SYMMETRY PROPERTIES OF LEPTONS IN THE ZERO MASS LIMIT *)

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A B S T R A C T

The symmetry properties of all known leptons are studied for those processes in which their mass differences can be neglected. It is found that in this approximation both the electromagnetic interaction H_{elect} and the weak interaction H_{weak} are invariant under a $U_2 \times U_2$ group of transformations. Thus, in the zero mass limit, all lepton systems are characterized by six quantum numbers (in addition to charge, momentum, etc.). The electromagnetic interaction is known to be further invariant under a discrete symmetry operator C (charge conjugation), or P (parity). It is proposed that the weak interaction is also invariant under a discrete symmetry operator, called D , which exchanges charged leptons and neutral leptons. The C symmetry is violated by H_{weak} , and the D symmetry is violated by H_{elect} and by the mass differences.

Applications of these symmetry properties to high energy lepton reactions are discussed.

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I. INTRODUCTION

It is well known that there exists a remarkable symmetry between muons and electrons if their mass difference ($m_\mu - m_e$) is neglected. It has been shown by Feinberg and Gürsey ¹⁾ that in this approximation the free Hamiltonian, the electromagnetic interaction and the weak interaction are all invariant under a U_2 group of transformations among the leptons. The purpose of this note is to make a more complete study of the symmetry properties of leptons by neglecting the mass differences between all leptons. As will be shown in the next Section, in the zero mass limit all known interactions are invariant under a $U_2 \times U_2$ group. Thus, all lepton systems are characterized by six quantum numbers (in addition to charge, momentum, etc.). The application of this symmetry property leads to a number of relations between different leptonic processes which are valid to all orders in the electromagnetic and the weak interactions, provided the zero mass limit is a good approximation.

The question of zero mass limit in electrodynamics has been investigated in the literature ^{2),3)}. If radiative corrections are included, the power series expansions of most differential cross-sections contain logarithmic singularities as the mass of the charged particle $\rightarrow 0$. The underlying reason for having such a mass singularity is the appearance of degenerate states in the zero mass limit. Similarly to the infra-red divergence, these singularities can be completely removed for every term in the power series expansion of any transition probability, provided it is averaged over an appropriate ensemble of degenerate states. For these averages, the zero mass limit becomes a good approximation if

$$\frac{m_\ell}{E} \ll 1 \quad (1)$$

is satisfied, where m_ℓ is the mass of the charged lepton and E is, for most processes, simply the energy of the lepton. As discussed in Refs. ²⁾ and ³⁾, for any given Feynman graph the evaluation of its mass singularity is a relatively simple task. Thus, the zero mass symmetry

properties can also be applied to each individual graph which contains mass singularity, provided that the singular part is calculated separately and Eq. (1) is satisfied.

At present, there exists no fundamental theory of weak interactions. From all existing weak reactions, it appears that, so far as the leptons are concerned, only the combination

$$j_\lambda = i \psi_e^\dagger \gamma_4 \gamma_\lambda (1 + \gamma_5) \psi_{\nu_e} + i \psi_\mu^\dagger \gamma_4 \gamma_\lambda (1 + \gamma_5) \psi_{\nu_\mu} \quad (2)$$

occurs in the weak interaction Hamiltonian H_{weak} , where ψ_e , ψ_μ , ψ_{ν_e} , ψ_{ν_μ} denote, respectively, the field operators for the leptons e^- , μ^- , ν_e , ν_μ , and $\gamma_1, \dots, \gamma_5$ are the usual five anticommuting Hermitian matrices. In the following, we consider the general case

$$H_{\text{weak}} = \text{arbitrary function of } j_\lambda \text{ and } j_\lambda^\dagger, \quad (3)$$

and make no further restriction about its specific form.

Lacking a complete theory, it is not possible to make definite statements concerning the properties of mass singularity in weak interactions. However, from the general discussions given in Ref. ²⁾, and the form of H_{weak} given by Eq. (3), it is expected that to arbitrary high orders in weak interactions there also should exist no mass singularity in any appropriate ensemble averages, provided the ultra-violet divergence difficulties are solved. Furthermore, since the symmetry property is valid to all orders in all interactions, its applications to high energy leptonic processes may serve as a test of our basic assumption Eq. (3).

II. SYMMETRY PROPERTIES

The Hamiltonian density H of the known leptons can be written as :

$$H = H_{\text{free}} + H_{\text{elect}} + H_{\text{weak}} . \quad (4)$$

In order to study the zero mass limit, it is useful to decompose the free particle Hamiltonian density H_{free} into two terms

$$H_{\text{free}} = H_0 + H_m \quad (5)$$

where

$$H_m = m_e \psi_e^\dagger \gamma_4 \psi_e + m_\mu \psi_\mu^\dagger \gamma_4 \psi_\mu . \quad (6)$$

The weak interaction part of the Hamiltonian density is given by Eq. (3), and the electromagnetic part is given by

$$H_{\text{elect}} = e J_\lambda A_\lambda \quad (7)$$

where

$$J_\lambda = i \psi_e^\dagger \gamma_4 \gamma_\lambda \psi_e + i \psi_\mu^\dagger \gamma_4 \gamma_\lambda \psi_\mu \quad (8)+$$

and A_λ describes the electromagnetic field.

In Eq. (2), the neutrino fields can be chosen to satisfy

$$\gamma_5 \psi_{\nu_\ell} = \psi_{\nu_\ell} \quad (9)$$

where $\ell = e$ or μ . Similarly, the charged lepton fields ψ_ℓ can be decomposed into

$$\psi_\ell = \psi_{\ell,L} + \psi_{\ell,R} \quad (10)$$

4.

where

$$\psi_{\ell,L} = \frac{1}{2} (1 + \gamma_5) \psi_{\ell}$$

and

$$\psi_{\ell,R} = \frac{1}{2} (1 - \gamma_5) \psi_{\ell} .$$

Theorem : The Hamiltonian density $(H_0 + H_{\text{elect}} + H_{\text{weak}})$ is invariant under a $U_2 \times U_2$ group which transforms

$$\phi_L \equiv \begin{pmatrix} \psi_{e,L} \\ \psi_{\mu,L} \end{pmatrix} \rightarrow u \phi_L , \quad (11)$$

$$\phi_\nu \equiv \begin{pmatrix} \psi_{\nu_e} \\ \psi_{\nu_\mu} \end{pmatrix} \rightarrow u \phi_\nu$$

and

$$\phi_R \equiv \begin{pmatrix} \psi_{e,R} \\ \psi_{\mu,R} \end{pmatrix} \rightarrow v \phi_R , \quad (12)$$

but leaves the field operators of all other particles invariant. The matrices u and v are two arbitrary (2×2) unitary matrices.

This theorem can be easily proved by noticing that both J_λ and j_λ are invariant under these two independent U_2 transformations. The special case $u=v$ has been discussed by Feinberg and Gürsey¹⁾.

From this theorem, it follows that $(H_0 + H_{\text{elect}} + H_{\text{weak}})$ commutes with the following eight Hermitian operators :

$$N_L = \int (\phi_L^\dagger \phi_L + \phi_\nu^\dagger \phi_\nu) d^3r, \quad (13)$$

$$N_R = \int \phi_R^\dagger \phi_R d^3r, \quad (14)$$

$$L_i = \frac{1}{2} \int (\phi_L^\dagger \sigma_i \phi_L + \phi_\nu^\dagger \sigma_i \phi_\nu) d^3r \quad (15)$$

and

$$R_i = \frac{1}{2} \int \phi_R^\dagger \sigma_i \phi_R d^3r \quad (16)$$

where the σ_i ($i=1,2,3$) are the usual three anticommuting (2×2) Pauli matrices. These operators obey the following commutation relations

$$[L_a, L_b] = i \epsilon_{abc} L_c, \quad (17)$$

$$[R_a, R_b] = i \epsilon_{abc} R_c \quad (18)$$

where $\epsilon_{abc} = +1$ or -1 , depending on whether (a,b,c) is an even or odd permutation of $(1,2,3)$; otherwise, $\epsilon_{abc} = 0$. All other commutators between these eight operators are zero. In the zero mass limit, all lepton systems are characterized by six quantum numbers (in addition to charge, momentum, etc.) :

$$N_L, N_R, L, L_3, R \text{ and } R_3 \quad (19)$$

6.

where N_L , N_R , L_3 and R_3 denote the eigenvalues of the corresponding operators, $L(L+1)$ and $R(R+1)$ are, respectively, the eigenvalue of the operators $\underline{L}^2$ and $\underline{R}^2$. All non-leptons are also eigenstates of these operators but with eigenvalues = 0.

The mass term H_m breaks the symmetry. The full Hamiltonian H commutes only with

$$N \equiv N_L + N_R \quad (20)$$

and

$$M \equiv L_3 + R_3 \quad (21)$$

which are the two familiar leptonic numbers that remain conserved in the presence of the mass term. From the discussions given in the introduction, we expect the zero mass limit to be important for any high energy leptonic processes. The use of these operators $\underline{L}$, $\underline{R}$, etc., gives a number of relations between the amplitudes of different leptonic processes.

III. APPLICATIONS

It is convenient to classify the twelve single-lepton states into two $L = \frac{1}{2}$ doublets with $N_L = 1$ ($N_R = R = 0$)

$$\begin{pmatrix} e_L \\ \mu_L \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \quad (22)$$

two $L = \frac{1}{2}$ doublets with $N_L = -1$ ($N_R = R = 0$)

$$\begin{pmatrix} \bar{\mu}_L \\ -\bar{e}_L \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \bar{\nu}_\mu \\ -\bar{\nu}_e \end{pmatrix}, \quad (23)$$

one $R = \frac{1}{2}$ doublet with $N_R = 1$ ($N_L = L = 0$)

$$\begin{pmatrix} e_R \\ \mu_R \end{pmatrix} \quad (24)$$

and one $R = \frac{1}{2}$ doublet with $N_R = -1$ ($N_L = L = 0$)

$$\begin{pmatrix} \bar{\mu}_R \\ -\bar{e}_R \end{pmatrix} \quad (25)$$

where ℓ_s ($\ell = e$ or μ) denotes the charged lepton ℓ^- with helicity s and $\bar{\ell}_s$ denotes its antiparticle (i.e., one with opposite charge and opposite helicity). A particularly useful operator is

$$S = \exp [i \pi (L_2 + R_2)] \quad (26)$$

which exchanges the upper and the lower members of the doublets.

For simple reactions, the use of this exchange operator and the four conservation laws of N_L , N_R , L_3 , R_3 are sufficient to obtain almost all physically interesting results. For example, let us consider either the reaction :

$$\ell_1(\underline{q}) + A \rightarrow \ell_2(\underline{p}) + B, \quad (27)$$

or

$$A + B \rightarrow \ell_1(\underline{q}) + \ell_2(\underline{p}) + C \quad (28)$$

where A, B, C denote arbitrary systems of non-leptons and $\ell_1(\underline{q})$ [or $\ell_2(\underline{p})$] can be any one of the twelve single lepton states with momentum $\underline{q}$ [or $\underline{p}$].

Let $a[\ell_1; \ell_2]$ be the zero mass limit of the amplitude for reaction (27). The leptons ℓ_1 and ℓ_2 must have the same quantum numbers N_L, N_R, L, L_3 and R, R_3 ; otherwise, $a[\ell_1; \ell_2] = 0$. The use of S relates the amplitude for $\ell_1 = \uparrow$ with that for $\ell_1 = \downarrow$ member of the same doublet⁴⁾. Thus, we find

$$a[\ell_1(\uparrow); \ell_2(\uparrow)] = a[\ell_1(\downarrow); \ell_2(\downarrow)]. \quad (29)$$

For example, if $\ell_1(\uparrow) = \nu_e$ and $\ell_2(\uparrow) = e_L$ then Eq. (29) becomes :

$$a[\nu_e; e_L] = a[\nu_\mu; \mu_L]. \quad (30)$$

Similar expressions can be easily written down for all other choices of $\ell_1(\uparrow)$ and $\ell_2(\uparrow)$.

In an identical way, we can analyze reaction (28). Let $b[\ell_1, \ell_2]$ be the zero mass limit of its amplitude. The two leptons must have the same L and R , but their N_L, N_R, L_3 and R_3 must be of opposite signs; otherwise, $b[\ell_1, \ell_2] = 0$. The further use of S leads to the identity⁴⁾:

$$b[\ell_1(\uparrow), \ell_2(\downarrow)] = -b[\ell_1(\downarrow), \ell_2(\uparrow)]. \quad (31)$$

For example, if $l_1(\uparrow) = e_L$ and $l_2(\downarrow) = -\bar{e}_L$, Eq. (31) becomes

$$b[e_L, \bar{e}_L] = b[\mu_L, \bar{\mu}_L]. \quad (32)$$

Next, we discuss the reaction

$$l(\underline{q}) + A \rightarrow l_1(\underline{p}) + l_2(\underline{p}') + l_3(\underline{p}'') + B \quad (33)$$

where A and B are two arbitrary systems of non-leptons. Let the amplitude of reaction (33) in the zero mass limit be $c[l; l_1, l_2, l_3]$. For any given initial state l , the four conservation laws of N_L, N_R, L_3, R_3 limit the number of possible choices of l_1, l_2, l_3 for which the amplitude is not zero. These choices can be further classified into two classes :

- (α) the leptons l, l_1, l_2, l_3 all have the same quantum number $L = \frac{1}{2}$, or all have $R = \frac{1}{2}$. By using the properties of $\underline{L}$ and $\underline{R}$, we find

$$C_\alpha[l(\uparrow); l_1(\uparrow), l_2(\uparrow), l_3(\downarrow)] + C_\alpha[l(\uparrow); l_1(\uparrow), l_2(\downarrow), l_3(\uparrow)] \\ + C_\alpha[l(\uparrow); l_1(\downarrow), l_2(\uparrow), l_3(\uparrow)] = 0, \quad (34)$$

where the subscript α indicates that Eq. (34) is valid only for case (α). The use of S leads to

$$C[l(\uparrow); l_1(\uparrow), l_2(\uparrow), l_3(\downarrow)] = -C[l(\downarrow); l_1(\downarrow), l_2(\downarrow), l_3(\uparrow)],$$

$$C[l(\uparrow); l_1(\uparrow), l_2(\downarrow), l_3(\uparrow)] = -C[l(\downarrow); l_1(\downarrow), l_2(\uparrow), l_3(\downarrow)],$$

and

$$C[l(\uparrow); l_1(\downarrow), l_2(\uparrow), l_3(\uparrow)] = -C[l(\downarrow); l_1(\uparrow), l_2(\downarrow), l_3(\downarrow)]. \quad (35)$$

- (β) Two of the leptons, say, ℓ and ℓ_1 have the same quantum number $L = \frac{1}{2}$ and other two ℓ_2 and ℓ_3 have $R = \frac{1}{2}$.
 [Or, ℓ , ℓ_1 have $R = \frac{1}{2}$ and ℓ_2 , ℓ_3 have $L = \frac{1}{2}$.] In this case we have, instead of Eq. (34),

$$c_{\beta}[\ell(\uparrow); \ell_1(\uparrow), \ell_2(\uparrow), \ell_3(\downarrow)] = -c_{\beta}[\ell(\uparrow); \ell_1(\uparrow), \ell_2(\downarrow), \ell_3(\uparrow)] \quad (36)$$

where the subscript β indicates that it holds only for case (β).
 The equations listed in (35) remain valid.

As an example, the complete set of identities for $\ell = e_L$ is given below (arranged in descending order of practical importance) :

$$c[e_L; e_L, e_L, \bar{e}_L] = c[e_L; e_L, \mu_L, \bar{\mu}_L] + c[e_L; \mu_L, e_L, \bar{\mu}_L], \quad (37)$$

$$c[e_L; e_L, e_R, \bar{e}_R] = c[e_L; e_L, \mu_R, \bar{\mu}_R], \quad (38)$$

$$c[e_L; e_L, e_L, \bar{\nu}_e] = c[e_L; e_L, \mu_L, \bar{\nu}_{\mu}] + c[e_L; \mu_L, e_L, \bar{\nu}_{\mu}], \quad (39)$$

$$c[e_L; e_L, \nu_e, \bar{e}_L] = c[e_L; e_L, \nu_{\mu}, \bar{\mu}_L] + c[e_L; \mu_L, \nu_e, \bar{\mu}_L], \quad (40)$$

$$c[e_L; \nu_e, e_R, \bar{e}_R] = c[e_L; \nu_{\mu}, \mu_R, \bar{\mu}_R], \quad (41)$$

$$c[e_L; e_L, \nu_e, \bar{\nu}_e] = c[e_L; e_L, \nu_{\mu}, \bar{\nu}_{\mu}] + c[e_L; \mu_L, \nu_e, \bar{\nu}_{\mu}], \quad (42)$$

$$c[e_L; \nu_e, \nu_e, \bar{e}_L] = c[e_L; \nu_e, \nu_{\mu}, \bar{\mu}_L] + c[e_L; \nu_{\mu}, \nu_e, \bar{\mu}_L], \quad (43)$$

$$c[e_L; \nu_e, \nu_e, \bar{\nu}_e] = c[e_L; \nu_e, \nu_{\mu}, \bar{\nu}_{\mu}] + c[e_L, \nu_{\mu}, \nu_e, \bar{\nu}_{\mu}] \quad (44)$$

and all other unrelated amplitudes $c[\underline{e}_L; \underline{l}_1, \underline{l}_2, \underline{l}_3] = 0$.

These identities are valid for arbitrary non-leptonic systems A and B. For different identities, A and B can (and sometimes must) be different. All above identities are also valid for any given momentum distribution. The different momentum distributions are related by the antisymmetric relation

$$\begin{aligned} c[\underline{l}; \underline{l}_1(\underline{p}), \underline{l}_2(\underline{p}'), \underline{l}_3(\underline{p}'')] &= -c[\underline{l}; \underline{l}_2(\underline{p}'), \underline{l}_1(\underline{p}), \underline{l}_3(\underline{p}'')] \\ &= -c[\underline{l}; \underline{l}_1(\underline{p}), \underline{l}_3(\underline{p}''), \underline{l}_2(\underline{p}')] \end{aligned} \quad (45)$$

In Eqs. (34)-(44), all amplitudes $c[\underline{l}; \underline{l}_1, \underline{l}_2, \underline{l}_3]$ refer to that for the momentum distribution given by (33). Thus, e.g., $c[\underline{e}_L; \underline{e}_L, \underline{\mu}_L, \underline{\bar{\mu}}_L]$ denotes $c[\underline{e}_L; \underline{e}_L(\underline{p}), \underline{\mu}_L(\underline{p}'), \underline{\bar{\mu}}_L(\underline{p}'')]$, which for $\underline{p} \neq \underline{p}'$, is not related to $c[\underline{e}_L; \underline{\mu}_L, \underline{e}_L, \underline{\bar{\mu}}_L] = c[\underline{e}_L; \underline{\mu}_L(\underline{p}), \underline{e}_L(\underline{p}'), \underline{\bar{\mu}}_L(\underline{p}'')]$.

As a final example, we discuss the reaction

$$\underline{l}_1(\underline{q}) + \underline{l}_2(-\underline{q}) \rightarrow \underline{l}_3(\underline{p}) + \underline{l}_4(\underline{p}') + A \quad (46)$$

where A is, again, any system of non-leptons. Let $d[\underline{l}_1, \underline{l}_2; \underline{l}_3, \underline{l}_4]$ be the zero mass limit of the amplitude for reaction (46). As before, the conservation laws of N_L, N_R, L_3 and R_3 limit the different choices of lepton states. These states can be further classified as follows:

- (α) The four leptons $\underline{l}_1, \underline{l}_2, \underline{l}_3, \underline{l}_4$ all have the same quantum number $L = \frac{1}{2}$, or all have $R = \frac{1}{2}$. By using the symmetry properties, we have

$$\begin{aligned} d_\alpha[\underline{l}_1(\uparrow), \underline{l}_2(\downarrow); \underline{l}_3(\uparrow), \underline{l}_4(\downarrow)] + d_\alpha[\underline{l}_1(\uparrow), \underline{l}_2(\downarrow); \underline{l}_3(\downarrow), \underline{l}_4(\uparrow)] \\ - d_\alpha[\underline{l}_1(\uparrow), \underline{l}_2(\uparrow); \underline{l}_3(\uparrow), \underline{l}_4(\uparrow)] = 0 \end{aligned} \quad (47)$$

For example, when $\ell_1(\uparrow) = \ell_3(\uparrow) = e_L$ and $\ell_2(\uparrow) = \ell_4(\uparrow) = \bar{\mu}_L$, Eq. (47) becomes

$$d[e_L, \bar{e}_L; e_L, \bar{e}_L] - d[e_L, \bar{e}_L; \mu_L, \bar{\mu}_L] - d[e_L, \bar{\mu}_L; e_L, \bar{\mu}_L] \quad (48)$$

$$= 0.$$

- (β) The two initial leptons ℓ_1, ℓ_2 have the same quantum number $L = \frac{1}{2}$, but the two final ones ℓ_3, ℓ_4 both have $R = \frac{1}{2}$; or, the initial leptons both have $R = \frac{1}{2}$ and the final ones both have $L = \frac{1}{2}$. In this case, we have

$$d_\beta[\ell_1(\uparrow), \ell_2(\downarrow); \ell_3(\uparrow), \ell_4(\downarrow)] = -d_\beta[\ell_1(\uparrow), \ell_2(\downarrow); \ell_3(\downarrow), \ell_4(\uparrow)] \quad (49)$$

- (γ) One of the initial leptons, say ℓ_1 , has $L = \frac{1}{2}$ and the other one ℓ_2 has $R = \frac{1}{2}$. Thus, one of the final leptons, say ℓ_3 , must have $L = \frac{1}{2}$ and the other one ℓ_4 has $R = \frac{1}{2}$. We find

$$d_\gamma[\ell_1(\uparrow), \ell_2(\uparrow); \ell_3(\uparrow), \ell_4(\uparrow)] = -d_\gamma[\ell_1(\uparrow), \ell_2(\downarrow); \ell_3(\uparrow), \ell_4(\downarrow)] \quad (50)$$

where the subscripts α, β, γ indicate the cases in which these equations are valid.

The application of S leads to the following identities which are valid for all cases :

$$d[l_1(\uparrow), l_2(\uparrow); l_3(\uparrow), l_4(\uparrow)] = d[l_1(\downarrow), l_2(\downarrow); l_3(\downarrow), l_4(\downarrow)],$$

$$d[l_1(\uparrow), l_2(\downarrow); l_3(\uparrow), l_4(\downarrow)] = d[l_1(\downarrow), l_2(\uparrow); l_3(\downarrow), l_4(\uparrow)]$$

and

$$d[l_1(\uparrow), l_2(\downarrow); l_3(\downarrow), l_4(\uparrow)] = d[l_1(\downarrow), l_2(\uparrow); l_3(\uparrow), l_4(\downarrow)].$$

All these relations are valid to all orders of H_{elect} and H_{weak} . They can be applied to any Feynman graph (or sum of graphs) for a high energy leptonic process if it does not contain mass singularity or if its mass singularity has already been separated out.

IV. DISCRETE SYMMETRIES

Throughout this paper, we study only those symmetry properties of leptons that are independent of space and time. In the zero mass limit, there are twelve field operators: $\psi_{\nu_e}, \psi_{\nu_\mu}, \psi_{e,L}, \psi_{e,R}, \psi_{\mu,L}, \psi_{\mu,R}$ and their conjugate operators $\psi_{\nu_e}^c, \psi_{\nu_\mu}^c, \dots, \psi_{\mu,R}^c$. In the representation $\gamma_1, \gamma_2, \gamma_3$ are real and γ_4, γ_5 are imaginary, the conjugate field operator of ψ_ℓ is related to its Hermitian conjugate $\psi_\ell^\dagger$ by

$$(\psi_\ell^c)_a = (\psi_\ell^\dagger)_a \quad (51)$$

where $a = 1, 2, 3, 4$, and $\ell = (\nu_e)$, or (ν_μ) , or $(e, L), \dots$, or (μ, R) . The operators $\psi_{\nu_e}, \psi_{\nu_\mu}, \psi_{e,L}, \psi_{\mu,L}$ and $\psi_{e,R}^c, \psi_{\mu,R}^c$ satisfy the same set of anticommutation relations (which is different from that satisfied by $\psi_{\nu_e}^c, \psi_{\nu_\mu}^c, \psi_{e,L}^c, \psi_{\mu,L}^c$ and $\psi_{e,R}, \psi_{\mu,R}$). The quantization rules and H_0 are invariant under a U_6 group of transformations among these six field operators at the same space time point. This U_6 symmetry is broken by the interactions of leptons.

It can be shown that the only subgroup of U_6 which leaves the currents J_λ and j_λ invariant is the $U_2 \times U_2$ group given by Eqs. (11) and (12), according to which these six operators are grouped into three doublets, ϕ_L, ϕ_ν and ϕ_R^c . We know that the electromagnetic interaction is invariant under charge conjugation C which connects ϕ_L with ϕ_R^c ,

$$C \phi_L C^\dagger = \phi_R^c \quad (52)$$

and

$$C^2 = 1 \quad (53)$$

The electromagnetic current changes its sign under C , but

$$C H_{elect} C^\dagger = H_{elect} \quad (54)$$

The charge conjugation symmetry is broken by the weak interaction. The similarity in the roles of the three doublets ϕ_L , ϕ_ν and ϕ_R^c leads us to introduce another discrete operator D , defined by

$$D \phi_L D^\dagger = \phi_\nu \quad (55)$$

and

$$D^2 = 1$$

The leptonic current j_λ transforms into j_λ^* under D ; i.e.,

$$D j_\lambda D^\dagger = j_\lambda^* \quad (56)$$

where $j_\lambda^* = j_\lambda^\dagger$ for $\lambda \neq 4$, but $-j_\lambda^* = j_4^\dagger$ for $\lambda = 4$. It is natural to impose the condition

$$D H_{weak} D^\dagger = H_{weak} \quad (57)$$

The D symmetry is broken by the electromagnetic interaction.

The symmetry property of C requires that all (leptonic or non-leptonic) electromagnetic current should change sign under C . Similarly, the requirement of D symmetry [Eq. (57)] requires that all non-leptonic currents in weak interactions⁵⁾ should obey a similar transformation under D as that given by Eq. (56).

The D symmetry leads to identities which, e.g., equate the amplitude for reaction

$$\nu_e + A \rightleftharpoons e_L + B \quad (58)$$

with that for

$$e_L + A^D \rightleftharpoons \nu_e + B^D \quad (59)$$

in the zero mass limit, where A and B are arbitrary systems of non-leptons, and A^D , B^D are their respective D conjugate system. For example, if $A=n$, $B=p$, then $A^D=p$ and $B^D=n$. The consequence of D symmetry ⁶⁾ is valid to all orders of H_{weak} , but is violated by H_{elect} and by the mass differences.

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- 4) Throughout this section, we use $\ell_i(\uparrow)$ and $\ell_i(\downarrow)$ to denote, respectively, the two $L_3 = +\frac{1}{2}$ and $L_3 = -\frac{1}{2}$ members (or, $R_3 = +\frac{1}{2}$ and $R_3 = -\frac{1}{2}$ members) of any $L = \frac{1}{2}$ doublet (or, $R = \frac{1}{2}$ doublet).
- 5) This implies a symmetry requirement also for the strongly interacting particles, which can be satisfied in many models of strong interactions. A full discussion will be given elsewhere.
- 6) To lowest order in weak interaction and for leptonic interaction with non-strange particles, D symmetry is the same as the $|\Delta I| = 1$ rule. See T.D. Lee and C.N. Yang, Phys.Rev. 119, 1410 (1960).