yu-Li Pan and R.P. Ely, Phys. Rev. Lett. 13, 277 (1964).
- 5) In fact, static model considerations had already predicted a $T = 2$, $J^P = \frac{3}{2}^+ \pi \Sigma$ resonance⁶⁾. (Notice that here, unlike the $\pi \Xi$ case where the nearby lying $\bar{K} \Lambda$ and $\bar{K} \Sigma$ channels can be invoked to modify the original static model predictions, the $T = 2 \pi \Sigma$ channel is rather well isolated.) A similar argument comes from the fact that, when $\frac{3}{2}^+$ baryons are "explained" dynamically, in the exact unitary symmetry limit, by $\frac{1}{2}^+$ baryon exchange in pseudoscalar meson - $\frac{1}{2}^+$ baryon scattering, the force is attractive in the 27 representation (see references 7 and 8). Of course, when only a decuplet of $\frac{3}{2}^+$ baryons is wanted, one manages so that this attractive force is not sufficient to produce a pole in the corresponding amplitude. But it seems reasonable to suppose that, if a $T = 2 \pi \Sigma$ resonance exists, then it has $J^P = \frac{3}{2}^+$.
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- 9) Of course, one can think of another possible solution in which all $\frac{3}{2}^+$ baryons would enter a single representation, which would then have to be 27 (see reference 10). But then the Ω would have isospin 1 which, if not excluded, is less favoured than $T = 0$ (a doubly charged Ω should be easier to see experimentally than a singly charged one), and other particles would be predicted in regions where there seems to be none. (The mass formula for this isolated 27-plet does not strongly disagree with 1415 MeV as the mass of the Y_2^* , but it implies a strangeness + 1 baryon around 1100 MeV.)
- 10) S.L. Glashow and J.J. Sakurai, Nuovo Cim. 25, 337 (1962).
- 11) Another possible candidate for $J^P = \frac{3}{2}^+$ is the $Y_1^*(1660 \text{ MeV})$: the experimental situation is not settled yet in so far as its parity is concerned (see reference 12).
- 12) R. Armenteros, Report at the Dubna International Conference on High Energy Physics (1964).
- 13) This result is already mentioned in S. Coleman, S.L. Glashow and D.J. Kleitman, Phys. Rev. 135, B779 (1964).
- 14) As 27 appears only once in the tensorial product of 10 by 8, one has only one mixing parameter.
- 15) J.J. de Swart, Rev. Mod. Phys. 35, 916 (1963).
- 16) V. Gupta and V. Singh, Phys. Rev. 136, B782 (1964).
- 17) B. Diu, H.R. Rubinstein and J.L. Basdevant, to be published in Nuovo Cimento.

- 18) To be definite, let us give the way we have calculated the reduced widths. We start from a partial wave amplitude for pseudoscalar meson - $\frac{1}{2}^+$ baryon scattering (see, for example, reference 19):

$$T_{\frac{3}{2}^+} = \frac{W^2 e^{i\delta_{\frac{3}{2}^+}} \sin \delta_{\frac{3}{2}^+}}{q^3 [(W+M)^2 - \mu^2]}$$

where W is the total centre-of-mass energy, q the corresponding momentum, $\delta_{\frac{3}{2}^+}$ the phase-shift, and M and μ the masses of the $\frac{1}{2}^+$ baryon and pseudoscalar meson respectively. The elastic unitarity condition then writes:

$$\text{Im } T_{\frac{3}{2}^+}^{-1} = -\frac{q^3}{W^2} [(W+M)^2 - \mu^2]$$

We then define the reduced width g^2 of a $\frac{3}{2}^+$ resonance of mass M^* as the residue at the corresponding pole:

$$T_{\frac{3}{2}^+} \simeq -g^2 / [W - M^* + i\Gamma/2]$$

and relate the width Γ to it by the formula:

$$\Gamma = 2g^2 \frac{q^3}{M^{*2}} [(M^*+M)^2 - \mu^2]$$

Notice that g^2 has dimensions μ^{-2} . The values found when taking the mass of the pion as 1 are then: ²⁰⁾

$$\begin{aligned} g_{N^*\pi N}^2 &\simeq 3.1 \times 10^{-2} \quad \text{for } \Gamma_{N^*\pi N} \simeq 125 \text{ Mev.} \\ &\simeq 2.5 \times 10^{-2} \quad \text{for } \Gamma_{N^*\pi N} \simeq 100 \text{ Mev.} \end{aligned}$$

$$g_{Y_1^* \pi \Lambda}^2 \simeq 1.7 \times 10^{-2} \quad \text{for} \quad \Gamma_{Y_1^* \pi \Lambda} \simeq 50 \text{ Mev.}$$

$$g_{Y_1^* \pi \Sigma}^2 \lesssim 3.1 \times 10^{-3} \quad \text{for} \quad \Gamma_{Y_1^* \pi \Sigma} \lesssim 2 \text{ Mev.}$$

$$g_{\Xi^* \pi \Xi}^2 \simeq 6.1 \times 10^{-3} \quad \text{for} \quad \Gamma_{\Xi^* \pi \Xi} \simeq 7 \text{ Mev.}$$

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