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ESNUCLEAR CORRELATION FUNCTIONS AND MUON CAPTUREL.L. Foldy ^{*)} and J.D. Walecka ^{**)}

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ABSTRACT

Properties of certain nuclear correlation functions which enter into the theory of muon capture in complex nuclei in the closure approximation are examined. The reliability of certain suggested methods for evaluating these correlation functions involving the Fermi gas approximation, and refinements of it, are assessed both from the point of view of internal theoretical self-consistency and by comparison with pertinent empirical information available from photodisintegration data on nuclei.

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The application of the closure approximation ^{1),2)} in the theory of muon capture by complex nuclei allows the total capture rate to be expressed in terms of Fourier transforms of certain two-particle nuclear correlation functions. The properties of these functions have been discussed at some length by Primakoff ¹⁾, Tolhoek ²⁾, Klein and Wolfenstein ³⁾, Luyten et al. ⁴⁾, and Bell and Løvseth ⁵⁾, but some of their relatively simple aspects appear to have passed unnoticed and are the subject of this note.

1. GENERAL CONSIDERATIONS

The contribution of the (effective) vector part of the muon capture interaction in the closure approximation is proportional to $1 - \frac{A}{2Z}F(\nu)$ where F is the function :

$$F(\nu) = -\frac{2}{A} \int \frac{d\hat{\nu}}{4\pi} \langle 0 | \sum_{i \neq j} \tau_i^{(+)} \tau_j^{(-)} e^{i\vec{\nu} \cdot (\vec{r}_i - \vec{r}_j)} | 0 \rangle \quad (1a)$$

$$= -\frac{2}{A} \int \frac{d\hat{\nu}}{4\pi} \left\{ \langle 0 | \sum_{i,j} \tau_i^{(+)} \tau_j^{(-)} e^{i\vec{\nu} \cdot (\vec{r}_i - \vec{r}_j)} | 0 \rangle - Z \right\} \quad (1b)$$

$$= \frac{2Z}{A} - \frac{2}{A} \int \frac{d\hat{\nu}}{4\pi} \sum_b \left| \langle b | \sum_j \tau_j^{(+)} e^{-i\vec{\nu} \cdot \vec{r}_j} | 0 \rangle \right|^2 \quad (1c)$$

Here A and Z are the mass and atomic numbers of the capturing nucleus, $|0\rangle$ represents its ground state and $|b\rangle$ any state, the sum on $|b\rangle$ running over all states, $\tau_i^{(+)}$, $\tau_i^{(-)}$, and $\vec{r}_i$ are isospin and position operators

2.

for the i^{th} nucleon, $\vec{v} = v \hat{v}$ where v is an average momentum of the emitted neutrino in the closure sense, and the integration on $\hat{v}$ is over the directions of the vector $\vec{v}$. Its magnitude may be estimated to lie in the neighbourhood of (80 ± 10) MeV/c, so that if R represents the radius of the nucleus, vR varies from about 0.9 in ^4He to 3.6 in ^{238}U . The function $F(v)$ can also be written as

$$F(v) = - \int \frac{d\hat{v}}{4\pi} \int d\vec{r} G(\vec{r}) e^{i\vec{v} \cdot \vec{r}} = - \int d\vec{r} G(\vec{r}) \frac{\sin vr}{vr} \quad (2)$$

where

$$G(\vec{r}) = \frac{2}{A} \langle 0 | \sum_{i \neq j} \tau_i^{(+)} \tau_j^{(-)} \delta(\vec{r} - \vec{r}_i + \vec{r}_j) | 0 \rangle \quad (3)$$

is then the two-particle correlation function referred to above.

We make the following observations concerning the function $F(v)$:

- (I) If the ground state of the nucleus is an eigenstate of isotopic spin T with $T_3 = -T$, then it follows from $T_- |0\rangle = 0$ and (1c) that

$$F(0) = - \int d\vec{r} G(\vec{r}) = \frac{2Z}{A} \quad (4)$$

which is equivalent to one of Tolhoek's "detailed normalization conditions".

(II) It is clear from (3) that the function $G(\vec{r})$ will vanish rapidly for any finite nucleus when r exceeds twice the radius R of the nucleus. In fact, on the basis of a model in which one has nucleons moving in a potential well, one expects that $G(r)$ will be bounded asymptotically by an exponential function $e^{-\lambda r}$ where $\lambda (> 0)$ is of the order $(2ME_B)^{1/2}/\hbar$ with M the mass of a nucleon and E_B is the binding energy of the most weakly bound nucleon. Thus, with E_B of the order of 8 MeV, λ will be of the order of 3×10^{-13} cm corresponding to a momentum of about 130 MeV/c.

If indeed $G(\vec{r})$ is asymptotically bounded by such an exponential, it then follows from the second form of Eq. (2) that $F(\nu)$ considered as a function of ν will be analytic within the strip of the complex ν plane lying between $\nu = -i\lambda$ and $\nu = i\lambda$. Since $F(\nu)$ must be an even function of ν , one concludes that it possesses a power series expansion

$$F(\nu) = F(0) - a\nu^2 - b\nu^4 - \dots \quad (5)$$

with radius of convergence $\geq \lambda$. The fact that $F(0) - F(\nu)$ behaves as ν^2 as $\nu \rightarrow 0$ is a stronger condition than Tolhoek's detailed normalization condition. The value of ν of interest in muon capture, 80 MeV/c, will generally lie within the circle of convergence of (5) *).

*) This may fail to be the case in such nuclei as ^2H and ^9Be which have an anomalously low threshold for nucleon emission. It is also possible that a subunit of the nucleus (deuteron or alpha particle, etc.) may have a sufficiently small binding energy that $(2ME_B)^{1/2}/\hbar$, with M the (reduced) mass of the subunit and E_B its binding energy, is less than the corresponding value for the most weakly bound nucleon, in which case λ will be expected to be of this order of magnitude rather than that estimated for nucleon emission. Even when the pertinent value of ν for muon capture lies outside the radius of convergence of the power

(continued next page)

- (III) For finite nuclei with ground state $T = 0$, one can make a reasonably accurate estimate of the value of the coefficient a in (5) by the use of empirical information. Starting from (1c), using isotopic spin invariance and the Wigner-Eckart theorem, we easily derive

$$a = \frac{2}{A} \int \frac{d\vec{\nu}}{4\pi} \sum_b |\langle b | \sum_j \tau_j^{(1)} \vec{\nu} \cdot \vec{r}_j | 0 \rangle|^2$$

$$= \frac{1}{A} \int \frac{d\vec{\nu}}{4\pi} \sum_b |\langle b | \sum_j \tau_j^{(1)} \vec{\nu} \cdot \vec{r}_j | 0 \rangle|^2 \quad (6)$$

where the sum on $|b\rangle$ now effectively runs over the $T = 1$ states of the parent nucleus ^{**}). Following an argument given in a previous paper, we

*) series, the integral (2) will still represent the analytic continuation of the power series to the pertinent value of ν , but information about the small ν behaviour of $F(\nu)$, which is the point of our later discussion, may not have much relevance to the estimation of $F(\nu)$ at the point required. We must also recognize the possibility that a nucleus may be a spontaneous alpha emitter or be spontaneously fissionable. In this case the appropriate E_B for determining λ will be negative, and the function $F(\nu)$ may not be strictly defined at all. If a muon capture rate is definable at all in such cases (and presumably it would have some observable meaning when the lifetime for muon capture is much shorter than the lifetime for alpha emission or fission) we speculate that the appropriate $G(\vec{r})$ and hence $F(\nu)$ are to be calculated by artificially closing the objectionable decay channel through some mathematical artifice.

**) Note that in the preceding argument, the fact that $F(0) - F(\nu)$ behaves as ν^2 for small ν is a consequence of the integration over all directions of $\vec{\nu}$. On the assumption that the ground state is $T = 0$, it follows from (6) that $F(0) - F(\vec{\nu}) = 1 - F(\vec{\nu})$ is of second degree in the components of $\vec{\nu}$ as $\vec{\nu}$ approaches zero even before the integration over the direction of $\vec{\nu}$ is carried out.

recognize the matrix elements occurring in the last form of (6) as those involved in the photoeffect in nuclei which allows the coefficient a to be expressed as an integral over the total photoabsorption cross-section (averaged over all nuclear orientations) $\sigma_\gamma(E)$ for photons of energy E :

$$a = \frac{1}{\pi^2 \alpha A} \int_0^\infty \frac{\sigma_\gamma(E)}{E} dE \quad (7)$$

where $\alpha = 1/137$ is the fine-structure constant. This equation is strictly true if $\sigma_\gamma(E)$ is the cross-section in the long wavelength dipole approximation, but in most cases the integral does not differ much if one inserts the total cross-section, or corrections to $\sigma_\gamma(E)$ can be applied. On the basis of certain nuclear models, Levinger^{6),7)} and Khokhlov⁸⁾ have shown that the integral in (7) can be represented by a formula of the form :

$$\int_0^\infty \frac{\sigma_\gamma(E)}{E} dE = c r_0^2 A^{4/3} = c r_0 R A \quad (8)$$

where $r_0 = 1.2 \times 10^{-13}$ cm and we take $R = r_0 A^{1/3}$ c is a numerical constant which has a value in the neighbourhood of 0.02. The formula (8) fits empirical data with reasonable accuracy ($\sim 15\%$) with $c = 0.021$, which choice yields for the coefficient a the value

$$a = 0.44 R / K_F = 0.67 A^{1/3} / K_F^2 \quad (9)$$

where K_F is the Fermi momentum determined by $K_F r_0 = (9\pi/8)^{1/3}$.

- (IV) It has been shown that for an infinite Fermi gas of non-interacting nucleons with isospin $T = 0$:

$$F(\nu) = F_{FG}(\nu) = \begin{cases} 1 - \frac{3}{2} \left(\frac{|\nu|}{2K_F} \right) + \frac{1}{2} \left(\frac{|\nu|}{2K_F} \right)^3, & \nu^2 < 4K_F^2 \\ 0, & \nu^2 > 4K_F^2 \end{cases} \quad (10)$$

Clearly this result contradicts our conclusion concerning the analyticity properties of $F(\nu)$ in that it possesses a branch point singularity at $\nu^2 = 0$. This must clearly be a consequence of the fact that for such an infinite system $G(\vec{r})$ is not asymptotically bounded by an $e^{-\lambda r}$ ($\lambda > 0$) as assumed earlier for finite nuclei. For the Fermi gas it is given indeed by

$$\begin{aligned} G_{FG}(r) &= -\frac{K_F^3}{6\pi^2} \left[3x^{-3} (\sin x - x \cos x) \right]^2 \\ &= -\frac{K_F^3}{6\pi^2} \left[\frac{3j_1(x)}{x} \right]^2, \quad x = K_F r \end{aligned} \quad (11)$$

We conclude from this that (10) cannot possibly yield a good approximation to $1 - F(\nu)$ for actual nuclei when $\nu \lesssim 1/R$. An argument given by Bell and Løvseth⁵⁾ that for "real extended nuclear matter" $1 - F(\nu)$ is linear in ν as $\nu \rightarrow 0$ must therefore be of little relevance to muon capture in actual nuclei where νR is never much larger than unity; this accords with the doubts expressed by these authors themselves.

- (V) It has been proposed ^{1),2)} to take into account the finite nuclear size in a Fermi-gas "model" by appropriately modifying the large r behaviour of the spatial correlation function $G_{FG}(r)$ for an infinite Fermi gas. Effectively such schemes amount to multiplying $G_{FG}(r)$ by a function $H(r)$ which (for R large) is unity for $r = 0$ and drops to zero for $r \gtrsim 2R$. The problem is to determine the "correct" form for $H(r)$, if, indeed, one exists. We examine this question in the following section of this paper, but note here that if we simply choose $H(r)$ to be unity for $r < R_c$ and zero for $r > R_c$, then for R_c large compared to $2K_F$ we obtain

$$a = \frac{R_c}{2\pi K_F} \simeq 0.16 \frac{R_c}{K_F} \quad (12)$$

which is in accordance with the form (9) but with a coefficient which is too small if we identify R_c with R . For agreement it would be necessary to choose $R_c \simeq 3R$ which is unreasonable since from (3), $G(r)$ should be vanishingly small as soon as r exceeds $2R$. This shows that a sensible result can only be obtained (if at all) by attention to the detailed character of $H(r)$ as a function of r for all r and not simply by exploiting the gross fact that it drops off from unity at the origin to zero at an $r \simeq 2R$.

Before passing to these considerations, we remark that insofar as the effective axial vector and induced pseudoscalar interaction contributions to muon capture are concerned one must deal with the Fourier transform of another correlation function :

$$F_A(\nu) = -\frac{2}{A} \int \frac{d\vec{v}}{4\pi} \langle 0 | \sum_{i \neq j} \tau_i^{(+)} \tau_j^{(+)} \sigma_i \cdot \sigma_j e^{i\vec{v} \cdot (\vec{r}_i - \vec{r}_j)} | 0 \rangle \quad (13)$$

where σ_i is a component of the ordinary spin of the i^{th} nucleon. The same arguments as given earlier again require that $F_A(\nu)$ be an analytic function of ν^2 and therefore that $F_A(0) - F_A(\nu)$ behaves as ν^2 for $\nu \rightarrow 0$. It is not so easy, however, to evaluate the coefficient since it cannot be directly related to photonuclear information. In the case of weak spin dependence of the nuclear forces where the Wigner supermultiplet approximation can be applied, one can show that for nuclei where $A = 4n$ (n integer) the ground state belongs to the identity representation of SU_4 , that is, is a scalar supermultiplet, from which follows ⁹⁾ $F_A(\nu) = F(\nu)$. This last equality has been almost universally invoked in any case where the closure approximation has been applied to muon capture. In recent work the present authors ⁹⁾ have given some evidence which suggests that this equality is not strongly violated in the presence of spin-orbit and spin-spin interactions in ^{16}O , but at present it is not clear whether tensor interactions may lead to a more severe break-down. For ^4He and ^{12}C it is known that $F_A(0)$ may differ by as much as 25% from $F(0)$ and a more severe break-down is to be expected for light nuclei in which $A \neq 4n$. Any closer examination on the theoretical side of the properties of $F_A(\nu)$ requires the assumption of a detailed nuclear model and quite extensive calculation. We do not enter into this question further.

2. THE "FINITE FERMI GAS"

We consider a bounded volume Ω of space occupied by a collection of $\frac{1}{2}A$ protons and $\frac{1}{2}A$ neutrons, non-interacting, and with each single particle spatial state of energy less than the Fermi energy $E_F = \hbar^2 K_F^2 / 2M$ occupied by a pair of protons and a pair of neutrons, the members of each pair having opposite spin projections. Let the orthonormal single particle space wave functions be represented by $\phi_n(\mathbf{r})$ and the associated single particle energy by E_n . Then from Eq. (1a) the function $F(\nu)$ can be written as

$$F(\nu) = \frac{4}{A} \left(\frac{d\nu}{4\pi} \sum_n \sum_{n'} \int_{\Omega} d\vec{r}_1 \int_{\Omega} d\vec{r}_2 \phi_n^*(\vec{r}_1) \phi_{n'}^*(\vec{r}_2) e^{i\vec{\nu} \cdot (\vec{r}_1 - \vec{r}_2)} \phi_n(\vec{r}_2) \phi_{n'}(\vec{r}_1) \right) \quad (14)$$

where the sum on n and n' is such that $E_n, E_{n'} \leq E_F$. The "Fermi gas approximation" is obtained by replacing $\phi_n(\vec{r})$ by a plane wave $e^{i\vec{k}_n \cdot \vec{r}}/\Omega^{1/2}$, taking E_n to be $\hbar^2 k_n^2/2M$ and writing

$$F(\nu) = \frac{4}{\Omega^2 A} \int \frac{d\hat{\nu}}{4\pi} \sum_{\vec{k}_n} \sum_{\vec{k}_{n'}} \int_{\Omega} d\vec{r}_1 \int_{\Omega} d\vec{r}_2 e^{i(\vec{k}_n - \vec{k}_{n'} + \hat{\nu}) \cdot (\vec{r}_1 - \vec{r}_2)} \quad (15)$$

The plane waves, of course, will only be orthogonal if the region Ω is a rectangular box with the wave functions satisfying periodic boundary conditions but we ignore this fact for the moment. If we take Ω to be a spherical region of radius R , replace the sums on $\vec{k}_n$ and $\vec{k}_{n'}$ by integrations over the Fermi sphere, and carry out these latter integrations as well as that over $\hat{\nu}$, Eq. (15) takes the form

$$F(\nu) = -\Omega \int d\vec{r}_1 \int d\vec{r}_2 D(\vec{r}_1) D(\vec{r}_2) G_{FG}(r) \frac{\sin \nu r}{\nu r} \quad (16)$$

Here $r = |\vec{r}|$, $\vec{r} = |\vec{r}_1 - \vec{r}_2|$, and $G_{FG}(r)$ is the spatial correlation function (11) for the infinite Fermi gas. The integrations on $\vec{r}_1$ and $\vec{r}_2$ are now to be extended over all space while the function $D(r)$ is defined to be $1/\Omega$ inside the sphere Ω and zero outside. Equation (16) is then of the form suggested by Primakoff¹⁾ and Tolhoek²⁾ as the modified Fermi gas function $F(\nu)$ for a finite Fermi gas, except that these authors consider $D(r)$ to represent the actual density distribution inside a finite nucleus which hence may assume a somewhat more general form than that given above; the difference is not important for our purposes. Now letting $\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$, converting the integrations on $\vec{r}_1$ and $\vec{r}_2$ to integrals over $\vec{r}$ and $\vec{R}$, and carrying out the integration on $\vec{R}$ and over the directions of $\vec{r}$, we obtain

$$\begin{aligned}
 F(\nu) &= \frac{K_F^3}{6\pi^2} \int d\vec{r} \frac{R \sin \nu r}{\nu r} H(r) F_{FG}(r) \\
 &= \frac{6K_F}{\pi} \int_0^\infty \frac{R \sin \nu r}{\nu r} H(r) j_1^2(K_F r) dr
 \end{aligned} \tag{17}$$

where

$$H(r) = \begin{cases} 1 - \frac{3}{2} \left(\frac{r}{2R} \right) + \frac{1}{2} \left(\frac{r}{2R} \right)^3, & r < 2R \\ 0, & r > 2R \end{cases} \tag{18}$$

is then to be identified (according to this treatment) with the cut-off function introduced in the previous section. Evaluating (17) in the limit of large R one finds

$$F(\nu) = 1 - \frac{3R}{8\pi K_F} \nu^2 - \dots \tag{19}$$

whence

$$a = \frac{3}{8\pi} \frac{R}{K_F} = \frac{3}{8\pi} \left(\frac{9\pi}{8} \right)^{\frac{1}{3}} \frac{A^{\frac{1}{3}}}{K_F^2} = 0.182 \frac{A^{\frac{1}{3}}}{K_F^2} \tag{20}$$

Except to notice that this is of the form of the empirical result (9) but with a coefficient which is too small, we pass without pausing for discussion to evaluate the coefficient a in a different way following a corresponding calculation of Levinger and Kent¹⁰⁾. We return to Eq. (14) but now expand first in powers of ν to obtain

$$a = \frac{2}{3A} \sum_n \sum_{n'} \int_{\Omega} d\vec{r}_1 \int_{\Omega} d\vec{r}_2 (\vec{r}_1 - \vec{r}_2)^2 \phi_n^*(\vec{r}_1) \phi_{n'}^*(\vec{r}_2) \phi_n(\vec{r}_2) \phi_{n'}(\vec{r}_1) \quad (21)$$

On writing $(\vec{r}_1 - \vec{r}_2)^2 = (r_1^2 + r_2^2) - 2\vec{r}_1 \cdot \vec{r}_2$ and using the orthogonality of the ϕ_n to carry out one of the spatial integrations and one of the summations in those terms which involve r_1^2 and r_2^2 , there results

$$a = \frac{4}{3A} \left\{ \sum_n \int_{\Omega} d\vec{r}_1 r_1^2 |\phi_n(\vec{r}_1)|^2 - \sum_n \sum_{n'} \int_{\Omega} d\vec{r}_1 \int_{\Omega} d\vec{r}_2 (\vec{r}_1 \cdot \vec{r}_2) \phi_n^*(\vec{r}_1) \phi_{n'}^*(\vec{r}_2) \phi_n(\vec{r}_2) \phi_{n'}(\vec{r}_1) \right\} \quad (22)$$

Now the insertion of plane waves for the ϕ_n yields

$$\begin{aligned} a &= \frac{4}{3A} \left\{ \frac{1}{(2\pi)^3} \int d\vec{k} \int_{\Omega} d\vec{r} r^2 - \frac{1}{(2\pi)^6} \int d\vec{k} \int d\vec{k}' \int_{\Omega} d\vec{r}_1 \int_{\Omega} d\vec{r}_2 (\vec{r}_1 \cdot \vec{r}_2) e^{i(\vec{k}' - \vec{k}) \cdot (\vec{r}_1 - \vec{r}_2)} \right\} \\ &= \frac{R^2}{5} \left\{ 1 - \frac{10}{\pi} \int_0^\infty j_2^2(K_F \rho) H(\rho) d\rho \right\} \end{aligned} \quad (23)$$

which agrees with the result of Levinger and Kent¹⁰⁾. Finally, evaluating the integral in the limit of large R , we find

$$a = \frac{3R}{4\pi K_F} \left[\ln 4K_F R + (8-1) + O\left(\frac{1}{R}\right) \right] \quad (24)$$

where $\gamma = 0.577$ is Euler's constant. This is clearly different from the result obtained earlier which would follow from inserting plane waves directly into (21). What is most startling is that even the asymptotic behaviour with R is essentially different in the two cases!

The reason for the discrepancy is easily traced to the fact that plane waves are not orthogonal in a spherical volume; this may be verified by examining the term which represents the difference in the two methods of calculating a and seeing that it vanishes if the ϕ_n are indeed orthogonal. It is very dismaying, however, to find that the ambiguity arising from this inconsistency leads to completely different asymptotic forms for the coefficient a .

There remains the question whether one or the other of the results (20) and (24) for a is more reliable, and how reliable it might be. To explore this point, we have calculated the value obtained for the coefficient a when the region Ω is taken to be a cube of side length L . In this case we have orthogonality of the plane waves in this cubical box and the two methods of calculation should agree. Following the method of Levinger and Kent but with careful attention to the sum over plane waves, we find for the asymptotic form

$$a = \frac{3}{4\pi} \frac{L}{k_F} [\ln 4k_F L + O(1)] \quad (25)$$

This clearly suggests that (24) is the "more reliable" result for the spherical box since it has the asymptotic form suggested by a completely consistent calculation. The presumption would be that a varies asymptotically as $A^{1/3} \ln A$ but that the coefficient depends on the shape of the region Ω , which is not unreasonable. If one accepts (24) as being appropriate to a spherical region and assumes for A in the logarithm term the value 100 as representative of the region of nuclei of interest, one finds

$$a = 0.70 \frac{R}{K_F} = 1.06 \frac{A^{\frac{1}{3}}}{K_F^2} \quad (26)$$

which disagrees with the empirical findings by a factor of almost two.

Our conclusion from the above is that the use of Eq. (16) to obtain the function $F(\nu)$ for heavy nuclei is highly questionable since theoretically it appears to give the wrong asymptotic behaviour (for large R) of the coefficient of the term in ν^2 . Furthermore it is in quantitative disagreement with the empirical result contained in Eq. (12). As for the method of Levinger and Kent, it is indeed not obvious that it is reliable as a way of computing the coefficient a even for asymptotically large R nor is it clear that it possesses a unique generalization for calculation of the complete function $F(\nu)$; it also does not agree with empirical information.

Apparently the only consistent way to treat a spherical Fermi gas would be to use the correct single particle wave functions for a spherical region. Such calculations, insofar as the integral (8) needed to obtain the coefficient a is concerned, have been carried out for a set of values of A lying in the region of physical interest by Levinger and Kent¹⁰⁾ and by Khokhlov⁸⁾ and yield results which are in agreement with the empirical values^{*)}. Thus if Eq. (24) is indeed the correct asymptotic form for a spherical region, this suggests that actual nuclei correspond to A values which are too small for the asymptotic result to be applied. Unfortunately the use of correct functions in a spherical box requires even more involved calculations if one is to obtain not only the coefficient a but the whole function $F(\nu)$. Such calculations for the value of ν appropriate

*) It is not clear to us why this agreement should exist for while it is known that the effect of internucleonic interactions is to increase both the dipole absorption energy and $\int_0^\infty \sigma_\gamma(E) dE$, we do not see why these effects should cancel in $\int_0^\infty \frac{\sigma_\gamma(E)}{E} dE$. Some discussion of this point has been given in Ref. 7).

to muon capture have been carried out by Luyten et al.⁴⁾ for ^{16}O and ^{40}Ca but yield muon capture rates in these nuclei which are too large by a factor of two. The source of this discrepancy has been discussed previously⁹⁾ by the present authors. Whether such calculations would be more successful for heavier nuclei is not at all clear. The one case where the asymptotic A dependence of $F(z)$ might be obtained analytically without too great effort in a self-consistent way is for the artificial example of a cubical box.

We close by remarking that we have limited our discussion here to the general properties of the correlation functions relevant to muon capture in heavy nuclei. Actual computation of muon capture rates involves an estimation of the appropriate average neutrino momentum which we do not enter into here. For further discussion of such features of the closure approximation method, reference should be made to the papers quoted in the introduction.

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