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LOW ANGULAR MOMENTUM
NUCLEON-NUCLEON SCATTERING

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ABSTRACT

The two states of elastic nucleon-nucleon scattering containing S waves - the single channel $J^P = 0^+$, $I = 1$ state and the two channel $J^P = 1^+$, $I = 0$ state - are treated within the framework of relativistic partial wave dispersion relations. A ND^{-1} ansatz is used for the partial wave amplitudes, where the left-hand singularities are generated by one-boson-exchange. A technique suggested by Schmidt's treatment of Fredholm integral equations is used for solving the integral equations.

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I. INTRODUCTION

The forces generated by one-boson-exchange appear to provide a fair description of elastic nucleon-nucleon scattering in the low-energy region ¹⁻³⁾ ($E_{\text{Lab}} \lesssim 350$ MeV). Considering only one-boson-exchange forces, one assumes that the exchange of a multimeson system contributes significantly only when a resonance is formed in it. This resonance is then treated as a particle to be exchanged. Relativistic partial wave dispersion relations provide a dynamical scheme for the treatment of two-body scattering processes at low energies. Within this framework, we have investigated the $J^P = 0^+$, $I = 1$ state and the two channel $J^P = 1^+$, $I = 0$ state of the nucleon-nucleon system, using forces generated by π , σ , η , ρ , ω and ϕ exchange. The treatment of these low angular momentum states can serve as a test case for the different sets of parameters used in Refs. ¹⁻³⁾. In addition to them we have considered another set of coupling constants chosen according to general principles.

A new technique is introduced for solving the integral equations which appear where the ND^{-1} ansatz for the partial wave amplitudes is used. This technique is especially suitable for many channel processes.

In Section II we introduce the basic definitions and integral equations and develop in detail the technique used for their solution. The forces we considered are described in Section III, and the additional set of coupling constants we used is discussed.

In Section IV we give qualitative arguments on the relative contributions of the one-boson-exchange forces in the various partial waves considered and discuss the quantitative results of the calculations.

In Appendix A we collect some formulae of E. Schmidt's method, which underlies our technique for the solution of the integral equations.

Finally, Appendix B contains a kinematic integral.

II. BASIC EQUATIONS

The partial wave decomposition of the relativistic nucleon-nucleon scattering amplitude is well known ⁴⁾. It can be conveniently performed using the helicity formalism ⁵⁾. We consider the partial wave amplitudes as functions of the variable z , which in the physical region is equal to the momentum squared of one nucleon in the centre-of-mass system.

$$N_1 + N_2 \rightarrow N_1' + N_2'$$

$$P_1 + P_2 = P_1' + P_2'$$

$$P_i^2 = M^2$$

$$i = 1, 2$$

(II.1)

$$P_i'^2 = M^2$$

$$S = (P_1 + P_2)^2$$

$$z = \frac{1}{4} (S - 4M^2)$$

This paper deals only with the two states of the nucleon-nucleon system containing S waves. These are:

$$J^P = 0^+, I=1: \quad h(z)$$

$$J^P = 1^+, I=0: \quad H(z) = \begin{pmatrix} h_{-}(z) & h_{-}(z) \\ h_{+}(z) & h_{+}(z) \end{pmatrix} \quad (II.2)$$

In the matrix H for the coupled triplet amplitudes, h_{-} , h_{+} and h_{+} describe the angular momentum transitions $J-1 \leftrightarrow J-1$, $J-1 \leftrightarrow J+1$ and $J+1 \leftrightarrow J+1$ respectively.

As we are interested only in the energy region up to $E_{\text{Lab}} \approx 350$ MeV we will neglect production processes and use purely elastic unitarity conditions:

$$\text{Im } h(x+i0) = \rho(x) |h(x+i0)|^2$$

$$\text{Im } H(x+i0) = \rho(x) H^\dagger(x+i0) H(x+i0) \quad (\text{II.3})$$

for

$$x \geq 0$$

where $\rho(x)$ is a kinematical function

$$\rho(x) = \frac{1}{2} \sqrt{\frac{x}{x+M^2}} \quad (\text{II.4})$$

which together with the unitarity relation (II.3) fixes our normalization of the partial wave amplitudes. The S matrix elements are related to the partial wave amplitudes (II.2) by the equations

$$S_0^{(1)}(x+i0) = 1 + 2i \rho(x) h(x+i0) \quad (\text{II.5})$$

$$S_1^{(3)}(x+i0) = 1 + 2i \rho(x) H(x+i0)$$

They can therefore be parametrized by ⁶⁾

$$S_0^{(1)}(x+i0) = e^{2i\delta_0}$$

$$S_1^{(3)}(x+i0) = \begin{pmatrix} \sqrt{1-\rho_1^2} e^{2i\theta_0^1} & i\rho_1 e^{i(\theta_0^1+\theta_2^1)} \\ i\rho_1 e^{i(\theta_0^1+\theta_2^1)} & \sqrt{1-\rho_1^2} e^{2i\theta_2^1} \end{pmatrix} \quad (\text{II.6})$$

4.

The phase shifts δ_0 , θ_0^1 , θ_2^1 and the mixing parameter ϱ_1 are all real functions of x .

The partial wave amplitudes (II.2) are considered as analytical functions in the cut z plane, with cuts on the real axis for $0 \leq x < +\infty$ and $-\infty < x \leq -\frac{1}{4}m^2$. But, as was shown by Goldberger, Grisaru, MacDowell and Wong⁽⁴⁾, in addition to the dynamical singularities, the coupled triplet amplitudes have a purely kinematical branch point at $z = -M^2$. This could be avoided, at least in principle, by going back to the helicity amplitudes and treating those as analytical functions. Following such a procedure, however, one would run into difficulties with the threshold properties of the partial waves. Therefore we prefer to keep this kinematical singularity in the partial wave amplitudes since it is in any case far removed from the considered energy region, compared to the left-hand singularities which we will actually take into account.

We turn now to the formulation of the N/D equations. Let us begin with the simpler case of the uncoupled singlet amplitude in order to illustrate in some detail our method for solving the integral equations. Because of unitarity the amplitude is bounded on the positive real axis by a constant. This is assumed to be true going to infinity in every direction of the complex z plane.

We write

$$h(z) = \frac{n(z)}{d(z)} \quad (\text{II.7})$$

where the functions n and d display the assumed analyticity properties of $h(z)$

$$n(z) = n(k) + \frac{z-k}{\pi} \int_{-\infty}^{-\frac{1}{4}m_{\pi}^2} dx' \frac{\sigma(x') d(x')}{(x'-k)(x'-z)}$$

$$d(z) = 1 - \frac{z-k}{\pi} \int_0^{\infty} dx' \rho(x') \frac{n(x')}{(x'-k)(x'-z)} \quad (\text{II.8})$$

$$-\frac{1}{4}m_{\pi}^2 < k \leq 0$$

We will specify the subtraction point k later. $\sigma(x)$ is the discontinuity of the partial wave amplitude h on the left-hand cut

$$\sigma(x) = \Delta h(x) \quad \text{for } -\infty < x \leq -\frac{1}{4}m_{\pi}^2 \quad (\text{II.9})$$

We make the assumption that σ vanishes for $x \rightarrow -\infty$; then $b(z)$ is defined as

$$b(z) = \frac{1}{\pi} \int_{-\infty}^{-\frac{1}{4}m_{\pi}^2} dx' \frac{\sigma(x')}{x'-z} \quad (\text{II.10})$$

Inserting the d equation into the n equation provides an integral equation for n alone. Using (II.10) we get:

$$\frac{n(z)}{z-k} = \frac{b(z) - b(k) + n(k)}{z-k} + \frac{1}{\pi} \int_0^{\infty} dx' \rho(x') \frac{b(x') - b(z)}{x'-z} \frac{n(x')}{x'-k} \quad (\text{II.11})$$

This, because of our assumption (II.10), is an integral equation of the Fredholm type for $\frac{n(z)}{z-\lambda}$. We will use a method due to E. Schmidt (see Appendix A) to solve this integral equation. The idea is to subtract a suitably chosen separable kernel from the original one, such that the difference of the kernels is "small" enough for the Neumann series of the related resolvent operator to exist. Then the solution of the integral equation (II.11) can be calculated from this resolvent purely algebraically. The special structure of the kernel in Eq. (II.11) and the integral representation (II.10) of $b(z)$ suggest an approximation of $b(z)$ by a series of poles. This gives

$$\tilde{b}(z) = \sum_{\ell=1}^L \frac{r_{\ell}}{z-x_{\ell}} ; \quad x_{\ell} < -\frac{1}{4} m_{\pi}^2 \quad (\text{II.12})$$

with the condition (on the residues r_{ℓ} and the positions x_{ℓ} of the poles) that

$$\int_0^{\infty} |b(x) - \tilde{b}(x)|^2 dx < \epsilon \quad (\text{II.13})$$

If ϵ is sufficiently small then the degenerate kernel

$$\begin{aligned} k(x, z) &= \frac{1}{\pi} \rho(x) \frac{\tilde{b}(x) - \tilde{b}(z)}{x - z} \\ &= -\frac{1}{\pi} \rho(x) \sum_{\ell=1}^L \frac{r_{\ell}}{(x-x_{\ell})(z-x_{\ell})} \end{aligned}$$

will lead to a small enough difference between the original kernel and the degenerate kernel exhibited above. Then the solution of the integral equation (II.11) is given by formula (A.8) of Appendix A. Now it is easy, for the $b(x)$ we will consider, to find a $\tilde{b}(x)$ such that the

difference $|b(x) - \tilde{b}(x)|$ is smaller than 10^{-3} in the energy range from 0 to 30 GeV. From a physical point of view it is therefore justified to identify $\tilde{b}(x)$ with $b(x)$. This implies putting $F_1(\lambda) = 0$ in the solution (A.8) of Appendix A. But if we do so our procedure is equivalent to inserting the n equation of (II.8) into the d equation putting $\sigma(x) = \Delta \tilde{b}(x)$. This reduces the integral equation for d into a system of L linear equations. We shall also fix the subtraction point at threshold. Then we get:

$$n(z) = n(0) + \sum_{\ell=1}^L R_{\ell} \frac{z}{z-x_{\ell}}$$

$$d(z) = 1 - \frac{z}{\pi} n(0) F(0, z) - \frac{z}{\pi} \sum_{i=1}^L R_i F(x_i, z) \quad (\text{II.14})$$

where the residues R_i are determined by

$$R_j = \frac{r_j}{x_j} \left[1 - \frac{x_j}{\pi} n(0) F(0, x_j) \right] - \frac{1}{\pi} \sum_{i=1}^L R_i F(x_i, x_j) r_j$$

$$j = 1, 2, \dots, L \quad (\text{II.15})$$

The functions $F(x, y)$ are defined in Appendix B. Because we have subtracted n and d at threshold, the subtraction constant $n(0)$ is simply related to the singlet scattering length a_s

$$n(0) = -2M a_s \quad (\text{II.16})$$

The procedure developed above can now be conveniently generalized to the two channel problem of the triplet amplitudes. Using the matrix extension by Bjorken ⁷⁾ we write

$$H(z) = N(z) D(z)^{-1} \quad (\text{II.17})$$

where N and D are now 2×2 matrices. Then

$$N(z) = N(0) + \frac{z}{\pi} \int_{-\infty}^{-\frac{1}{4} m_{\pi}^2} dx' \frac{\Sigma(x') D(x')}{x'(x'-z)} \quad (\text{II.18})$$

$$D(z) = 1 - \frac{z}{\pi} \int_0^{\infty} dx' \rho(x') \frac{N(x')}{x'(x'-z)}$$

Here $\Sigma(x)$ is the discontinuity matrix of $H(z)$ on the left-hand cut. It is assumed to vanish as $x \rightarrow -\infty$. Then

$$B(z) = \frac{1}{\pi} \int_{-\infty}^{-\frac{1}{4} m_{\pi}^2} dx' \frac{\Sigma(x')}{x'-z} \quad (\text{II.19})$$

Because of the threshold behaviour of the partial wave amplitudes the matrix $N(0)$ introduced by the subtraction at $z = 0$ has the form

$$N(0) = \begin{pmatrix} -2M a_t & 0 \\ 0 & 0 \end{pmatrix} \quad (\text{II.20})$$

with a_t being the triplet scattering length. Again, as in the single channel case, we approximate $B(z)$ by a number of poles $\tilde{B}(z)$ such that on the right-hand cut the difference between B and $\tilde{B}$ is sufficiently small to identify $\tilde{B}$ with B

$$B(z) \Rightarrow \tilde{B}(z) = \sum_{i=1}^L \frac{1}{z - x_i} r(i) \quad (\text{II.21})$$

Inserting $N(z)$ of (II.18) into $D(z)$ one obtains the solution

$$N(z) = N(0) + \sum_{k=1}^L \frac{z}{z - x_k} R(k) \quad (\text{II.22})$$

$$D(z) = 1 - \frac{z}{\pi} N(0) F(0, z) - \frac{z}{\pi} \sum_{i=1}^L R(i) F(x_i, z)$$

where the new residue matrices $R(i)$ are defined by the set of linear matrix equations

$$R(j) + \frac{1}{\pi} \sum_{i=1}^L r(i) F(x_j, x_i) R(i) = \frac{r(j)}{x_j} \left[1 - \frac{x_j}{\pi} F(0, x_j) * N(0) \right] \quad (\text{II.23})$$

Because of the subtraction performed at threshold with the subtraction matrix (II.20) the amplitudes $H_{11} = h_-$ and $H_{12} = H_{21} = h_+$ have the required threshold behaviour, independently of any approximation to

10.

the left-hand cuts. But this is not true for $H_{22} = h_+$. There the additional condition

$$\sum_{k=1}^L \frac{R_{22}(k)}{X_k} = 0 \quad (\text{II.24})$$

must be fulfilled in order to have $h_+(z) \sim z^2$ at threshold. We shall come back to this point in the following section.

III. INPUT FORCES

The assumption underlying the one-boson-exchange approximation to the left-hand singularities of the partial wave amplitudes is that only the resonances observed in the multimeson states contribute significantly to the spectral functions of the multimeson exchange terms. Treating the exchanged resonances as elementary particles with the corresponding quantum numbers, one meets for spin $J \geq 1$ the familiar unbounded partial wave projections. We believe, however, that this divergence difficulty is due to our very approximate treatment of the spectral functions and is not an inherent one. Clearly we know nothing about the damping which should occur in the spectral functions. But it is nevertheless hoped that the influence of this unknown damping mechanism, at least in the low energy part of the physical region, can be simulated by a simple cut-off. We shall consider only spin 0 and spin 1 exchange and multiply all contributions of the exchanged vector mesons by a common cut-off of the form $1/(1+z/x_c)$. This leads then to integral equations for the N functions which are of the Fredholm type. The particles we exchanged are π , η , σ , ρ , ω and ϕ . While η , ρ , ω and ϕ are well-established resonances, the $J^P = 0^+ \ 2\pi$ states called σ (σ_0 with $I = 0$, σ_1 with $I = 1$) have not yet been definitely observed as resonances. The force generated by the exchange of the $J^P = 0^+$, $I = 0 \ 2\pi$ system plays a crucial role and in all dynamical treatments of nucleon-nucleon scattering it had to be introduced. In a completely degenerate parity doublet theory ⁸⁾ the strength of an exchanged σ_0 in the 1S_0 state, for example, would be a factor $4M^2/m^2$ times that of the exchanged π meson!

Although we may be allowed to introduce the σ_0 as a particle in case it is a true resonance, in the case of a strong $\pi\pi \ S$ wave enhancement, however, we would have to introduce a continuum contribution. Here we consider the resonance case only. This need not be understood as a restriction but just as a convenient parametrization. Durso and Signell ⁹⁾ have shown the effective equivalence of the σ_0

mechanism and that of the BCL ¹⁰⁾ 2π S wave contribution in the Amati-Leader-Vitale treatment ¹¹⁾ of the higher NN partial waves. Scotti and Wong ¹⁾ also have treated both the σ_0 exchange and a 2π S wave continuum contribution and found that the parameters of the other exchanged particles remained very close to each other in the two treatments. In recent years there have been found several new boson resonances ¹²⁾ all with masses $\gtrsim 1$ GeV. Their contribution is presumably taken into account to a good approximation by introducing both the S wave scattering lengths as subtraction constants. The role of the ϕ meson can be absorbed in the ω exchange. We are aware of the fact that the usually adapted argument concerning the smallness of the Pauli-coupling of ω and ϕ to $N\bar{N}$ is not fully conclusive. In the language of SU_3 symmetry we can only conclude from the isoscalar magnetic form-factor that the octet component is very small because there only the octet contributes in the sum over the intermediate states ($|8\rangle\langle 8|$). In the nuclear force problem, however, both octet and singlet do appear ($|8\rangle\langle 8| + |1\rangle\langle 1|$). As we are considering only a few partial waves we cannot deal with this question and use the parameters of Refs. 1-5). Furthermore a set of coupling constants is used which is deduced from considerations other than NN scattering. We take a small $\gamma_{N\bar{N}}$ coupling constant, $g_{\gamma}^2/4\pi = 2$, in agreement with the commonly accepted F/D ratio in the pseudoscalar-baryon-baryon coupling. Universality connects the Dirac $Q_{N\bar{N}}$ coupling with the $Q_{\pi\pi}$ coupling and yields $g_{\rho_1}^2/4\pi = 1.8$. The ratio $(g_{\rho_2}/g_{\rho_1})^2$ can be deduced to be $(g_{\rho_2}/g_{\rho_1})^2 = (\mu_p - \mu_n)^2 = 13.5$ from a simple vector meson exchange model for the nucleon isovector form factors. As parameters for the σ_0 we take those determined by Durso and Signell. They found the effective equivalence of a 2π S wave contribution in the ALV treatment ¹¹⁾ of higher NN partial waves to the exchange of a σ_0 with $m_{\sigma_0} = 400$ MeV and $g_{\sigma_0}^2/4\pi = 4.72$. As ϕ and ω have the same quantum numbers and as ϕ is a very heavy particle we considered only the exchange of ω with $g_{\omega_1}^2/4\pi$ as a free parameter. In the table below we collect the different sets of parameters used. The masses are given in MeV.

	π	σ_0	η	σ_1	ρ	ω	Φ
	m_π	m_{σ_0}	m_η	m_{σ_1}	m_ρ	m_ω	m_Φ
	$\frac{g_\pi^2}{4\pi}$	$\frac{g_{\sigma_0}^2}{4\pi}$	$\frac{g_\eta^2}{4\pi}$	$\frac{g_{\sigma_1}^2}{4\pi}$	$\frac{g_{\rho_1}^2}{4\pi}$	$\frac{g_{\omega_1}^2}{4\pi}$	$\frac{g_{\Phi_1}^2}{4\pi}$
	$\frac{g_\pi^2}{4\pi}$	$\frac{g_{\sigma_0}^2}{4\pi}$	$\frac{g_\eta^2}{4\pi}$	$\frac{g_{\sigma_1}^2}{4\pi}$	$\frac{g_{\rho_2}^2}{4\pi}$	$\frac{g_{\omega_2}^2}{4\pi}$	$\frac{g_{\Phi_2}^2}{4\pi}$
Scotti-Wong	135 14	437 3.05	548 12.1	---	591 5.08 45.6	780 2.77 0	1020 2.26 0
Bryan	138 11.7	560 9.4	548 7.0	770 6.5	760 2.72 8.8	782 21.5 0	-----
Sawada et al.	137 14.4	600 13.1	---	---	760 6.75 26.2 ^{*)}	782 11.8 0.04	-----
F.P.	137 14	400 4.7	548 2	---	760 1.8 24.3	782 4 0	-----

*) $g_{\rho_1} g_{\rho_2}$ negative.

Our prescription for evaluating the contributions of the one-boson-exchange terms is to take into account at fixed S only the residues of the poles at $t = m_1^2$ and $u = m_1^2$ where m_1 is the mass of the exchanged particle i . The various coupling constants are defined by the following effective Lagrangian:

$$\begin{aligned} \mathcal{L}_{int} = & g_\pi \bar{\Psi} \gamma_5 \vec{\tau} \Psi \vec{\pi} + g_\eta \bar{\Psi} \gamma_5 \Psi \eta \\ & + g_{\sigma_0} \bar{\Psi} \Psi \sigma_0 + g_{\sigma_1} \bar{\Psi} \vec{\tau} \Psi \vec{\sigma}_1 \\ & + g_{\rho_1} \bar{\Psi} \gamma_\mu \frac{\vec{\tau}}{2} \Psi \vec{\rho}^\mu + \frac{g_{\rho_2}}{2M} \bar{\Psi} \sigma_{\mu\nu} \frac{\vec{\tau}}{2} \Psi \frac{1}{2} (\partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu) \\ & + g_{\omega_1} \bar{\Psi} \gamma_\mu \Psi \omega^\mu + g_{\phi_1} \bar{\Psi} \gamma_\mu \Psi \phi^\mu \end{aligned}$$

with

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \sigma_{\mu\nu} = \frac{i}{2} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$$

Finally we have to deal with the threshold behaviour of the D wave amplitude in the coupled triplet state. It is well known ¹³⁾ that in higher partial waves the approximation of the left-hand singularities by one-particle-exchange terms only is in contradiction with the partial wave dispersion relations. The reason is that the one-particle-exchange terms have the threshold properties themselves and because of this cannot cancel the positive definite contribution of the unitarity integral at threshold. Therefore one has to modify the one-particle-exchange terms by some continuum contribution in order to produce the correct threshold behaviour in the solution of the dispersion equations.

Clearly such a modification can be done in various ways. We will do it in the simplest way, representing the contributions of the uncorrelated multimeson exchange by one pole only whose position is fixed a priori and whose residue is then determined by requiring that the solution fulfills Eq. (II.24). We have investigated the influence of the position of this additional pole and we found that changing the position x_p of the pole between $-30m_\pi^2 < x_p < -2m_\pi^2$ changes the phases by less than ± 0.02 rad, if we allow a slight variation of the cut-off x_c (about 20%).

IV. RESULTS AND DISCUSSION

Although the dynamical cuts of the one-boson-exchange amplitudes determine the corresponding partial wave amplitudes in a rather complicated way via integral equations, it is nevertheless worthwhile to consider the partial wave projections $h(x)$ of the one-boson-exchange terms on the right hand cut as a rough estimate of their influence. In Figs. 1 to 4 the partial wave projections of the one-boson-exchange amplitudes are plotted. One should notice that $h(z)$ may be different from $b(z)$ (II.10) by a constant, such that $\lim_{x \rightarrow \infty} h(x) \neq 0$. But as in the integral equations only the difference $h(x) - h(z)$ appears, this constant actually does not contribute. [The same is valid in the triplet case for $H(z)$ and $B(z)$.] In all plots we have put $\epsilon_{BNN}^2/4\pi = 1$ for all exchanged bosons B.

The main attraction in the 3S_1 channel is produced by σ_0 and ρ exchange and the repulsion by ω (and ϕ) exchange and by σ_1 exchange, if the latter is included. The 3S_1 - 3D_1 mixing (tensor force) receives its main contribution from π , η and ρ exchange, with the π acting in opposite direction to η and ρ . In the 3D_1 channel the exchanged π is already dominant, though the exchange of the other particles cannot be neglected. In the 1S_0 partial wave, σ_0 (and σ_1) exchange and the Pauli-coupled ρ exchange are strongly attractive. The repulsion, however, is generated by the Dirac-coupled ρ and ω (ϕ) exchange.

We found that ten poles were sufficient to give a very good approximation of $b(z)$ and $B(z)$. The positions x_p of the poles on the left hand-cut were given a priori and the residues were determined by minimizing the square of the difference $b(z) - \tilde{b}(z)$ in the interval $0 < z < 500 m_\pi^2$ according to (II.13). In the case of the triplet state the same was done for every element of the matrix $B(z) - \tilde{B}(z)$. The S wave scattering lengths are taken from experiment: $a_t = 5.38f$ and $a_s = -21 f$.

We want now to discuss the resulting 1S_0 and 3S_1 - 3D_1 phases calculated with the different sets of coupling constants introduced in Section III.

We compare our results with the phases of the modified phase shift analysis ¹⁴⁾. The 3S_1 - 3D_1 mixing parameter ϱ_1 is compared with that of the phenomenological potential approach of Breit et al. ¹⁵⁾. There exist essentially two very different curves for ϱ_1 representing the YLAN1 and the YLAN3M results, which show that this mixing parameter is far from being determined.

We start with the discussion of our results, using the set of coupling constants of Scotti and Wong. The 3S_1 and 3D_1 phases turn out to be nearly within the experimental errors, when the cut-off is chosen to be $x_c = 5 m_\pi^2$. The mixing parameter ϱ_1 , however, is very different from both the YLAN1 and YLAN3M values. It passes through zero at $E_{\text{Lab}} = 80$ MeV. This is due to the big Q and η coupling constants. A change of the cut-off to $x_c = 20 m_\pi^2$ would make the 3S_1 wave slightly better, but it would make the modulus of the 3D_1 phase too big (by a factor 1.5 at 300 MeV). The 1S_0 phase agrees with the experimental values rather well, if the cut-off is chosen between $10 m_\pi^2$ and $30 m_\pi^2$. A cut-off of $5 m_\pi^2$ would give too big a δ_0 phase (zero at 310 MeV). As the relative contributions of the vector mesons to the 1S_0 ($I = 1$) and the 3S_1 - 3D_1 ($I = 0$) channels are different and as Scotti and Wong have used a different cut-off for every vector meson, it is justified to have a somewhat different cut-off for the two states, i.e., $x_c = 5 m_\pi^2$ for the triplet and $x_c = 10 m_\pi^2$ for the singlet state. The investigation of the D functions shows that in the triplet state no ghost occurs, whereas there is one in the 1S_0 state at $z \approx -50 m_\pi^2$.

Bryan's set of coupling constants fits the 3S_1 and 3D_1 phases quite well, the mixing parameter being rather big, if a cut-off of $x_c = 5 m_\pi^2$ is chosen. The 1S_0 phase δ_0 , however, can only be reproduced if a very large cut-off of $x_c = 60 m_\pi^2$ is chosen. This is due to the introduction of an isovector scalar particle (σ_1), which

is strongly attractive in the 1S_0 channel and its influence has to be compensated by the repulsive ω exchange, already taken into account, with a very big coupling. The very different cut-off parameters $5 m_\pi^2$ and $60 m_\pi^2$ for the triplet and singlet states respectively do not speak in favour of Bryan's isovector scalar particle.

The set of coupling constants proposed by Sawada et al. reproduces the observed 3S_1 phase rather poorly. The zero of it is around 700 MeV for cut-offs between $5 m_\pi^2$ and $50 m_\pi^2$. This could be due to the fact that Sawada et al. have a very big σ_0 coupling and a different sign for the Dirac and Pauli-coupling constant of the ρ meson, which makes the interference term attractive. The 1S_0 phase has its zero also at too high an energy. The qualitative behaviour of both phases, however, is correct.

We have furthermore introduced another set of coupling constants with only $g_{\omega 1}$ left as a free parameter. This set produces the experimentally observed phases quite well, with a cut-off $x_c = 10 m_\pi^2$ and $g_{\omega 1}^2/4\pi = 4$. But also this set yields a ghost in the 1S_0 state at $z \approx -50 m_\pi^2$ while no ghost appears in the coupled triplet state. Comparing this set of coupling constants with that of Scotti and Wong, we have to mention that their set describes both low and high angular momentum states, while we have considered only the two states $J^P = 1^+$, $I = 0$ and $J^P = 0^+$, $I = 1$. One of the reasons why we investigated the low angular momentum states was to find a set of coupling constants which, though possibly insufficient in describing higher partial waves, can be used via SU_3 to treat the S wave effective ranges of hyperon-nucleon scattering. This would be possible if we assign the σ_0 to an SU_3 singlet⁸⁾.

The investigation of the N and D functions of the triplet state at negative values of z shows the deuteron pole. Its properties have been already published in an earlier paper¹⁶⁾.

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A P P E N D I X A

The Method of E. Schmidt

For completeness we collect here the essential formulae of the procedure due to E. Schmidt for solving an inhomogeneous integral equation of the Fredholm type. The details and proofs can be found, for example, in the book of Riesz and Nagy¹⁷⁾. It is convenient to use a Hilbert space notation. The equation to be solved is

$$f = g + \lambda K f \quad (\text{A.1})$$

where K is a completely continuous operator and λ an arbitrary complex number. The solution is obtained by the resolvent operator

$$f = (1 - \lambda K)^{-1} g \quad (\text{A.2})$$

Let us also introduce the solving operator $F(\lambda)$ which is simply connected with the resolvent operator by

$$1 + \lambda F(\lambda) = (1 - \lambda K)^{-1} \quad (\text{A.3})$$

For all λ having a modulus

$$|\lambda| < \frac{1}{\|K\|}$$

The result is:

$$F(\lambda)g = F_1(\lambda)g + \sum_{i,j=1}^N (\psi_i, [1 + \lambda F_1(\lambda)]g) T_{ij}(\lambda) [1 + \lambda F_1(\lambda)] \varphi_j \quad (\text{A.8})$$

$$T^{-1} = 1 - \lambda C$$

$$C_{ij} = (\psi_i, [1 + \lambda F_1(\lambda)] \varphi_j)$$

A P P E N D I X BKinematic Integral

In this appendix we give an explicit expression for the function $F(x,y)$ entering the N/D equations. It is defined by the following integral

$$F(z, \lambda) = \frac{1}{2} \int_0^{\infty} dx' \sqrt{\frac{x'}{x'+M^2}} \frac{1}{(x'-z)(x'-\lambda)}$$

M real, z, λ arbitrarily complex.

Performing the integration we get:

$$F(z, \lambda) = - \frac{\omega(z) - \omega(\lambda)}{z - \lambda}$$

$$\omega(z) = \sqrt{\frac{z}{z+M^2}} \left\{ \ln \left(\sqrt{\frac{z}{M^2}} + \sqrt{\frac{z}{M^2} + 1} \right) - i \frac{\pi}{2} \right\}$$

The cuts of the square roots are chosen as

We define the physical sheet requiring both $\sqrt{z}$ and $\sqrt{z+M^2}$ to have positive imaginary parts for $z = x+iy$ with $x, y > 0$.

$$(A^{-1}X)(I + X^{-1}A^{-1})^{-1} = \left(\sum_{i=1}^n \frac{1}{\lambda_i} \frac{1}{1 + \lambda_i^{-1}} \right)^{-1}$$

$$\frac{1}{\lambda_i} = \frac{1}{\lambda_i} \frac{1}{1 + \lambda_i^{-1}} = \frac{1}{\lambda_i + 1}$$

$$\left(\sum_{i=1}^n \frac{1}{\lambda_i + 1} \right)^{-1} = \left(\sum_{i=1}^n \frac{1}{\lambda_i + 1} \right)^{-1} = \left(\sum_{i=1}^n \frac{1}{\lambda_i + 1} \right)^{-1}$$

$$\frac{1}{\lambda_i + 1}$$

$$\frac{1}{\lambda_i + 1} = \frac{1}{\lambda_i + 1} = \frac{1}{\lambda_i + 1}$$

R E F E R E N C E S

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FIGURE CAPTIONS

Fig. 1 to Fig. 4 : Partial wave projections of the OBE amplitudes.

(Notice that without the constant contribution the partial wave projections of σ_0 , σ_1 , π and η vanish as $\ln z/z$ when z goes to infinity.)

$$\sigma_0 : \sigma_0(J = 0^+, I = 0) \text{ exch. amp. } g_{\sigma_0}^2/4\pi = 1$$

$$\sigma_1 : \sigma_1(J = 0^+, I = 0) \quad " \quad " \quad g_{\sigma_1}^2/4\pi = 1$$

$$\pi : \pi \text{ exch. ampl. } g_{\pi}^2/4\pi = 1$$

$$\eta : \eta \quad " \quad " \quad g_{\eta}^2/4\pi = 1$$

$$\rho, V : \rho \quad " \quad " \quad , \text{ pure vector-coupling contribution, } g_{\rho_1}^2/4\pi = 1$$

$$\omega, V : \omega \text{ exch. ampl., pure vector-coupling contribution, } g_{\omega_1}^2/4\pi = 1$$

$$\rho, T : \rho \text{ exch. ampl., pure tensor-coupling contribution, } g_{\rho_2}^2/4\pi = 1$$

$$\omega, T : \omega \text{ exch. ampl., pure tensor-coupling contribution, } g_{\omega_2}^2/4\pi = 1$$

$$\rho, JTV : \rho \text{ exch. ampl., interference of tensor and vector coupled contributions, } g_{\rho_1} \cdot g_{\rho_2}/4\pi = 1$$

$$\omega, JTV : \omega \text{ exch. ampl., interference of tensor and vector coupled contributions, } g_{\omega_1} \cdot g_{\omega_2}/4\pi = 1$$

* λ behind the meson symbol means that the OBE amplitude has been multiplied by a factor λ before being plotted.

The vector meson contributions have been calculated without cut-off.

Fig. 5 to Fig. 7 : Phase shifts for different sets of coupling constants as introduced in Section III.

S.W. Scotti and Wong's set.

B.S. Bryan's set.

F.P. Set of coupling constants deduced from other sources than NN scattering.

x_c is the cut-off parameter.

The experimental points are taken from Ref. ¹⁴).

Fig. 8 : The 3S_1 - 3D_1 mixing parameter Q_1 .

The curves YLAN3M and YLAN1 are deduced from phenomenological potentials ¹⁵).

FIG. 3

FIG 1.

FIG.6

FIG. 7

FIG.8