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A MODEL OF WEAK INTERACTIONS MEDIATED BY SCALAR BOSONS

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ABSTRACT

By introducing two new neutral leptons, two massive scalar bosons and a new absolutely conserved quantum number, an attempt is made to formulate a renormalizable theory of the weak interactions.

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INTRODUCTION

The CERN high energy neutrino experiment ¹⁾ has shown quite conclusively that the four-Fermi weak interaction is of a non-local type. At present the simplest way of explaining these results and the V-A nature of the interaction seems to be by introducing an intermediate vector boson W^+ with a mass of about 1.5 GeV which mediates these interactions. A field theory of such an interaction is, however, unrenormalizable in the usual sense and despite the recent introduction of powerful techniques in dealing with such theories ²⁾, the problems involved are not yet fully understood.

We believe, therefore, that it is interesting to see whether these phenomena could be explained within the framework of local quantum field theory by a model which is renormalizable in the sense of a perturbation theory expansion ³⁾. The only interactions of spin 1/2 fermions that fall under this category are direct ones with scalar bosons or neutral vector mesons coupled to a conserved current. As we must have charged currents we shall concentrate on the former case. Section I will be devoted to a discussion of the interaction Lagrangian and its invariances and a perturbation theory calculation of purely leptonic processes. In section II we discuss the effect of strong interactions between the scalar bosons mediating the weak interactions and the possibility of thereby reducing the effect of neutral currents. Section III deals with semi-leptonic and non-leptonic weak interactions and, finally in section IV, we discuss the differences between this model and the usual phenomenological current-current interaction.

I. The first case we treat is that of purely leptonic weak interactions. We shall need to introduce two new leptons μ_o^* and e_o^* having masses equal to those of the μ and the e and carrying an additional property which we shall call "weakness". We can then form two triplets $(\bar{\mu}, \nu_\mu, \mu_o^*)$ and $(\bar{e}, \nu_e, e_o^*)$ analogous to the (n, p, Λ) system where the first two members have weakness zero and the third has weakness -1. Consider now a boson doublet $\begin{pmatrix} B_+ \\ B_o \end{pmatrix}$ with weakness +1, analogous to the K mesons, and let them have a very large mass, $\gtrsim 5$ GeV.

We call l the lepton field and l^* the field of a weakness carrying lepton. The interaction, which we take to be of the form

$$\mathcal{L}_I = \sum_i \bar{l}^* O_i l B + \text{h.c.} \quad (\text{I.1})$$

will not contain any direct $l-l$ or l^*-l^* coupling, as we assume weakness is conserved at every vertex. The interaction is also assumed to be invariant under the following transformation

$$\begin{aligned} l &\rightarrow \gamma_5 l & m_e &\rightarrow -m_e \\ l^* &\rightarrow -\gamma_5 l^* & m_{e^*} &\rightarrow -m_{e^*} \end{aligned} \quad (\text{I.2})$$

for every fermion field, separately. The interaction Lagrangian of the form (I.1) which satisfies this invariance explicitly and furthermore is renormalizable is the following

$$\begin{aligned} \mathcal{L}_I = \frac{g}{\sqrt{2}} [&\bar{\mu} (1-\gamma_5) \mu_o^* B_+ + \bar{\nu}_\mu (1-\gamma_5) \mu_o^* B_o + \\ &+ \bar{e} (1-\gamma_5) e_o^* B_+ + \bar{\nu}_e (1-\gamma_5) e_o^* B_o + \text{h.c.}] \end{aligned} \quad (\text{I.3})$$

Let us apply it immediately to the calculation of the Feynman diagram for μ decay (Fig. 1). As we have supposed there are no charged weakness carrying leptons, there is also no crossed diagram for μ decay. In addition we assume conservation of both lepton number and muon number. The matrix element for Fig. 1 is then given by

$$\begin{aligned}
 T_{fi} = & g^4 \int d^4q \left\{ (q^2 - M^2) [(p_1 - p_3 - q)^2 - M^2] [(p_1 - q)^2 - m_{\mu^*}^2] [(p_2 + q)^2 - m_{e^*}^2] \right\}^{-1} \times \\
 & \times \frac{1 - \gamma_5^{(1)}}{\sqrt{2}} \left[\gamma^{(1)} (p_1 - q) + m_{\mu^*} \right] \frac{1 + \gamma_5^{(1)}}{\sqrt{2}} \times \\
 & \times \frac{1 - \gamma_5^{(2)}}{\sqrt{2}} \left[\gamma^{(2)} (p_2 + q) + m_{e^*} \right] \frac{1 + \gamma_5^{(2)}}{\sqrt{2}}
 \end{aligned} \quad (I.4)$$

where the superscript (1) denotes that the operator is taken between $\bar{\nu}_\mu$ and μ and (2) between $\bar{e}$ and ν_e . As the lepton masses and momenta are negligible compared to the boson mass we can integrate immediately by setting them equal to zero obtaining thereby

$$T_{fi} = - \frac{ig^4 \pi^2}{4M^2} \left[\gamma_\alpha^{(1)} (1 + \gamma_5^{(1)}) \gamma_\alpha^{(2)} (1 + \gamma_5^{(2)}) + \mathcal{O}\left(\frac{m_e^2}{M^2}\right) \right] \quad (I.5)$$

Comparison with the usual current-current interaction matrix element leads to

$$G_F / \sqrt{2} = g^4 \pi^2 / (4M^2 (2\pi)^4) = G^4 / (4M^2) \quad (I.6)$$

where $G^2 = g^2 / 4\pi$. Taking $G_F = 10^{-5} / m_p^2$ we have $G^4 = 2 \sqrt{2} \times 10^{-5} M^2 / m_p^2 \approx 3 \times 10^{-3}$ for $M \approx 10$ GeV.

It is of course not surprising that we obtain the V-A type of interaction for μ decay as it follows quite generally from the Lagrangian's invariance (I.2) that any four lepton interaction amplitude must be of the form
(ε is some characteristic energy, G_F is omitted)

$$\bar{\ell}_1 \bar{\ell}_2 [\gamma_\alpha (1+\gamma_5)]^{(1)} [\gamma_\alpha (1+\gamma_5)]^{(2)} \ell_3 \ell_4 + O\left(\frac{m_\ell^2}{M^2}, \frac{\varepsilon^2}{M^2}\right)$$

Similarly the amplitudes for four lepton* and two lepton-two lepton* interactions are of the form

$$\bar{\ell}_1^* \bar{\ell}_2^* [\gamma_\alpha (1-\gamma_5)]^{(1)} [\gamma_\alpha (1-\gamma_5)]^{(2)} \ell_3^* \ell_4^* + O\left(\frac{m_{\ell^*}^2}{M^2}, \frac{\varepsilon^2}{M^2}\right)$$

$$\bar{\ell}_1 \bar{\ell}_2^* [1-\gamma_5]^{(1)} [1+\gamma_5]^{(2)} \ell_3^* \ell_4 + O\left(\frac{m_\ell m_{\ell^*}}{M^2}, \frac{\varepsilon^2}{M^2}\right)$$

Among the possible interactions between leptons we cannot exclude those which would result from neutral currents in a current-current interaction. There is, however, no clear evidence for or against them in purely leptonic interactions.

Finally, we notice that if the charged and neutral boson have different masses, M_1 and M_2 respectively, in expression (I.5) we have

$$\frac{\log M_1^2/M_2^2}{M_1^2 - M_2^2}$$

in place of $1/M^2$. The corrections are $O(m_\ell^2)/(M_<^2)$ where $M_<$ is $\text{Min}(M_1, M_2)$.

II. In semi-leptonic weak interactions, there is very good evidence against the existence of neutral currents ⁴⁾. In our model this corresponds to a great enhancement of the charged boson-antiboson state with regard to the neutral one. Let us assume that there is an interaction of the renormalizable type, a part of which could be direct ⁵⁾, e.g.,

$$\mathcal{L}_I = \lambda (\bar{B} B)^2$$

A model for the two lepton to two-boson transition matrix J th partial wave is then

$$A_{\ell\bar{\ell} \rightarrow B\bar{B}}^J = A_{\ell\bar{\ell} \rightarrow B\bar{B}}^{J \text{ B.A.}} \frac{C_{B\bar{B}}^J}{C_{B\bar{B}}^{J \text{ B.A.}}} = A_{\ell\bar{\ell} \rightarrow B\bar{B}}^{J \text{ B.A.}} R_J \quad (\text{II.1})$$

where B.A. means the Born approximation value and C is the boson-antiboson transition matrix. This "ansatz" has the correct phase and the correct Born approximation limit.

If we then consider the process $\mu^- + \bar{\nu}_\mu \rightarrow e^- + \bar{\nu}_e$ (see Fig. 1), with $p = \frac{p_1 - p_3}{2}$, $p' = \frac{p_2 - p_4}{2}$ unitarity in the t channel readily gives us the imaginary part of the V-A amplitude

$$\begin{aligned} \text{Im } T_{fi} = & \frac{i}{2} G^4 (2\pi)^4 \int \frac{|\vec{q}| d\Omega_q}{4(\vec{q}^2 + M^2)^{3/2}} \left\{ \sum_{J, J', m, m'} [\gamma_m(1+\gamma_5)]^{(1)} [\gamma_{-m'}(1+\gamma_5)]^{(2)} \right. \\ & \times \sqrt{\frac{JJ'(J+1)(J'+1)}{(2J+1)(2J'+1)}} (Q_{J-1} - Q_{J+1}) Y_{J,m}(\hat{q}\hat{p}) R_J \times \\ & \left. \times [(Q_{J'-1} - Q_{J'+1}) Y_{J',m'}(\hat{q}\hat{p}') R_{J'}]^+ \right\} \end{aligned}$$

where the Q 's are Legendre functions of the second kind, $\hat{q}p = \vec{q}\vec{p}/\sqrt{q^2 p^2}$, and $-1 \leq m, m' \leq +1$. Performing the integration we get

$$\begin{aligned}
 \text{Im } T_{fi} = & \frac{i G^4 (2\pi)^4 |\vec{q}|}{8 (q^2 + M^2)^{3/2}} \left[\sum_{m, m'} [\gamma_m (1 + \gamma_5)]^{(1)} [\gamma_{m'} (1 + \gamma_5)]^{(2)} \right. \\
 & \times (Q_{J-1} - Q_{J+1})^2 |R_J|^2 \frac{J(J+1)}{2J+1} D_{m, m'}^{(J)}(\hat{p}\hat{p}') + O\left(\frac{m_e^2}{M^2}\right) \Big] \quad (\text{II.2})
 \end{aligned}$$

Up to terms $O(m_e^2/M^2)$ only $m = m'$ contributes to $\text{Im} T$, that is to say the matrix element is of the form $\gamma_\alpha^{(1)}(1 + \gamma_5^{(1)}) \gamma_\alpha^{(2)}(1 + \gamma_5^{(2)})$, as we already know from invariance arguments. We may then write a dispersion relation for each partial wave amplitude, assuring that no subtractions are required. If for all J , $|R_J|^2 = \text{constant}$, we have of course expression (I.4), the perturbation theory amplitude, which we shall call $T^{\text{P.T.}}$ multiplied by this constant. As one can easily see, the main contribution to T comes from small J 's.

Let us illustrate what the effects on the amplitude of a strong boson-boson interaction might be by considering the $J = 1$ partial wave with $|R_J|^2 = 0$ except over a suitable region, which we will take, e.g., to be $M \leq |\vec{q}| \leq 2M$, where $|R_J|^2 = \eta$. Using a dispersion relation and integrating we find for this case

$$T \approx T^{J=1} \approx \eta T^{\text{P.T.}}/10$$

By (II.1) a reasonable approximation to η would be $\sin^2 \mathcal{J}_1$, where $\mathcal{J}_1$ is the actual boson-boson $J = 1$ scattering phase shift, divided by $\sin^2 \mathcal{J}_1^{\text{B}}$ as calculated in Born approximation. Assuming that there is a resonance region in which η is large, possibly as large as one hundred, which gives the dominant contribution to the amplitude, we can roughly take η as constant in the resonance region and zero elsewhere. As we see this leads to a $J = 1$ amplitude considerably larger than the whole perturbation theory amplitudes.

By enhancing many of the partial waves of the scattering in charged $B\bar{B}$ states and dampening those of neutral $B\bar{B}$ states we could appreciably alter the neutral currents. (Note that the rate depends on $|R_J|^4 = \eta^2$.)

That there should be strong boson-boson interactions is not in itself surprising as one supposes that there must be some mechanism present capable of generating the large mass they are presumed to possess ⁶⁾.

III. For semi-leptonic reactions most of what has been said in the first two chapters remains true. Within pure perturbation theory or dispersion theory the only difference can be found in the strong interactions of one fermion line. The invariance argument ceases to hold for this line so that the matrix element must be of the form

$$T_{fi} \propto \gamma_{\alpha}^{(1)} (1 + \gamma_5^{(1)}) \left[\sum_i P_{i,\alpha}^{(2)} H_i(s,t) \right] + O\left(\frac{m_e m_N}{M^2}\right) \quad (\text{III.1})$$

where

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ t &= (p_1 - p_3)^2 \\ P_{i,\alpha} &= (p_1 + p_3)_{\alpha}, \gamma_{\alpha}, \gamma_5(p_1 + p_3)_{\alpha}, \gamma_5 \gamma_{\alpha} \end{aligned}$$

Another possible $P_{i,\alpha}$, namely $(p_1 - p_3)_{\alpha}$ gives a contribution to the correction term. This differs from the usual current-current expression by the dependence on s . Let us take as an example neutron β decay in perturbation theory (Fig. 2). The amplitude reads for this process

$$\begin{aligned} T_{fi} &= g^4 \int d^4 q \left\{ [(p_2 + q)^2 - m_{e_0}^2] [q^2 - M^2] [(p_4 - p_2 - q)^2 - M^2] \right\}^{-1} \\ &\times \frac{1 - \gamma_5^{(1)}}{\sqrt{2}} \left[\gamma_{\alpha}^{(1)} (p_2 + q) + m_{e_0} \right] \frac{1 + \gamma_5^{(1)}}{\sqrt{2}} F^{(2)}(q^2, \tilde{s}(q), t) \end{aligned} \quad (\text{III.2})$$

with

$$\tilde{s} = (p_1 - q)^2$$

For negligible lepton masses $m_{e_0}^*$ and m_e the $\gamma \cdot q$ in the lepton propagator picks out the V-A interaction from the nucleon bubble, represented by

$$F = F_1 + \gamma_9 F_2 + \gamma_5 F_3 + \gamma_5 \gamma_9 F_4 \quad (\text{III.3})$$

if the integral exists as we assume. In order to have an explanation of the equality of the effective β decay and μ decay vector coupling constant, a hypothesis analogous to that of the conserved vector current ⁷⁾ (CVC) has to be introduced ; this is possible only if the s dependence in H_1 and H_2 of (III.1) is weak. This dependence on s is determined by the masses appearing in the reaction, the boson masses, the nucleon mass and some effective mass m_F which represents the dependence of F_1, F_2 on $\tilde{s}$ (e.g., in form of a fermion pole). As long as the boson masses are large compared to m_N, m_F , we have (apart from the additional t dependence) the situation of the pure leptonic case where s appeared in terms $O(s/M_Z^2) \approx O(m_N^2/M_Z^2) \ll 1$. Then there is an explanation for the required weak dependence on s . No simple reason for this property of the amplitude can be given, however, if one of the boson masses is of the order of the nucleon mass or m_F .

The experimentally established $\Delta I = 1/2$ dominance in non-leptonic weak interactions requires neutral currents of equal importance as the charged ones. The strong boson-boson interaction between the charged and the neutral boson in our model excludes neutral currents for leptonic and non-leptonic reactions. Hence, an explanation of $\Delta I = 1/2$ in terms of some schizon theory ⁸⁾, is impossible ; it has to be a consequence of the strongly interacting part of the matrix element only. It should be noted that the dynamical differences between such a model and a schizon theory are outside the present experimental possibilities, anyhow.

IV. In concluding, we would like to discuss some of the experimental implications of our theory. The new leptons we have introduced would be very difficult to detect as electromagnetic interactions mask their production in, e.g.,

$$e^+ + e^- \rightarrow e_o^* + \bar{e}_o^*$$

The $(g-2)$ of the μ is altered (see Fig. 3) by an amount of order $G^2(m_\mu^4/M^4)$ which is unobservable. Furthermore, once produced, the μ_o^* decays into three neutral particles. In semi-leptonic processes, one must fix the value of the intermediate mass m_F at a rather large value (≥ 4 GeV) in order to assure that this state is not produced in conjunction with a μ_o^* in the CERN neutrino experiment, the limit being

$$2E_\nu m_N \sim m_F^2$$

where m_N is the nucleon mass and E_ν the incident neutrino energy. Deviations from V-A in nuclear β decay, however, are still $O(m_e/M)$ and hence unobservable. They could, however, be measurable in μ capture.

Lastly, we mention the difficulty of explaining with the present model the production of charged lepton pairs in the CERN experiment. A possibility is obtained by introducing a gross disparity between the charged boson and the neutral boson mass, e.g., take the one to be about 10 GeV and the other only 1 GeV or a little more so that the neutral boson would actually be produced ^{*)}

*) We must take the charged boson mass large in order not to have a reaction competing with the direct process in which no bosons are produced.

(see Fig. 4)⁹⁾. Calculations have not been performed for this process, but it appears possible that one could reproduce the CERN result by a suitable choice of parameters^{*)}.

In conclusion, we do not wish to suggest that this model is more economical in its assumptions than the intermediate vector boson, nor more elegant, however we have carried out this investigation to emphasize the possibility that the situation in the weak interactions is even more complicated than it presently appears.

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^{*)} A small boson mass leads, however, to a possible s dependence of the amplitude, as explained in section III.

R E F E R E N C E S

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Yasutaka Tamikawa, Phys. Rev. 108, 1615 (1957).
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- 5) This presumes that $B-\bar{B}$ scattering is extremely charge dependent. If
one wishes to attribute isotopic spin to the boson doublet, we must
assume the existence of a second boson doublet

$$\begin{pmatrix} \widetilde{B}_0 \\ \widetilde{B}_- \end{pmatrix}$$

with $W = +1$ in order to construct an isotopic spin rotation invariant
charged state. We would then have to enhance the $I = 0$ $B\bar{B}$ state
and dampen all other states.

- 6) G. Feinberg (preprint).
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Fig. 1

Fig. 2

Fig. 3

Fig. 4