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AN SU_3 ESTIMATE OF THE EFFECTIVE NON-LEPTONIC PARITY VIOLATING
COUPLING IN STRANGENESS CONSERVING WEAK PROCESSESW. Kummer ^{*)}
CERN - GenevaA B S T R A C T

We assume that non-leptonic weak interactions contain a product of SU_3 octets which are the natural generalization of the octet currents in semileptonic interactions. If the representation 27 is suppressed this leads to a prediction of the effective coupling in the weak nucleon-pion vertex which contributes to non-leptonic strangeness conserving (NLSC) weak processes. We discuss both non-derivative and derivative effective coupling. In the latter case which seems the more important one, an argument can be given that the effective parity violating vertex coupling for the NLSC reactions which follows from the experimental parameters of hyperon decays, is likely to be not very different for any other effective coupling.

^{*)} As from October 1964 at : Institut für Theoretische
Physik der Technischen Hochschule Wien, Vienna, Austria.

derivative type. We take into account symmetry breaking in the latter case by inserting in the hyperon decays the physical masses of the particles. This seems reasonable from the form of the PV matrix element which consists formally of the product of a vector current with an axial vector current, because of a theorem by Bouchiat and Meyer⁷⁾ for the vector current in semileptonic interactions. Another assumption is that the non-leptonic matrix element contains a weak interaction of the current-current type. The current shall depend on the strangeness conserving and strangeness violating member of the octet in the ratio given by the Cabibbo angle²⁾. Since the rule $\Delta I = \frac{1}{2}$ holds rather well in hyperon decays, we assume the suppression of the representation 27 in the symmetric current-current product. This gives a definite ratio between the sixth component of the octet which contributes to hyperon decays³⁾, the third component ($\Delta I = 1, \Delta S = 0$), and the singlet ($\Delta I = 0, \Delta S = 0$).

We also use RP invariance⁵⁾ which is correct to the extent that the 30% contribution of f coupling in strong interactions and the weak axial vector current can be neglected in the case of derivative coupling. A short analysis using the experimental data (cf., the Appendix) shows that (for the PV part) RP invariance is compatible with experimental values for the decay of hyperons to an excellent degree as long as the asymmetry parameter of the decay $\Sigma^+ \rightarrow p \pi^0$ is assumed to be one.

In the first Section we shall define the amplitudes of the hyperon decays and of the weak NLSC vertex in terms of SU_3 amplitudes under the assumptions that the representation of dimension 27 has not to be taken into account and CP invariance holds. The second Section contains the numerical results for the PV part of non-derivative coupling where no further assumptions are needed. The third Section is devoted to the discussion of derivative coupling. It contains numerical results for the effective PV derivative NLSC coupling constant.

1. OCTET CURRENTS AND NON-LEPTONIC MATRIX ELEMENTS

The octet current which is coupled "universally" to leptons is usually written as

$$J_{\pm} = I_{\pm} \cos \vartheta + S_{\pm} \sin \vartheta \quad (1.1)$$

where $I_{\pm}$ is the strangeness conserving current and S is the strangeness violating one. From experiments follows ²⁾ $\vartheta \simeq \pi/12$. We will assume that this ratio between octet members remains the same if the current is replaced by an object involving more than two strongly interacting particles. Then in the matrix notation of Okubo ⁸⁾ the terms contributing to the weak non-leptonic amplitude are of the form

$$\begin{aligned} & \alpha \left[\text{Tr} (\sigma J_{+}) \text{Tr} (\tau J_{-}) + \text{Tr} (\sigma J_{-}) \text{Tr} (\tau J_{+}) \right] + \\ & + \beta \left[\text{Tr} (\mu J_{+} \varrho J_{-}) + \text{Tr} (\mu J_{-} \varrho J_{+}) \right] \end{aligned} \quad (1.2)$$

where by σ, τ and μ, ϱ the different combinations of the matrices N and $\bar{N}$ for the baryons and of π (matrix of pseudoscalar mesons) ⁹⁾ are indicated, and

$$J_{\pm} = (\lambda_1 \mp i \lambda_2) \cos \vartheta + (\lambda_4 \mp i \lambda_5) \sin \vartheta$$

The matrices λ_i are the three-dimensional traceless Hermitian matrices of Ref. ¹⁾.

The validity of the $\Delta I = \frac{1}{2}$ rule implies that the representation 27 does not contribute to the non-leptonic matrix element. It follows that terms of the first type in Eq. (1.2) must be excluded and in terms of the second type ϱ can only be the unit matrix in three dimensions.

The matrix element depends therefore only on

$$\{J_+, J_-\} = 8/3 + 3^{-1/2}(1 + 3\cos 2\vartheta)\lambda_8 + (1 - \cos 2\vartheta)\lambda_3 + 2\sin 2\vartheta\lambda_6 \quad (1.3)$$

It contains

$$\begin{aligned} \mathcal{C} = & \mathcal{C}_1 \bar{h}(\bar{N}N\lambda\pi) + \mathcal{C}_2 \bar{h}(\bar{N}N\pi\lambda) + \mathcal{C}_3 \bar{h}(N\bar{N}\lambda\pi) + \\ & + \mathcal{C}_4 \bar{h}(N\bar{N}\pi\lambda) + \mathcal{C}_5 \bar{h}(\bar{N}\lambda N\pi) + \mathcal{C}_6 \bar{h}(\bar{N}\pi N\lambda) + \\ & + \mathcal{C}_7 \bar{h}(N\bar{N}) \bar{h}(\lambda\pi) + \mathcal{C}_8 \bar{h}(\bar{N}\lambda) \bar{h}(N\pi) + \mathcal{C}_9 \bar{h}(\bar{N}\pi) \bar{h}(N\lambda) \end{aligned} \quad (1.4)$$

where λ is to be replaced by λ_6 for hyperon decays, and by λ_3 (for $\Delta I=1$), λ_8 , or the unit matrix for NLSC reactions. Among the nine terms in Eq. (1.4) one is linearly dependent on the others.

The two types of effective interaction Hamiltonians read (G is the usual Fermi coupling constant which is split off for convenience)

$$H_{nd} = 2^{-1/2} G \bar{N}(A + B\gamma_5)N\pi \quad (1.5)$$

and

$$H_{der} = 2^{-1/2} G \bar{N}\gamma^\mu(a + b\gamma_5)N m_\pi \partial_\mu \pi \quad (1.6)$$

$\mathcal{C}$ is an abbreviation for (A,B) or (a,b) . We label the amplitudes by the indices :

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reaction

index

Λ	Σ _±	Σ ₀	Ξ	s	08	03
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As long as no further invariance principle is used there is no difference between the equations for the different amplitudes. They are for the sixth component ⁵⁾

$$\begin{aligned} 6^{1/2} \ell_{\Lambda} &= 2\ell_4 - \ell_6 - \ell_2 \\ 2^{1/2} \ell_{\Sigma_0} &= \ell_6 - \ell_2 \\ 6^{1/2} \ell_{\Xi} &= \ell_4 + \ell_5 - 2\ell_2 \end{aligned} \tag{1.7}$$

$$\begin{aligned} \ell_{\Sigma_+} &= -\ell_6 - \ell_8 \\ \ell_{\Sigma_-} &= -\ell_2 - \ell_8 \end{aligned} \tag{1.8}$$

for the singlet

$$2^{1/2} \ell_s = \ell_3 + \ell_4 + \ell_6 \tag{1.9}$$

for the eighth component

$$6^{1/2} \ell_{08} = \ell_3 + \ell_4 - 2\ell_6 \tag{1.10}$$

and for the third component

$$2^{1/2} \mathcal{C}_{03}^{(1)} = \mathcal{C}_3 + \mathcal{C}_4 + 2\mathcal{C}_7 \quad (1.11)$$

$$\mathcal{C}_{03}^{(2)} = \mathcal{C}_3 - \mathcal{C}_4 \quad (1.12)$$

Equations (1.9) to (1.12) refer to matrix elements for the nucleon-pion vertex in NLSC reactions. We understand $\mathcal{C}_S$ and $\mathcal{C}_{03}$ to be the amplitudes a and b in the derivative Hamiltonian ($\vec{\tau}$ are the Pauli matrices, N is now the nucleon isospinor)

$$H_{der} = 2^{-1/2} G \bar{N} \vec{\tau} \gamma^\mu (a + b \gamma_5) N \partial_\mu \vec{\pi} \quad (1.13)$$

$\mathcal{C}_{03}^{(1)}$ and $\mathcal{C}_{03}^{(2)}$ are the coefficients (A, B) or (a, b) in

$$\begin{aligned} H_{03,nd}^{(1)} &= 2^{-1/2} G \bar{N} (A^{(1)} + B^{(1)} \gamma_5) N \pi_0 \\ H_{03,nd}^{(2)} &= 2^{-1/2} G \bar{N} (\tau_- \pi_- + \tau_+ \pi_+) (A^{(2)} + B^{(2)} \gamma_5) N \end{aligned} \quad (1.14)$$

$$\begin{aligned} H_{03,der}^{(1)} &= 2^{-1/2} G \bar{N} \gamma^\mu (a^{(1)} + b^{(1)} \gamma_5) N m_\pi \partial_\mu \pi_0 \\ H_{03,der}^{(2)} &= 2^{-1/2} G \bar{N} [\tau_- (\partial_\mu \pi_-) + \tau_+ (\partial_\mu \pi_+)] \gamma^\mu (a^{(2)} + b^{(2)} \gamma_5) N m_\pi \end{aligned} \quad (1.15)$$

For derivative coupling some of these expressions vanish if the nucleons are on the mass shell. This is not true for a transition within the nucleus. In any case the amplitudes remain a measure of the effective coupling constant, even, as pointed out in the Introduction, if the actual weak transition in the nucleus is described better by an effective four fermion interaction between nucleons.

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In terms of our particle matrices ⁹⁾ charge conjugation means

$$\begin{aligned} CMC^{-1} &= \bar{N}^T \\ C\pi C^{-1} &= \pi^T \end{aligned} \quad (1.16)$$

so that for the PV amplitudes CP invariance leads to

$$\begin{aligned} A_1 &= -A_2 & a_1 &= a_2 \\ A_3 &= -A_4 & a_3 &= a_4 \\ A_8 &= -A_9 & a_8 &= a_9 \\ A_5 &= A_6 = A_7 = 0 \end{aligned} \quad (1.17)$$

whereas for the PC ones

$$\begin{aligned} B_1 &= B_2 & b_1 &= b_2 \\ B_3 &= B_4 & b_3 &= b_4 \\ B_8 &= B_9 & b_8 &= b_9 \end{aligned} \quad (1.18)$$

follow. From Eq. (1.17) and Eqs. (1.7) for $\mathcal{C}$ replaced by A ³⁾

$$\sqrt{3}A_{\Sigma^0} + A_{\Lambda} = 2A_E \quad (1.19)$$

can be concluded. It is the only linear relation among hyperon decay amplitudes which is a consequence of CP invariance alone.

2. PARITY VIOLATING NLSC VERTEX WITH NON-DERIVATIVE COUPLING

Within our considerations of NLSC vertices only a reaction with $\Delta I = 1$ has a non-vanishing PV amplitude if the coupling does not contain derivatives. Equations (1.14), (1.11) and (1.12) together with Eq. (1.17) give

$$\begin{aligned} H_{03,nd,pv}^{(1)} &= 0 \\ H_{03,nd,pv}^{(2)} &= 2^{-1/2} G \bar{N} (\tau_- \pi_- + \tau_+ \pi_+) N 2A_3 \end{aligned} \quad (2.1)$$

By means of Eq. (1.7) with φ replaced by A and Eq. (1.17), we get

$$2A_3 = 2\sqrt{2} (2A_{Z_0} - \sqrt{3}A_E) \quad (2.2)$$

Under the extreme assumption that the coupling is purely non-derivative, relation (1.19) selects uniquely from the amplitudes A which are the possible solutions of the analysis of experimental values, the set [cf., Eq. (A.1), we have neglected final state interactions so that the amplitudes are real]

$$\begin{aligned} A_\Lambda &= (4.2 \pm 0.3) 10^{-2} \text{ GeV}^2 \\ A_{Z_0} &= (3.61 \pm 0.07) 10^{-2} \text{ GeV}^2 \\ A_E &= (5.3 \pm 0.2) 10^{-2} \text{ GeV}^2 \end{aligned}$$

Then the numerical value for (2.2) is

$$2A_3 = (5.4 \pm 1.4) 10^{-2} \text{ GeV}^2 \quad (2.3)$$

Sakita ⁵⁾ has pointed out that the vanishing of the asymmetry parameter in the decays $\Sigma^\pm \rightarrow n + \pi^\pm$ and the equal signs of the same parameters in the decays Ξ and Σ_0 is possible only if [cf., Eq. (1.8)]

$$a_{\Sigma^\pm} = -a_6 - a_8 = 0$$

From this equation one can conclude, as in Ref. ⁵⁾, that a relation similar to Eq. (1.19) holds for the amplitudes a and b . Comparison with the experimental numbers which we compiled in the Appendix shows, however, that the stronger conditions

$$\begin{aligned} a_\Lambda &= a_\Xi = \sqrt{3} a_{\Sigma_0} \\ -b_\Lambda &= b_\Xi = b_{\Sigma_0}/\sqrt{3} \end{aligned} \quad (3.4)$$

which follow from the same assumptions, are satisfied within the experimental errors, by the set of solutions

$$\begin{aligned} a_\Lambda &= 1.70 \pm 0.12 \\ a_{\Sigma_0} &= 1.03 \pm 0.02 \\ a_\Xi &= 1.83 \pm 0.07 \end{aligned} \quad (3.5)$$

For the corresponding values for the amplitudes b , the agreement is true within roughly 20%. It should be noted that already Eq. (3.3) is enough to select uniquely the solution (3.5). Thus RP invariance is by obscure reasons in hyperon decays true to a much better degree than one would expect according to the argument after Eq. (3.1). This encourages the application of this invariance principle to the determination of the effective coupling in our PV NLSC reactions.

We treat first the reaction with change of I spin, Eq. (1.15). From CP invariance [Eq. (1.17)] alone we arrive at

$$H_{\Sigma_0, \Delta, PV}^{(1)} = 2^{-1/2} G \bar{N} \gamma^\mu N m_\pi \partial_\mu \pi_0 \sqrt{2} a_4 \quad (3.6)$$

where we have used our freedom in the choice of amplitudes to put $a_7 = 0$. RP invariance, together with Eqs. (1.7) for a , and with the relation (3.3) and the values (3.5), gives

$$\sqrt{2}a_4 = 2a_{\Sigma_0} = 2.06 \pm 0.04 \quad (3.7)$$

Taking into account the weight of this member of the octet relative to the one of hyperon decay [cf., Eq. (1.3)] we get

$$\frac{G_{\text{eff}, \Delta I=1, PV}}{G_{\text{eff}}^{\Sigma_0}} = 0.26 \pm 0.02 \quad (3.8)$$

Since the effective coupling of the decay $\Sigma^+ \rightarrow p + \pi^0$ to which we have normalized is not very different from the Fermi coupling constant, the reaction with change of isospin is suppressed by a factor 1/4. This is mainly due to the angle ϑ in Eq. (1.3).

In the transition where isotopic spin is conserved the singlet and the octet contribute. CP invariance alone [cf., Eq. (1.17)] in Eqs. (1.10) and (1.7) leads with the solution (3.5) to

$$a_{08} = (a_4 - a_6)\sqrt{2/3} = a_\Lambda - a_{\Sigma_0}/\sqrt{3} = 1.11 \pm 0.09 \quad (3.9)$$

In this case by taking the set (3.5) instead of another combination of the solutions (A.2), we use RP invariance only to select the most probable solution.

In order to get the contribution of the singlet, we have to rely on all consequences of RP invariance. From Eqs. (1.9), (3.2), with the set of experimental values (3.5), the result is

$$a_S = (5a_{\Sigma_0} + \sqrt{3}a_\Sigma)/4 = 2.06 \pm 0.04 \quad (3.10)$$

The total effective coupling constant for the PV part of the isospin conserving NLSC transition is obtained by using the amplitudes (3.9) and (3.10) together with the appropriate coefficients from Eq. (1.3). One arrives at the large value

$$\frac{G_{\text{eff}, \text{der}, \text{PV}}^{\Delta I=0}}{G_{\text{eff}}^{\text{I}_0}} = 7.6 \pm 0.3 \quad (3.11)$$

It contains about the same amount of singlet and octet coupling.

CONCLUSIONS

Under the assumption that the non-leptonic parity violating weak interactions are built up by products of octet currents of the type known from semileptonic interactions, and that the representation 27 is suppressed in this product, we get the effective non-derivative coupling in the parity violating nucleon-pion vertex for non-leptonic strangeness conserving reactions. From a "universality" argument, we believe matrix elements containing derivative coupling more important. If this is true for non-leptonic interactions, the large PV coupling for the isospin conserving vertex [Eq. (3.11)] may not be far ($\simeq 30\%$) from the value of any other effective PV coupling.

A P P E N D I X

NUMERICAL ANALYSIS OF HYPERON DECAY EXPERIMENTS

The experimental values which we need in the text are essentially the S waves and P waves of non-leptonic hyperon decays. We assume as in Ref. ⁴⁾ an asymmetry parameter $\alpha_{\Sigma^0} = 1$. From the other asymmetry parameters ¹²⁾

$$\alpha_{\Lambda} = -0.61 \pm 0.05 \quad \alpha_E = 0.62 \pm 0.11$$

and with the lifetimes for the decays Λ, E, Σ^0 taken from the tables by Roos ¹³⁾ the possible amplitudes in a matrix element of the form (1.5) are (phases are neglected, the numbers are given in GeV^2)

$$\begin{array}{ll} 10^2 A_{\Lambda} = \begin{array}{l} 4.2 \pm 0.3 \\ -1.4 \pm 0.2 \end{array} & 10 B_{\Lambda} = \begin{array}{l} -2.7 \pm 0.4 \\ 7.8 \pm 0.5 \end{array} \\ 10^2 A_{\Sigma^0} = 3.61 \pm 0.07 & 10 B_{\Sigma^0} = 3.70 \pm 0.07 \quad (\text{A.1}) \\ 10^2 A_E = \begin{array}{l} 5.3 \pm 0.2 \\ 1.8 \pm 0.4 \end{array} & 10 B_E = \begin{array}{l} 3.0 \pm 0.6 \\ 8.5 \pm 0.3 \end{array} \end{array}$$

These values are valid up to a relative sign between different reactions. For one and the same amplitude, numbers in one line belong together.

Assuming pure derivative coupling as in a matrix element of the type (1.6) the (dimensionless) amplitudes are

$$a_A = \begin{matrix} 1.70 \pm 0.12 \\ -0.58 \pm 0.09 \end{matrix}$$

$$b_A = \begin{matrix} 0.95 \pm 0.15 \\ -2.78 \pm 0.19 \end{matrix}$$

$$a_{\Sigma_0} = 1.03 \pm 0.02$$

$$b_{\Sigma_0} = -1.24 \pm 0.02$$

$$a_{\Xi} = \begin{matrix} 1.83 \pm 0.07 \\ 0.64 \pm 0.14 \end{matrix}$$

$$b_{\Xi} = \begin{matrix} -0.87 \pm 0.19 \\ -2.50 \pm 1.0 \end{matrix}$$

(A.2)

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$$\begin{array}{ccc}
 \frac{\pi_0}{\sqrt{2}} + \frac{1}{\sqrt{6}} & \pi^+ & K^+ \\
 \pi^- & -\frac{\pi_0}{\sqrt{2}} + \frac{1}{\sqrt{6}} & K^0 \\
 \bar{K}^+ & \bar{K}^0 & -\frac{2}{\sqrt{6}}
 \end{array}
 \quad
 N = C^{-1} \bar{N}^T C =
 \begin{array}{ccc}
 \frac{\Sigma_0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & Z^+ & P \\
 -Z^- & -\frac{\Sigma_0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\
 \bar{E}^- & -\bar{E}^0 & -\frac{2\Lambda}{\sqrt{6}}
 \end{array}$$

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INTRODUCTION

The application of the concept of unitary symmetry in elementary particle physics ¹⁾ to the theory of weak interactions was successful in two respects. The first one is the parametrization of semi-leptonic reactions in terms of a vector current and an axial vector current, which together represent one member of a unitary octet ²⁾. The ratio between the strangeness conserving and strangeness violating processes is given by one rotation parameter in SU_3 space. The second consequence of SU_3 symmetry relies on the additional assumption that in a non-leptonic reaction the two octet currents combine to form again an octet, with the irreducible representation of dimension 27 strongly suppressed - possibly by some dynamical mechanism. The component of the octet is specified further by CP invariance. The result of symmetry considerations depends on the type of effective interaction assumed between the two baryons and the meson which appear in observable reactions of this type. Non-derivative coupling leads, for the parity violating (PV) part, to a linear relation between the amplitudes of non-leptonic Λ , Σ , and Ξ decay ³⁾. The further assumptions of R invariance and the smallness of the asymmetry parameters in the decays $\Sigma^\pm \rightarrow n \pi^\pm$ is necessary in order to obtain a similar relation for the parity conserving (PC) part ⁴⁾ or for both amplitudes in the case of derivative coupling ⁵⁾. The relation for the PV and PC amplitudes together is satisfied experimentally rather well for non-derivative coupling ⁴⁾.

The recent experimental indications of non-leptonic strangeness conserving processes in nuclei ⁶⁾ make it desirable to find a relation between the amplitudes of these processes and the decay amplitudes of hyperons. It is obvious that this is possible only if the nucleon-pion vertex determines the effective coupling in such weak processes - at least as far as the PV part of the amplitude is concerned, which is the experimentally observable one. We cannot exclude the possibility that the result of our symmetry considerations is therefore true only to the extent that additional renormalization effects in a process involving more strongly interacting particles can be neglected.