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THE DEUTERON POLE IN THE NUCLEON-NUCLEON SCATTERING AMPLITUDE

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A B S T R A C T

This paper deals with the deuteron as a bound state in nucleon-nucleon scattering. The coupled $\ell = 0$ and $\ell = 2$ amplitudes of the $J = 1, I = 0$ NN state have been treated within the framework of relativistic partial wave dispersion relations using the matrix extension of the N/D technique in order to solve the coupled integral equations. As left-hand singularities we take into account the contributions of one-boson-exchange terms. Furthermore, the triplet scattering length is introduced as a subtraction constant.

I. INTRODUCTION

In field theory we expect any partial wave amplitude, when it is analytically continued below threshold, to have a pole on the real axis when there exists a stable particle with the same quantum numbers. This stable particle can be either an elementary particle or a bound state. From the point of view of relativistic partial wave dispersion relations, the interpretation of such a pole is quite different, depending on which of the two cases is present. If the particle is elementary the pole has to be added to the left-hand singularities and its position and residue can be chosen arbitrarily. In the case of a bound state, however, the pole is generated dynamically. [We have tacitly assumed that a pole corresponds to a bound state, when its position and residue can be calculated from given left-hand discontinuities using unitarity and dispersion relations.] In the N/D ansatz for the partial wave amplitude, which is conveniently used to solve the dispersion equation, the above distinction is reflected by the fact that an elementary particle pole enters as a singularity into the N function while a bound state pole develops as a zero of the D function.

The successful description of elastic nucleon-nucleon scattering¹⁻³⁾ with one-boson-exchange forces leads us to ask whether the deuteron can be produced by the same forces. We used the framework of relativistic partial wave dispersion relations and solved the coupled integral equations of the two channel $J = 1, I = 0$ nucleon-nucleon triplet state using the matrix extension of the N/D method proposed by Bjorken⁴⁾. Besides the forces generated by $\pi, \sigma, \eta, \rho, \omega$ and ϕ exchange, the triplet scattering length is introduced as a subtraction constant. This is not in contradiction with our definition of a bound state, since Goldberger, Grisaru, MacDowell and Wong⁵⁾ have shown how the scattering length can be calculated in principle by means of a relation existing between several partial waves at $\vec{p}_{\text{CMS}}^2 = -M^2$ (M is the nucleon mass). However, since our approximations to the left-hand

singularities are clearly unreliable in this region, we have to supply the scattering length as an additional parameter. Because the scattering length is explicitly built in, it is not very surprising to find the deuteron occurring as a bound state pole. But clearly the exact position of this pole is strongly dependent upon the input forces. From the residues at this pole the two constants entering the proton-neutron-deuteron vertex on the mass shell are determined.

In Section II we collect some necessary definitions and formulae concerning the deuteron.

Section III contains the partial wave decomposition of the deuteron pole and the dispersion relations used. Finally, in Section IV we compare and discuss the deuteron parameters resulting from the different sets of coupling constants determined in Refs. ¹⁻³⁾ and from one chosen according to some general principles.

II. DEFINITIONS

The relativistic proton-neutron-deuteron vertex on the mass shell is of the form

$$\begin{aligned} & \delta(q_p + q_n - q_D) \bar{u}(p) \langle n | f(0) | D \rangle \\ & = \delta(q_n + q_p - q_D) \frac{1}{(2\pi)^3} \bar{u}(n) \left[(\gamma_\xi^\dagger) F_1 + (q_n^\dagger) \frac{F_2}{2M} \right] C \bar{u}(p) \end{aligned} \quad (1)$$

The four-momentum vectors of the proton, neutron and deuteron are denoted by q_p , q_n and q_D respectively and ξ is the polarization vector of the deuteron. We use the metric $(ab) = a^0 b^0 - \vec{a} \cdot \vec{b}$ and γ matrices satisfying $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$; furthermore, $\bar{u}(p)u(p) = 2M$. Suppressing the spin indices as above, our states are normalized as $\langle p' | p \rangle = 2p_0 \delta(\vec{p}' - \vec{p})$. We write the non-relativistic deuteron wave function in the form

$$\Psi_{J_z}(\vec{r}) = \mathcal{N} \left\{ \frac{u(r)}{r} Y_0^0(\hat{r}) X_1^{J_z} + \frac{w(r)}{r} \sum_m \begin{pmatrix} 2 & 1 & 1 \\ J_z - m & m & J_z \end{pmatrix} Y_2^{J_z - m}(\hat{r}) X_1^m \right\} \quad (2)$$

where the notation used is obvious.

We introduce the Fourier-Bessel transforms of the radial functions

$$\begin{aligned} \tilde{u}(p) &= \frac{1}{\sqrt{2\pi}} \int_0^\infty j_0(pr) u(r) r dr \\ \tilde{w}(p) &= -\frac{1}{\sqrt{2\pi}} \int_0^\infty j_2(pr) w(r) r dr \end{aligned} \quad (3)$$

Then the ratio Q of the asymptotic part of the radial D and S wave functions is defined by

$$Q = \frac{\lim_{p^2 \rightarrow -\alpha^2} (p^2 + \alpha^2) \tilde{w}(p)}{\lim_{p^2 \rightarrow -\alpha^2} (p^2 + \alpha^2) \tilde{u}(p)} \quad (4)$$

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where $\alpha^2 = BM$, B is the binding energy of the deuteron and M the nucleon mass. Blankenbecler, Goldberger and Halpern⁶⁾ have derived the relation between Q and the constants F_1 and F_2 entering the proton-neutron-deuteron vertex (1). With the definitions we have used, it becomes

$$\frac{F_2}{F_1} = - \frac{3\sqrt{2} \frac{M}{B} Q + \left(1 + \frac{Q}{\sqrt{2}}\right)}{1 + \frac{Q}{\sqrt{2}}} \quad (5)$$

On account of the numerical values of α and Q , Eq. (5) can be simplified to

$$\frac{F_2}{F_1} \approx -3\sqrt{2} \frac{M}{B} Q \quad (5a)$$

III. THE $J = 1, I = 0$ PARTIAL WAVE AMPLITUDES

We consider the relativistic partial wave amplitudes for the $J = 1, I = 0$ triplet nucleon-nucleon state as functions of the variable $z = \vec{p}_{\text{CMS}}^2$

$$H(z) = \begin{pmatrix} h_{-}(z) & h_{\pm}(z) \\ h_{\pm}(z) & h_{+}(z) \end{pmatrix} \quad (6)$$

The function h_- is the S wave amplitude, h_+ the D wave amplitude and $h_{\pm}$ describes S-D transitions. Defining

$$\rho(x) = \frac{1}{2} \sqrt{\frac{x}{x+M^2}} \quad x \text{ real, } \geq 0$$

and neglecting production processes, we can write the unitarity relation in the form

$$\text{Im } H(x+i0) = \rho(x) H^+(x+i0) H(x+i0) \quad (7)$$

The residues of the deuteron pole, defined as

$$\lim_{z \rightarrow -\alpha^2} (z + \alpha^2) H(z) = \begin{pmatrix} r_- & r_{\pm} \\ r_{\pm} & r_+ \end{pmatrix} \quad (8)$$

can be calculated in terms of the constants F_1 and F_2 of the proton-neutron-deuteron vertex (1) using the helicity technique ⁷⁾

$$\begin{aligned} r_- &= \frac{1}{3} \{ R_{11} + 2R_{22} + 2\sqrt{2} R_{12} \} \\ r_+ &= \frac{1}{3} \{ 2R_{11} + R_{22} - 2\sqrt{2} R_{12} \} \\ r_{\pm} &= \frac{1}{3} \{ \sqrt{2} (R_{22} - R_{11}) - R_{12} \} \end{aligned} \quad (9)$$

where

$$\begin{aligned}
 R_{11} &= -\frac{1}{6} \frac{1}{2\pi} M^2 \left(F_1 + \frac{F_2}{2} \frac{B}{M} \right)^2 \\
 R_{12} &= -\frac{\sqrt{2}}{6} \frac{1}{2\pi} M^2 \sqrt{1 - \frac{B}{M}} F_1 \left(F_1 + \frac{B}{2M} F_2 \right) \\
 R_{22} &= -\frac{1}{3} \frac{1}{2\pi} M^2 \left(1 - \frac{B}{M} \right) F_1^2
 \end{aligned}
 \tag{10}$$

From the above, the ratio F_2/F_1 which determines Q through Eq. (5) is easily obtained

$$\frac{F_2}{F_1} = \frac{2M}{B} \left(\sqrt{2} \sqrt{1 - \frac{B}{M}} \frac{R_{12}}{R_{22}} - 1 \right)
 \tag{11}$$

The partial wave amplitudes h are analytic in the cut z plane. We use the matrix extension of the N/D method and write

$$H(z) = N(z) D(z)^{-1}
 \tag{12}$$

where N and D are now 2×2 matrices. Then, the relations

$$\begin{aligned}
 N(z) &= N(0) + \frac{z}{\pi} \int_{-\infty}^{-\frac{1}{4}m_\pi^2} dx' \frac{\Delta H(x') D(x')}{x'(x'-z)} \\
 D(z) &= I - \frac{z}{\pi} \int_0^\infty dx' \rho(x') \frac{N(x')}{x'(x'-z)}
 \end{aligned}
 \tag{13}$$

display the assumed analyticity properties. Since we subtracted at threshold ($z = 0$), the subtraction matrix $N(0)$ contains only one parameter because of the threshold behaviour of the partial wave amplitudes. This parameter is simply related to the triplet S wave scattering length a_t

$$N(0) = \begin{pmatrix} -2M a_t & 0 \\ 0 & 0 \end{pmatrix} \quad (14)$$

In order to solve the matrix N/D equations (13) numerically, we have approximated the partial wave projections of the one-boson-exchange terms in the physical region by a series of poles. The integral equations for the $J = 1, I = 0$ NN state then become a system of linear equations. We found that ten poles were sufficient to get a very good approximation for $0 < z < 800 \text{ m}\pi^2$. The positions of the poles on the left-hand cuts were given a priori; the residues were determined by minimizing the square of the difference between the one-boson-exchange terms and the pole approximation. The solutions of the N/D equation satisfy the unitarity condition (7) but the D wave has not the correct threshold behaviour, since our dispersion relations have only been subtracted once. In order to remove this defect, we have modified one of the residues of the poles, which approximates the D wave projection of the one-boson-exchange terms in such a way that the correct threshold behaviour appears. This is the simplest way of including very roughly necessary contributions of multimeson exchange in order to fulfil the threshold condition, which otherwise would be violated because of our approximate treatment of the left-hand discontinuities. We have furthermore introduced a common smooth cut-off for the vector mesons by multiplying the partial wave projections of all vector meson exchange terms by the factor $1/(1 + \frac{z}{x_c})$

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x_c being the cut-off parameter. We shall discuss all the method reviewed above in detail in a later paper, where we shall also deal with the S and D wave phase shifts and the mixing parameter.

The parameters entering our calculations are:

- 1) the triplet NN scattering length a_t ;
- 2) the masses and coupling constants to the $N\bar{N}$ system of the exchanged bosons;
- 3) the cut-off parameter x_c .

The scattering length is fixed experimentally to be $a_t = 5.38f$.

As coupling constants we have taken those of Refs. 1-3). In addition to these three sets of coupling constants, we have chosen another one on the basis of more general principles:

$$\begin{aligned} \frac{g_{\sigma_0}^2}{4\pi} &= 4.7, \quad m_{\sigma_0} = 400 \text{ MeV}; \quad \frac{g_{\pi}^2}{4\pi} = 14.4, \\ m_{\pi} &= 137 \text{ MeV}; \quad \frac{g_{\eta}^2}{4\pi} = 2.0, \quad m_{\eta} = 550 \text{ MeV}; \\ \frac{g_{\rho_1}^2}{4\pi} &= 1.8, \quad \frac{g_{\rho_2}^2}{4\pi} = 13.5, \quad m_{\rho} = 750 \text{ MeV}; \\ \frac{g_{\omega_1}^2}{4\pi} &= 4.0, \quad \frac{g_{\omega_2}^2}{4\pi} = 0.0, \quad m_{\omega} = 780 \text{ MeV} \end{aligned} \quad (15)$$

As Durso and Signell⁸⁾ have shown that the exchange of a σ_0 ($J^P = 0^+$, $I = 0$) is effectively equivalent to the 2π S wave contribution in the Amati-Leader-Vitale⁹⁾ treatment of the higher NN partial waves. SU_3 predicts $g_{\eta}^2/4\pi \approx 1 - 2$ and universality $g_{\rho_1}^2/4\pi \approx 1.8$. From a vector meson

model for the electromagnetic form factors of the nucleon, one can deduce the ratio $g_{\rho 2}^2 / g_{\rho 1}^2 = (\mu_p - \mu_n)^2 = 13.5$. Only $g_{\omega 1}$ is not predetermined. (Our ω is the combined ω and ϕ contribution.) It turns out that the deuteron parameters are nearly independent of $g_{\omega 1}$ in our set (15) $\pm \approx 5\%$ change for $1 < (g_{\omega 1}^2)/(4\pi) < 9$. We then have only one free parameter, the cut-off x_c . It is chosen so as to give good agreement for the phase shifts, but the deuteron parameters depend very weakly upon it. In the table below the calculated deuteron parameters are given: the binding energy B , the MND coupling constant F_1 and the asymptotic value of the D/S ratio Q for some values of the cut-off x_c .

	B in $\sqrt{\text{MeV}}$	F_1	Q	$\frac{x_c}{\frac{2}{m_\pi}}$
Scotti-Wong	2.42	2.18	0.020	5
	2.36	2.16	0.021	20
Bryan	2.40	2.20	0.023	5
	2.04	1.93	0.024	20
Sawada et al.	2.00	1.91	0.030	5
	2.10	1.93	0.026	20
set (15)	2.36	2.15	0.024	5
	2.34	2.15	0.023	20

The coupling constants for the ρ and η meson given by Scotti and Wong seem to be rather big. Therefore, the asymptotic D/S ratio Q is small because ρ and η exchange contribute in the opposite direction to π exchange, which is mainly responsible for the asymptotic D/S ratio. In an earlier calculation Wong¹⁰⁾ has used the low energy behaviour of the $3S_1$ wave (i.e., scattering length and effective range) in order to calculate the ratio Q . He gets a value of $Q = 0.029$. Gourdin, le Bellac, Renard and Tran Thanh Van¹¹⁾ have determined Q by comparison with photodisintegration data and get $Q = 0.0268$. We also calculated the deuteron constants taking only into account the π exchange. The result is: $B = 1.66$ MeV and $Q = 0.031$.

It may be of some interest to compare the ratio Q of this calculation with a result of Blankenbecler and Cook¹²⁾. They used dispersion relations for the proton-neutron-deuteron vertex and considered only the anomalous cut due to the exchange of a single π meson. This method^{*)} gives $Q = 0.037$.

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*) Blankenbecler and Cook have neglected terms vanishing on the mass shell but contributing to the dispersion integrals. These contributions shift their quoted value from 0.030 to 0.037.

R E F E R E N C E S

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