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ON THE PRINCIPLE OF SUPERPOSITION IN K^0 DECAYS

Bertel Laurent ^{*)}

CERN--Geneva

and

Institute for Theoretical Physics,
University of Stockholm, Stockholm

and

Matts Roos ⁺⁾

Institute for Theoretical Physics,
University of Copenhagen, Copenhagen

A B S T R A C T

Two alternatives for explaining the anomalous CP non-conserving 2π decays of K^0 mesons are discussed : the linear theory, violating CP invariance, and a CP invariant, non-linear theory, violating the principle of superposition. Experiments are proposed to distinguish between these two possibilities.

^{*)} Present address : Institute for Theoretical Physics,
Vanadisvägen 9, Stockholm.

⁺⁾ Present address : CERN, Geneva.

1. INTRODUCTION

The discovery of the anomalous 2π decays of K^0 mesons ¹⁾ has shown that the theory of K^0 decays is basically wrong in some respect. At first sight, it seems as if CP invariance would have to be given up altogether. This is, however, not necessary if one is willing to accept a violation of the principle of superposition instead. In a previous publication ²⁾ we have suggested such a model for the anomalous 2π decays. Moreover, this model makes definite predictions about the time distribution of 2π decays from K^0 mesons ²⁾. Below we give a more detailed presentation of these ideas. We also suggest further experiments in order to distinguish between our CP invariant, non-linear model and CP variant, linear models.

It is important to note the distinction between CP invariance and CP conservation. The lack of either one is usually referred to as CP violation. To avoid confusion we shall denote lack of CP invariance, CP variance, and lack of CP parity conservation, CP non-conservation.

In order to derive the law of CP parity conservation, it is sufficient to assume CP invariance and the principle of superposition. Thus for CP non-conservation it is necessary to violate either CP invariance or the principle of superposition. As an illustration, consider the well-known linear, CP invariant, theory for K^0 decays. Let $A_1(t)$ and $A_2(t)$ be the amplitudes for K_1 and K_2 mesons, respectively (in c.m.s. they are functions of time only). We define the CP transformation by

$$\begin{aligned} A_1 &\rightarrow A_1 \\ A_2 &\rightarrow -A_2. \end{aligned} \tag{1}$$

That the theory is invariant under (1) means that sets of values A_1 and A_2 at different times are related in a particular way. If A_1

2.

and A_2 at time t' have the values (a_1, a_2) and at time t'' the values (b_1, b_2) then invariance under (1) implies that to the values $(a_1, -a_2)$ at time t' correspond the values $(b_1, -b_2)$ at time t'' .

The principle of superposition now relates linear combinations of such sets of values. In particular, to the state $(a_1, 0)$ at time t' corresponds the state $(b_1, 0)$ at time t'' . Thus the state of a pure K_1 meson always remains a pure K_1 meson, and the state of a pure K_2 meson always remains a pure K_2 meson. In general, the principle of superposition requires that a mixture of a K_1 meson and a K_2 meson remains the same. This is conservation of CP parity.

In a non-linear theory things are quite different, since we cannot invoke the principle of superposition. It appears as if the invariant state $(a_1, 0)$, at time t' , still had an invariant state $(c_1, 0)$ as its correspondence at time t'' . Therefore, a pure K_1 meson still seems to remain a pure K_1 meson in the non-linear case. There is a snag, however: the non-linear equations of motion may be singular when $A_2 = 0$, in which case there is no CP conservation left. For the general state, with $A_2 \neq 0$, there is anyway nothing left of CP conservation. As a consequence, the terms " K_1 " and " K_2 " meson then lose their precise meaning.

No experiment done so far on the anomalous decays of K^0 mesons ^{3),4),6)} can distinguish between the two causes: CP variance or violation of the principle of superposition. We prefer not to enter into philosophical arguments about which of the principles is more fundamental: CP invariance, being part of the invariance under the full Lorentz group, or the principle of superposition, forming the basis of quantum mechanics. We only maintain that there is reason to pursue the experimental search, as long as there is the least doubt as to which of them is invalidated.

A question which immediately presents itself is whether a violation of one or the other principle should not have shown up in earlier experiments, for instance in electrodynamics. This cannot be answered until one assumes more about the particular mechanism responsible for the violation. One may, for instance, conceive of a general mechanism which is effective when a state and its antistate interfere, as is obviously the case in K^0 decay. If this is the case, one would not expect any effects in pure electromagnetic or strong processes, since states and their antistates are then prevented from interfering by conservation of charge, baryon number and strangeness. The particular model presented below, and our discussion of the reaction

$$p + \bar{p} \rightarrow K^0 + \bar{K}^0$$

is consistent with such an assumption.

Another possibility is that slight non-linearities do occur, not only in particle-antiparticle interferences, but that they can be detected only in extremely sensitive interference experiments, such as the ones considered.

2. BASIC ASSUMPTIONS

Abandoning the superposition principle one must still keep some of the features of the old theory. Always using the rest system of the K^0 (or $\bar{K}^0$) meson we still assume that the decay is governed by two complex functions of time, A_1 and A_2 . By this we mean that the decay rates into different channels at a certain time are given by the value of A_1 and A_2 and possibly also their first and higher time derivatives. Note that we do not here specify the connection between these functions and the different decay channels or the short and long-lived components. CP transformation is defined as in (1). The equations of motion are then

4.

$$-\dot{A}_1 = \alpha A_1, \quad (2)$$

$$-\dot{A}_2 = \beta A_2, \quad (3)$$

where α and β need not be constants but may be functions of A_1 and A_2 .

In order to keep Eqs. (2) and (3) as simple as possible, we shall assume that they are homogeneous, which means that α and β have to be homogeneous functions of degree zero in A_1 and A_2 . We have no other motivation for this than that the simplest inhomogeneous functions we could think of lead to results in obvious disagreement with experiment. The most general homogeneous equations are obtained if α and β are functions of s only, where

$$s \equiv \frac{A_2}{A_1}. \quad (4)$$

From Eqs. (2), (3) and (4) we obtain

$$\dot{s} = \gamma(s) \cdot s, \quad (5)$$

where

$$\gamma(s) = \alpha - \beta. \quad (6)$$

Our main purpose with this paper is to study Eq. (5). Let us first do this in two well-known cases.

In the linear CP invariant theory γ is a constant γ_0 ,

$$\gamma(s) = \frac{1}{\tau_1} - \frac{1}{\tau_2} - i\Delta m \equiv \gamma_0, \quad (7)$$

where τ_1 and τ_2 are the lifetimes of K_1 and K_2 respectively, and Δm is the K_2-K_1 mass difference. K_1 and K_2 (and the corresponding amplitudes A_1 and A_2) have here their usual meaning. The solution to (5) is then

$$s = s_0 e^{\gamma_0^* t}, \quad (8)$$

s_0 being an integration constant. When $t \rightarrow \infty$, clearly also $|s| \rightarrow \infty$. This means that $|A_1|$ goes to zero faster than $|A_2|$ and, contrary to the experiments 1), 3), 4), 6) the ratio of 2π to 3π decay rates then always decreases.

One way to account for the anomalous 2π decays is to introduce the linear CP variant theory, in which

$$\gamma(s) = \gamma_0 + \varepsilon s - \frac{\varepsilon'}{s}, \quad (9)$$

where ε and ε' are two constants, supposedly of the same order of magnitude. The important new feature in this theory is that $\gamma(s)$ has a zero. If we choose

$$|\varepsilon| \approx 2 \cdot 10^{-3} |\gamma_0| \quad (10)$$

the position of the zero is at $|s| \approx 500$. At large times $|s|$ does not go to infinity, but gets caught in the zero, where $\dot{s} = 0$ by Eq. (5).

The CP transformation (1), expressed in terms of s , is

$$s \rightarrow -s. \quad (11)$$

Obviously this theory is not CP invariant since Eq. (9) is not invariant under (11).

The necessary and sufficient condition for CP invariance is that $\gamma(s)$ be an even function of s . We shall now discuss the non-linear CP invariant case. Let a $K^0(\bar{K}^0)$ meson be characterized by $s = 1$ ($s = -1$) at time $t = 0$. If $\gamma(s)$ is even and has a zero at $s = +s_\infty$, then it also has a zero at $s = -s_\infty$. Thus it follows from CP invariance that if a beam which originally consisted of K^0 mesons, eventually reaches the stable point s_∞ , then a beam which originally consisted of $\bar{K}^0$ mesons must eventually reach the stable point $-s_\infty$.

By performing a regeneration experiment one can distinguish between $s = s_\infty$ and $s = -s_\infty$. The regeneration changes A_1 by the amount

$$\Delta A_1 = c A_2, \quad (12)$$

where c is a constant, typical for the particular arrangement used. It is reasonable to use the linear relation (12), because the change in A_1 caused by regeneration is much faster than that caused by non-linear effects. Immediately after the regeneration of a beam, which originally was pure $K^0(\bar{K}^0)$, the amplitudes A_2 and A_1 stand in the ratio $s = c+s_\infty$ ($s = c-s_\infty$). These two points, $s = c \pm s_\infty$, are not CP conjugate, and will in general give different decay rates. If the undisturbed (not regenerated) decay rates of $K^0(\bar{K}^0)$ mesons oscillate as s approaches s_∞ ($-s_\infty$), then the regenerated beam will also show such oscillations. The regenerated oscillations are characterized by opposite phase in the two cases, when the original beam was either K^0 or $\bar{K}^0$. In contrast, the linear CP variant theory predicts the same phase in the regeneration oscillations of originally K^0 and $\bar{K}^0$ beams.

We may also directly study the oscillations in the 2π decay rate of undisturbed K^0 and $\bar{K}^0$ beams. The non-linear CP invariant theory then predicts identical time distributions for K^0 and $\bar{K}^0$, whereas the linear CP variant theory predicts oscillations with opposite phase in the two cases *).

*) This has been independently pointed by J.S. Bell in a discussion with one of us.

These are the most general predictions we have been able to make in order to distinguish between non-linear, CP invariant theories and linear, CP variant theories. To proceed we shall investigate a particular dynamical model.

3. A DYNAMICAL MODEL

We shall base the discussion of the dynamical equation (5)

$$\dot{s} = \gamma(s) \cdot s, \quad (5)$$

on the following postulates :

A. The linear CP invariant theory is a good approximation,

$$\gamma(s) \approx \gamma_0 \quad (13)$$

in the region

$$\eta \ll |s| \ll \eta^{-1}, \quad (14)$$

where

$$\eta \approx 2 \cdot 10^{-3}. \quad (15)$$

B. The behaviour at zero and infinity is such that

$$\dot{s} \neq 0 \quad \text{for} \quad s \rightarrow 0 \quad (16)$$

and

$$(\dot{s}^{-1}) \neq 0 \quad \text{for} \quad s^{-1} \rightarrow 0. \quad (17)$$

8.

C. Analyticity

$\chi(s)$ analytic for $0 < |s| < \infty$; no branch singularities.

These postulates do not yet contain the assumption of CP invariance. The subsequent discussion will therefore concern both the linear CP variant theory and the non-linear CP invariant theory, until otherwise specified (Sections 6 and 7).

The value (15) under A is chosen to make the CP non-conservation appear at about the observed ^{1),4),6)} value of s .

Postulate B expresses the feeling that there exists a so far unknown mechanism, which prevents A_1 and A_2 from remaining zero, if they would momentarily vanish. Such a mechanism is required both in the linear CP variant theory and the non-linear CP invariant theory.

Under B we actually only need condition (17) for a discussion of K^0 and $\bar{K}^0$ decays. The inclusion of condition (16) puts A_1 and A_2 qualitatively on the same footing, but has no detectable effect on the decays. Note that the earlier discussed model ²⁾ does not fulfil condition (16).

Postulate C is the simplest generalization of linearity we could think of. It follows from C that $\dot{s}$ is unique and finite for $0 < |s| < \infty$.

4. THE SOLUTIONS OF THE DYNAMICAL EQUATION

According to postulate B, the function $\chi(s)s$ must not vanish when $s \rightarrow 0$. This excludes the existence of an essential singularity in $\chi(s)$ at $s = 0$ ^{*)}. On the other hand, $\chi^*(s)$ must for the same reason have a pole at $s = 0$. In the same way we can show that there is no essential singularity, but only a pole at $s = \infty$.

*) We are indebted to A. Martin for discussions on this point.

According to A, $\gamma(s)$ is practically a constant when s is far from the two poles. This means that the zeros must be located in the two separate regions $|s| < \eta$ and $|s| > 1/\eta$. These zeros cancel the effects of the poles when $\eta \ll |s| \ll 1/\eta$.

Let us now consider K^0 or $\bar{K}^0$ decay. From the well-known behaviour in the linear region we conclude that $|s| \geq 1$ for all times. Hence the effects of the pole at $s = 0$ and the zeros in $|s| < \eta$ cancel almost completely. We may therefore, to a good approximation, regard $\gamma(s)$ as analytic also in the point $s = 0$, and as having no zero in $|s| < \eta$. It follows that $\gamma(s)$ is a polynomial

$$\gamma(s) = A(s-s_1)(s-s_2) \dots (s-s_n). \quad (18)$$

Assume now that $\gamma(s)$ has only single zeros. This is not a severe restriction because the cases with multiple zeros can be regarded as limit cases. Equation (5) can then be solved for time, t

$$t = C' + a_0 \log s + \sum_{v=1}^n a_v \log(s-s_v), \quad (19)$$

where C' is an integration constant. The constants a_v are obtained from the partial fraction expansion

$$\frac{1}{As(s-s_1)\dots(s-s_n)} = \frac{a_0}{s} + \sum_{v=1}^n \frac{a_v}{s-s_v}. \quad (20)$$

From this equation it follows that

$$\sum_{v=0}^n a_v = 0. \quad (21)$$

Using Eq. (21) in Eq. (19) we see that $t \rightarrow C'$ when $s \rightarrow \infty$. Now t must always be real. If, therefore, the integration constant C' is chosen to have an imaginary part different from zero, s can never pass through infinity. In what follows we shall always assume that the initial

condition is such that s does not pass through infinity because such paths are singular. Physically this means that if s enters the point at infinity, very small disturbances will determine the path along which s departs from infinity.

Equation (19) can be written in a slightly more convenient form if we introduce the variable

$$r \equiv 1/s \quad (22)$$

and use Eq. (21). We then obtain

$$t = C + \sum_{v=1}^n a_v \log(r-r_v) \quad (23)$$

C is here an integration constant which in general differs from C' .

The initial condition, $t = 0$ when $r = 1$, is fulfilled to a good approximation if we put $C = 0$. This is so because $|r_v| \ll 1$ for all values of v .

5. BEHAVIOUR WHEN $t \rightarrow \infty$

We are now interested in what happens to r when $t \rightarrow \infty$. We shall discuss the following three possibilities

- 1) r approaches a limit point,
- 2) r approaches a limit cycle,
- 3) neither 1) nor 2) is fulfilled.

The only points where t in Eq. (23) can become definite, are the zeros r_v . In case 1) r must therefore approach a zero. If the system point has arrived in a zero it can never get out. In the following discussion of 2) and 3) we may therefore assume that the paths we treat do not go through zeros.

A limit cycle is a closed path which the system point approaches. Closed paths for r do, however, exist only under particular circumstances. When the system has completed one revolution in a closed orbit, t has,

Figure 1 : Path close to a closed path

according to Eq. (23), changed with an amount

$$\Delta t = 2 \pi i \sum' a_v, \quad (24)$$

where the sum goes over all enclosed branch points. Δt must, however, be a real quantity, hence

$$\text{Re} \sum' a_v = 0. \quad (25)$$

If our closed curve is a limit cycle, there must exist a part L , of a path, infinitesimally different from it (see Fig. 1). Denote by δ the distance, on the normal to the curve, by which L misses its starting point after one revolution. Using Eq. (25) we obtain for L

$$\text{Im} \Delta t_L = \delta \hat{n} \cdot \nabla (\text{Im } f) = -\delta \hat{t} \cdot \nabla (\text{Re } f), \quad (26)$$

where f is the right-hand side of Eq. (23). The last equality follows from the Cauchy differential equations. Now $\dot{t} \cdot \nabla (\operatorname{Re} f)$ must evidently be different from zero. Otherwise time would not change along the path and the system point would have an infinite velocity. This is forbidden except at $r = 0$. But we have explicitly excluded paths which pass through that point. As $\operatorname{Im} \Delta t_L = 0$ it follows that $\delta = 0$ and that L is a closed path. Thus our original curve cannot be a limit cycle, and so there are no limit cycles.

Finally, we must investigate the possibility that the path approaches neither a point nor a limit cycle. Consider a curve which starts at $|r| = 1$. It never returns to $|r| = 1$. We can therefore cut the Riemann sphere along $|r| = 1$ and consider only the part $|r| < 1$ (Fig. 2). The path must necessarily go on crossing branch lines, since it does not approach a limit point, because otherwise t could not increase monotonically. Sooner or later it must cross a branch line

Figure 2 : Path in the r plane

which it has crossed before. It must then go on crossing the same branch line at a point always closer to the branch point (further away from the boundary). Otherwise it would have crossed itself somewhere, which, of course, is not allowed. The path can, however, not go on crossing the same line indefinitely, because the crossing points would then approach a limit. This means either that the path approaches a limit cycle, or a limit point, which behaviours are both contrary to the assumptions. The only remaining possibility is that the path sooner or later ceases to cross the branch line and never returns to do so again. We can then apply the same reasoning to any further branch line which the path is still crossing. Finally, there is no branch line left, but that is an impossibility, because, as we already have pointed out, the path has to go on crossing branch lines.

We conclude that there is only one possible behaviour when $t \rightarrow \infty$. The system point $r(t)$ must approach a branch point. The problem of the behaviour at infinity is therefore reduced to the question of how the system point behaves close to a zero of $\chi(s)$. This can easily be investigated from case to case. We shall only make a few general remarks on the question.

When the system point is close enough to a simple zero, so that the behaviour is dominated by it, the point spirals in toward the zero. The spiralling frequency is given by

$$\omega = \text{Im} \frac{1}{\text{Residue } (1/s\chi)} . \quad (27)$$

If the spiral has low steepness, this expression can always be used to give an approximate value for the frequency, as soon as the path runs inside the closest of the other zeros of $\chi(s)$. The multiplicity of the zero around which the path is spiralling does then not matter.

When the system point comes very close to a multiple zero, it goes straight into it without spiralling.

6. THE $K^0\bar{K}^0$ SYSTEM

We shall now discuss the possible occurrence of CP non-conserving effects in the decay of the two-particle system, $K^0\bar{K}^0$. For our purpose it is sufficient to consider the linear region. Then the terms " K_1 " and " K_2 " are meaningful, and the amplitudes A^0 and $\bar{A}^0$ for K^0 and $\bar{K}^0$ may be defined in the usual way

$$\begin{aligned} A^0 &= 2^{-\frac{1}{2}}(A_1 + A_2), \\ \bar{A}^0 &= 2^{-\frac{1}{2}}(A_1 - A_2). \end{aligned} \quad (28)$$

In the previous discussion of the one-particle system we have assumed (postulate B) that a CP non-conserving mechanism becomes very strong and forces the state to change, when

$$A_1 = 0 \quad \text{or} \quad A_2 = 0. \quad (29)$$

The region where this force is weakest, Eq. (14), contains the cases

$$A^0 = 0 \quad \text{and} \quad \bar{A}^0 = 0. \quad (30)$$

Consider now the process ^{*)}

$$\bar{p} + p \rightarrow K^0 + \bar{K}^0. \quad (31)$$

CP conservation requires (31) to occur only when the initial system is in a triplet spin state. Assuming $\bar{p}$ and p to annihilate at rest, it follows that K^0 and $\bar{K}^0$ must be in a P state. For a P state, however, we have

^{*)} We are indebted to J. Prentki for pointing out that this process may be suitable for pure K_1 or K_2 production.

$$|\ell = 1, K^0, \bar{K}^0\rangle = -|\ell = 1, K_1, K_2\rangle. \quad (32)$$

Thus the process (31) cannot be distinguished from the process

$$\bar{p} + p \rightarrow K_1 + K_2. \quad (33)$$

If now CP is not conserved, it seems as if the non-conserving mechanism were minimal for process (31) but maximal for process (33) ^{*)}.

The paradox has a very natural solution, though. We may assume quite generally, that the CP non-conserving mechanism operates when a state and its antistate interfere, and that maximal non-conservation occurs when the amounts of state and antistate are about equal. For any detectable interference to occur, it is of course necessary that the state and the antistate are different.

In the reaction (31), however, the two-particle state $|K^0, \bar{K}^0\rangle$ is its own antistate : all additive quantum numbers are zero and a CP transformation takes it over into itself. This is true whether we write the state as on the left or on the right of Eq. (32). One would therefore not expect the CP non-conserving mechanism to produce any spectacular effects neither in process (31) nor in process (33). The apparent paradox is then solved.

Consider now the states

$$|\vec{p}K^0, -\vec{p}\bar{K}^0\rangle \quad (34)$$

and

$$|\vec{p}K_1, -\vec{p}K_2\rangle, \quad (35)$$

^{*)} We are indebted to L. Van Hove for pointing this paradox out to us.

where the particles have definite momenta, $\vec{p}$. These states are different and they do also behave differently with respect to the CP non-conserving mechanism. The state (34) is its own antistate while that is not the case with (35). A CP transformation of (35) takes it over into $-|\vec{p}K_2, -\vec{p}K_1\rangle$ which obviously is a different state. Another way of seeing this is to write

$$|\vec{p}K_1, -\vec{p}K_2\rangle = \frac{1}{2} \left\{ |\vec{p}K^0, -\vec{p}K^0\rangle - |\vec{p}\bar{K}^0, -\vec{p}\bar{K}^0\rangle - |\vec{p}K^0, -\vec{p}\bar{K}^0\rangle + |\vec{p}\bar{K}^0, -\vec{p}K^0\rangle \right\} . \quad (36)$$

The two first terms have strangeness +2 and -2, respectively, and they are antistates to each other. They should therefore be sensitive to the assumed mechanism.

A quantitative treatment of the process (31) can of course not be given at present when we do not even know how to treat the one-particle case.

7. PROPOSALS FOR EXPERIMENTS

We have seen that when $t \rightarrow \infty$ the point $s(t)$ comes to rest in a zero of $\chi(s)$, thus

$$\frac{A_2(t)}{A_1(t)} = s(t) \xrightarrow{t \rightarrow \infty} \text{const.} \quad (37)$$

There follow two important consequences of (37). First, the ratio of 2π decays to 3π decays should go towards a non-vanishing limit at large times. This is the observed ^{1),3),4),6)} lack of CP conservation which our non-linear model, as well as the linear CP variant models, set out to explain.

Second, the mass of the decaying K^0 meson should at large times be independent of the mode of decay (i.e., the "invariant mass" determined in 2π and 3π decays should yield the same K_2 mass) and independent on the time when the decay takes place. It is confirmed by the experiments ^{1),4),6)} that the mass determined by the anomalous 2π decays equals the K^0 mass determined in other decays ⁵⁾. No experiment has yet been done to determine the mass of the anomalously decaying K^0 meson with a precision of the order of the K_2-K_1 mass difference.

Neither of the above consequences distinguishes between a non-linear CP invariant theory and a linear CP variant theory. That we mention the mass prediction at all, is because it seems to be a consequence essentially of the assumption of homogeneity in the dynamical Eq. (5).

We have already seen in Section 2 that the assumption of CP invariance leads to a prediction about the phase of the oscillations in two K^0 and $\bar{K}^0$ experiments : a regeneration experiment and an experiment on the original time distribution of 2π decays. What remains to be said is only that the beam does not necessarily have to be originally pure K^0 or $\bar{K}^0$. The effect could also be studied with an incoherently mixed beam, in which it is possible to vary the proportions of K^0 and $\bar{K}^0$. In a beam of long-lived kaons from an initially incoherent mixture of equal amounts of K^0 and $\bar{K}^0$, the regenerated 2π oscillations are expected to vanish according to the non-linear CP invariant theory but not according to the linear CP variant theory ^{*)}.

To find further predictions we have to make more specific assumptions in addition to CP invariance. Since all such assumptions are necessarily quite ad hoc, one should not attach excessive importance to the ensuing predictions nor expect them to be quantitatively true.

*) We are indebted to R. Mermod and B. Jordan for this suggestion.

Assume first that $\gamma(s)$ has the simplest possible structure. In the linear CP variant case, a), this corresponds to one zero ; in the non-linear CP variant case, b), this corresponds to one pair of CP conjugate zeros. Under these assumptions Eq. (18) takes specific forms, which inserted in Eq. (5) give

$$a) \quad \dot{s} = A' s(s-s_1), \quad (38)$$

$$b) \quad \dot{s} = A'' s(s^2-s_1^2). \quad (39)$$

In the linear region Eqs. (38) and (39) become approximately

$$a) \quad \dot{s} \approx -A' s_1 s, \quad (40)$$

$$b) \quad \dot{s} \approx -A'' s_1^2 s, \quad (41)$$

and close to the zero at $s = s_1$,

$$a) \quad \dot{s} \approx A' s_1 (s-s_1), \quad (42)$$

$$b) \quad \dot{s} \approx 2A'' s_1^2 (s-s_1). \quad (43)$$

In both cases (a and b) the oscillation frequency is Δ_m in the linear region. Close to the zero (at long times) the frequency in case a) is therefore Δ_m , but in case b) it is $2\Delta_m$.

It may not be easy to distinguish between the frequencies Δ_m and $2\Delta_m$. In both cases, a) and b), the system point approaches the zero rather steeply, and in consequence the 2π oscillations are heavily damped. This is evident from Fig. 3 ; when the decay rates approach their final, constant ratio, the oscillations rapidly vanish. Furthermore, the quantity Δ_m is at present not better known (from measurements in the linear region) than up to a factor two ⁵⁾.

More specific (and more ad hoc) predictions on the time distribution of 2π decays can be made if one introduces assumptions about the decay rates. A non-linear model, in which the 2π decay rate is assumed proportional to $|A_1|^2$ and the 3π decay rate proportional to $|A_2|^2$, has been described before ²⁾. If we still want

$$|A_1|^2 + |A_2|^2$$

to be the probability of finding an undecayed particle in the beam, conservation of probability requires

$$-\frac{1}{2} \frac{d}{dt} (|A_1|^2 + |A_2|^2) = \text{Re } \alpha_0 |A_1|^2 + \text{Re } \beta_0 |A_2|^2, \quad (44)$$

where α_0 and β_0 are the constants obtained in the linear limit of $\alpha(s)$ and $\beta(s)$ in Eqs. (2) and (3). Equation (44) is satisfied if $\alpha(s)$ and $\beta(s)$ have the form

$$\alpha(s) = \alpha_0 + s f(s), \quad (45)$$

$$\beta(s) = \beta_0 - s^{-1} f^*(s). \quad (46)$$

The relation between the functions $f(s)$ and $\gamma^s(s)$ can easily be found. The particular choice of $f(s)$ in Ref. ²⁾ does not satisfy Eq. (44) exactly but only to a good approximation.

The non-linear CP invariant model can again be compared to the linear CP variant model, where the same assumptions about the decay rates have been made. In both cases there are oscillations which are determined up to a phase by the known ^{1),4),6)} branching ratio for anomalous 2π decays. In Fig. 3 we have drawn the envelopes for these oscillations. The curves a and b define the envelope for the non-linear CP invariant oscillations and curves c and d the envelope for the linear CP variant oscillations, as described in Ref. ²⁾.

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R E F E R E N C E S

- 1) J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, Phys.Rev. Letters 13, 138 (1964).
- 2) B. Laurent and M. Roos, Phys.Letters 13, 269 (1964) and 15, 104 (1965).
- 3) A. Abashian, R.J. Abrams, D.W. Carpenter, G.P. Fisher, B.M.K. Nefkens and J.H. Smith, Phys.Rev.Letters 13, 243 (1964).
- 4) X. de Bouard, D. Dekkers, B. Jordan, R. Mermod, T.R. Willits, K. Winter, P. Scharff, L. Valentin, M. Vivargent and M. Bott-Bodenhausen, Phys.Letters 15, 58 (1965).
- 5) A.H. Rosenfeld, A. Barbaro-Galtieri, W.H. Barkas, P.L. Bastien, J. Kirz and M. Roos, Revs.Modern Phys. 36, 977 (1964).
- 6) W. Galbraith, G. Manning, H.E. Taylor, B.D. Jones, J. Malos, A. Astbury, N.H. Lipman and T.G. Walker, Phys.Rev.Letters 14, 383 (1965).

Figure 3 : The time distribution of 2π decays. The curves a and b are the envelopes of the oscillation in the non-linear CP invariant theory; c and d are those for the linear CP variant theory.