EVIDENCE FOR A PION-NUCLEON INELASTIC S 31 RESONANCE

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From phase-shift analysis and from partial-wave dispersion relations, we have found the main content of the 800 MeV $\pi^+p$ shoulder to be an inelastic $S_{31}$ resonance. Such an object is required by $SU(6)$ in the $70^-$ multiplet to which the known $D_{13}$ resonance belongs.

We have previously published $\pi^+p$ phase-shifts up to 700 MeV $^1$. They were obtained by supplementing the experiments with peripheral calculations of the smaller partial waves $^2$. These phases have now been refined and extended to higher energies, using not only new experiments, but also greatly improved theoretical predictions based on partial wave dispersion relations.

Previous attempts to apply partial-wave dispersion relations at high energies have been frustrated by the lack of a theory of the inelastic processes. We have developed a semi-phenomenological technique, whereby the experimental phases and elasticities in a particular partial wave are simultaneously fitted by a parametrization which satisfies the dispersion relation exactly, and has the correct long-range forces $^2$. The short-range forces are treated as variables, subject to the unitary sum-rule $^3$. Further details have been given elsewhere $^4$. While this does not amount to a complete a priori prediction, it nevertheless gives strong consistency conditions on the data, enabling us to decide between rival sets of phases, and to interpolate from energies with complete experiments to those where they are incomplete. The difference from the previous work is that both rescattering and short-range forces have now been included, so that all partial waves can be predicted theoretically, and not only the small ones. Errors were placed on these theoretical predictions, and they were included in a second fit to the experiments. The results were then used to reevaluate the partial-wave dispersion relations, and so on until consistency was achieved. By this means we are able to obtain all the advantages of energy-dependent fits without the bias arising from their small number of parameters. The evaluation of the $S_{31}$ dispersion relation shown in Figure 1 was a 22 parameter fit (more than some people have in their entire solution), and it is
apparent from the graph that not all the experimental points are perfectly fitted even then. It must be emphasized that not everything can be fitted to dispersion relations, no matter how many parameters are used. The large number of parameters ensures that our results come from the dispersion relation itself, and not from the particular parametrization. The residual scatter in Figure 1 can probably be ascribed to the normalization errors of the experiments, especially at the lower energies.

Above 700 MeV, the \( \gamma^+ p \) experiments alone are not sufficient even to give well-defined phase-shifts, let alone unique ones. It is necessary either to analyse \( \gamma^+ p \), \( \gamma^- p \) and charge-exchange scattering simultaneously, or else to make extensive use of partial-wave dispersion relations. We did both. First, we performed phase searches at 870 and 990 MeV, where \( \gamma^+ \) and \( \gamma^- \) polarization\(^5\) and charge-exchange\(^6\) measurements are available, besides the differential cross-sections\(^7,8\). We found complete solutions with \( \chi^2 \) probabilities of 7.7% and 50% respectively, which appear to be almost, but not quite, unique\(^*\).

Partial-wave dispersion relations were then used to resolve the remaining ambiguities and interpolate to other energies. The resulting "theoretical data" (with errors on it) was added to the new Chilton experiments\(^3\) in a \( \gamma^+ p \) fit throughout this energy region.

The most interesting feature is in \( S_{31} \). The experimental points in Figure 1 show our latest phases and elasticities for this wave. We deliberately excluded the \( S_{31} \) theoretical predictions from the phase fits above 650 MeV, so as not to bias the conclusion. The results show an inelastic peak at 800 MeV, accompanied by a very rapid drop in the phase. The solid line shows a fit to the \( S_{31} \) dispersion relation using this data. The dotted line shows a dispersion relation prediction, made using only data below 700 MeV and a point at 990 MeV, before the numerical results of the Chilton experiments became available to us. The general

\(^*\) At 870 MeV there is obvious disagreement between Berkeley\(^7\) and Chilton\(^3\) \( \gamma^+ p \) differential cross-sections. We tried both, but our final fit includes only the latter.
form is plainly similar, though the inelastic peak is somewhat shifted. This shows that both the Chilton experiments, and the $S_{31}$ partial-wave dispersion relation, independently require this inelastic peak, so the evidence for it would appear rather convincing.

The interpretation is clearer if we plot the $S_{31}$ amplitude in the complex plane, normalized to its unitarity limit (Fig. 2). As is well known, a resonance would correspond to the amplitude describing a counterclockwise circle in the complex plane, as the energy increases. Its diameter determines the elasticity, according to the formula

$$\frac{\Gamma_{\text{in}}}{\Gamma_{\text{el}}} = \left(\frac{1}{\alpha}\right) - 1$$  \hspace{1cm} (1)

It appears that, in the 800 MeV region, $S_{31}$ does indeed describe part of a counterclockwise circle, but that this is displaced far to the left in the complex plane by the presence of a strongly repulsive background, so that the phase-shift does not pass through 90°. The dispersion relation analysis shows that this repulsive background comes from the left-hand cut. We can therefore eliminate most of it by subtracting the left-hand contribution from the real part. The broken line in Fig. 2 shows the result. The dotted line is a circle fitted visually to the most curved part of this trajectory. The points at higher energies were projected onto the nearest points of the circle. (This corresponds to assuming minimal energy-dependence of the background.) Interpreting the circle by the Breit-Wigner formula, the highest point is the mass of the resonance, the rightmost point determines its half-width, and the diameter gives the branching-ratio by Eq. (1). The result, in terms of total mass instead of kinetic energy, is

$$M = 1692 \text{ MeV}, \quad \Gamma_{\text{tot}} = 230 \text{ MeV}, \quad \frac{\Gamma_{\text{in}}}{\Gamma_{\text{el}}} = 1.27$$  \hspace{1cm} (2)
We emphasize, however, that these numbers (especially the branching-ratio) are very dependent on just how one fits a circle to Fig. 2, and should therefore be regarded with caution. Note that the mass, according to this definition, is considerably higher than the inelastic peak, which is at 1628 MeV total mass.

The other $T = 3/2$ partial waves (Fig. 3) are less interesting. $P_{37}$ is attractive and rising throughout the region, and has reached $15^\circ$ at 1 GeV. $P_{33}$, $D_{35}$ and $P_{31}$ acquire some inelasticity above 700 MeV. Together with the inelastic peak in $S_{31}$, these make up the observed shoulder. $D_{33}$ is already quite inelastic at lower energies $^1$.

In the $T = 1/2$ state, our fits at 698, 870 and 990 MeV can apparently only be explained in terms of genuine resonances in $S_{11}$ and $P_{15}$ (phases going through $90^\circ$), and a highly absorptive resonance in $D_{15}$ (phase going through $0^\circ$). The $S_{11}$ resonance is quite distinct from the virtual bound-state at the $N\gamma$ threshold found previously $^1$, and is confirmed by the $S_{11}$ dispersion relation. We intend to give details of these subsequently, when fits to the $T=\bar{p}$ and charge-exchange data at intervening energies are complete.

Resonances in $S_{31}$ and $S_{11}$ have been predicted by SU(6) $^9,10$. Together with the known $D_{13}$ resonance, they complete the strangeness-zero part of the 70$^-$ multiplet. The mass of the $S_{31}$ resonance disagrees with the SU(6) predictions $^9$, but this may be because of incorrect identification of the other members of the multiplet used as input in the mass-formula. It has been pointed out $^11$ that the $\Delta J/N\pi$ branching-ratios of the $S_{31}$ and $D_{13}$ resonances provide very important tests of "moving group" generalizations of SU(6).

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After completion of this work, we received a Saclay preprint [2] which reached the same qualitative conclusion about the $S_{31}$, $S_{11}$, $D_{15}$ and $P_{15}$ waves. In one respect their work is better than ours, for the $T = \frac{1}{2}$ part of their analysis contains additional energies. However, in other respects it is worse, for we have fitted partial-wave dispersion relations as well as the experiments. There are considerable quantitative differences between our phases and theirs, due to this.
REFERENCES


11) D.V. Volkov and M. Gell-Mann, private communications.

Figure 1: Our latest $S_{3/1}$ phases (a) and elasticities (b). The solid line is a dispersion relation fit, the dotted line a dispersion relation prediction.

Figure 2: Behaviour of $S_{3/1}$ in the complex plane. Solid line: complete amplitude. Broken line: right-hand cut contribution. Dotted line: approximate fit by a circle.

Figure 3: Dispersion relation fits to our other $T = 3/2$ phases (a) and elasticities (b).