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RADIATIVE CHARGED LEPTON PRODUCTION BY NEUTRINOSJ. Løvseth ^{*)} and J.D. Walecka ^{**)}CERN
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We have calculated the cross-section for production of high-energy photons from the reaction $\nu_l + n \rightarrow l^- + \gamma + p$. We first calculate for point weak interactions and then include the structure effects of the strong interaction of the form factors in two different ways: first by simply multiplying the point amplitude by an appropriate form factor and then by making a model of the form factors and attributing them to meson exchanges. The results for the ratio of the radiative to the "elastic" cross-section are relatively insensitive to the different approximations. Numerical results are given for photon spectra and integrated cross-sections in the case of incident ν_μ . The cross-section for production of a photon with energy greater than 100 MeV is about 1.5 % of the elastic cross-section when averaged over the CERN neutrino spectrum, and about 0.4% for photons with energy greater than 500 MeV.

1. INTRODUCTION

The existence of high energy mu-neutrino beams has made possible a study of the reaction ^{1),2)}

$$\nu_{\mu} + n \longrightarrow \mu^{-} + p \quad (1.1)$$

A natural companion of such a reaction is the radiative process

$$\nu_{\ell} + n \longrightarrow \ell^{-} + \gamma + p \quad (1.2)$$

which can be expected to lead to a high energy photon with a frequency of roughly $\alpha = 1/137$ with respect to the "elastic" process (1.1). Apart from its inherent interest as one of the fundamental processes in nature, this reaction becomes important as a source of background in the search for an intermediate vector boson which one might hope to see via the chain

$$\begin{aligned} \nu_{\mu} + Z &\longrightarrow \mu^{-} + W^{+} + Z \\ &\quad \quad \quad \searrow \\ &\quad \quad \quad e^{+} + \nu_e \end{aligned} \quad (1.3)$$

The signature of such an event in a spark chamber is a long non-interacting track (the μ^{-}) and a shower (from the e^{+}). It is known that such events occur with a frequency which is less than, or of the order of, 1% of the elastic events (1.1) ²⁾. Therefore, unless one can unambiguously distinguish an electron from a photon induced shower, the reaction (2.2) must be an important source of background.

Unfortunately, in contrast to the situation in electron scattering,

$$e^{-} + p \longrightarrow e^{-} + p + \gamma \quad (1.4)$$

the process (1.2) is not so easily calculated. The reason for this difficulty is that while in process (1.4) one can follow the electron charge by itself continuously through the diagram and thus take into account the radiation coming from the electron (the most important source of radiation

because of its mass) in a calculation involving only quantum electrodynamics, this is not possible in process (1.2). In process (1.2), if one follows the charge, it runs from the charged lepton through the weak vertex and out with the proton. Thus it is not possible to separate the radiation of the charged lepton in a gauge invariant fashion, and one is at the outset involved in problems of radiation coming from strongly interacting systems. One must thus relinquish at the beginning the idea of making a really quantitative calculation of the cross-section for process (1.2). However, because of its interest, one would like to have some estimates. If the weak interaction has the form

$$H_W = \frac{G}{\sqrt{2}} \left[\bar{\psi}_{(l)} \gamma_\lambda (1 + \gamma_5) \psi_{(l)} \right] \left[\bar{\psi}_{(p)} \gamma_\lambda (1 + \gamma_5) \psi_{(n)} \right] \quad (1.5)$$

$$G = \frac{1.01 \times 10^{-5}}{M^2}$$

and the nucleon can be treated as a point Dirac particle, then the differential photon spectrum for process (1.2) can be calculated analytically. The two Feynman graphs are illustrated in Fig. 1 and the calculation is carried out in Section 2. In this calculation, as well as the following, we give the ratio of the radiative cross-section to the elastic cross-section calculated under the same set of assumptions. This ratio should be much less sensitive to our approximations.

The first evident deficiency of such a simple calculation is that the weakly interacting leptons are coupled to a strong nucleon vertex which has a structure and thus form factors limiting the momentum transferred at the vertex. The main problem here is how to include these form factors in a gauge invariant fashion. One expects that because of its mass, the lepton will be responsible for most of the radiation. Thus one wants to treat process (A) of Fig. 1 as accurately as possible. One can take into account the effect of the form factors here by simply multiplying the amplitude by $F_V(q^2)$ where $q \equiv P - P'$. [We will assume here that $F_A(q^2) \equiv F_V(q^2)$.] In order to keep the amplitude gauge invariant,

however, we must multiply the whole amplitude by this form factor. This calculation is also carried out in Section 2. One can say that this calculation treats graph (A) of Fig. 1 correctly and then uses gauge invariance to dictate the contribution of graph (B).

The fundamental problem remains, however, that the charge flows through the weak vertex and thus one must make some model of this vertex in order to make a more complete, gauge invariant calculation. A reasonable model is to assume that the vector and axial vector form factors come from the exchange of a vector or axial vector meson (taken to have the same mass for simplicity) and that the induced pseudoscalar interaction comes from the exchange of a pion. One can thus write down a set of diagrams as indicated in Fig. 2. Since the weak magnetism part of the weak interactions, as well as the coupling of a pion to the leptons, is effectively a derivative coupling, one must put in counter terms [i.e., $\partial/\partial x_\mu \rightarrow \partial/\partial x_\mu - ieA_\mu$] to preserve gauge invariance. This is indicated in graphs (D) and (I) of Fig. 2. The only unknowns here are the anomalous magnetic moments of the mesons. Because of their large anomalous moments, one cannot forget the magnetic radiation of the nucleons, although to calculate this one must once more make a model, since the anomalous moments come from meson effects and it is not correct to use just the free nucleon-photon vertex when the nucleon is off the mass shell. Making a model, however, leads to graphs (B,C,G) and (H) of Fig. 2. We give the gauge invariant amplitude for the processes illustrated in Fig. 2 in the Appendix.

In this expression we have combined several terms and neglected all terms proportional to the lepton mass m_l (the diligent reader can easily supply all these extra terms if so inclined). The vector meson is taken to be coupled to the nucleon current through $\bar{u}(P') [\gamma_\lambda + \frac{\lambda_V}{2M} \sigma_{\lambda\rho} q_\rho] u(P)$ while the axial vector is coupled through $\bar{u}(P') \gamma_\lambda \gamma_5 u(P)$. The terms going as $(q_\mu q_\nu)/m^2$ in the vector meson propagators either give an effective induced pseudoscalar coupling or something proportional to the lepton mass *).

*) The matrix element of the effective axial vector nucleon current entering into the weak interactions in this model is divergenceless at momentum transfers large compared to the mass of the pion.

The expression is still sufficiently complicated so that evaluation of all the traces, even by computer, has so far proved intractable. One can again try to start from the idea that the most important contribution should be from the lepton radiation, and then evaluate the squares of the remaining terms to see if they are really small. One can treat graph (A) of Fig. 2 correctly and then keep the convection current parts of the graphs (B,...,J) in order to have a gauge invariant amplitude. This is also done in Section 2. In Section 3 we examine the corrections to this approximation and present a discussion of our results.

2. CALCULATIONS

A. Point Weak Interactions

In this Section we calculate the radiative production of charged leptons by the processes illustrated in Fig. 1. The weak vertex is taken to be just the point interaction of Eq. (1.5). Thus one finds by the usual Feynman rules ^{*)}

$$d\sigma = \frac{2G^2\alpha}{(2\pi)^4} \frac{1}{[-l \cdot P]} \delta^{(4)}(P+l-P'-l'-k) \frac{d\vec{k}}{2K} \frac{d\vec{l}'}{2\varepsilon'} \frac{d\vec{P}'}{2E'} |M|^2 \quad (2.1)$$

$$|M|^2 = \sum_{\text{spins}} \sum_{\text{pol.}} (\varepsilon \varepsilon' M \varepsilon') |e_\mu M_\mu|^2 \quad (2.2)$$

$$\begin{aligned} e_\mu M_\mu = & \left[\bar{u}(P) \gamma_\lambda (1+\gamma_5) u(P) \right] \left[\bar{u}(l') \not{\varepsilon} \frac{1}{i(\not{l}'+\not{k})+m_l} \gamma_\lambda (1+\gamma_5) u(l) \right] - \\ & - \left[\bar{u}(P') \not{\varepsilon} \frac{1}{i(\not{P}'+\not{k})+M} \gamma_\lambda (1+\gamma_5) u(P) \right] \left[\bar{u}(l) \gamma_\lambda (1+\gamma_5) u(l) \right] \end{aligned} \quad (2.3)$$

^{*)} We use $\hbar = c = 1$ and a metric $v_\mu = (\vec{v}, iv_0)$ with Hermitian γ matrices, $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2 \delta_{\mu\nu}$.

We note that one can just sum over incident neutrino spins with a fully polarized incident neutrino beam since the $(1 + \gamma_5)$ in the matrix element is a spin projection operator (there is no factor of $\frac{1}{2}$ which one would get by averaging over incident neutrino spins). One immediately verifies

$$K_\mu M_\mu = 0 \quad (2.4)$$

so that the amplitude is gauge invariant. The spin and covariant polarization sum can now be carried out and one finds

$$|M|^2 = -16(P \cdot l) \left\{ \frac{P' \cdot K}{l' \cdot K} + \frac{l' \cdot K}{P' \cdot K} + 2 + \left(P \cdot l + \frac{m_l^2}{2} \right) \left[\frac{m_l^2}{(l' \cdot K)^2} + \frac{M^2}{(P' \cdot K)^2} + \frac{2 P' \cdot l'}{(P' \cdot K)(l' \cdot K)} \right] \right\} \quad (2.5)$$

The invariant lepton and nucleon phase space integrals can be done by going to the frame where $\vec{P} + \vec{Q} + \vec{K} = 0$ (centre of momentum of the final lepton and nucleon) and taking the $\hat{z}$ axis along $\hat{K}$. The result is

$$d\sigma_8 = \frac{G^2 \alpha}{\pi^3} \frac{d\vec{K}}{2K} \frac{[(\Delta^2 - m_l^2 - M^2)^2 - 4m_l^2 M^2]^{1/2}}{2\Delta^2} \left\{ \frac{\beta_1 + \beta_2}{\beta_1 \beta_2} \left[\ln \frac{1 + \beta_1}{1 - \beta_1} + \ln \frac{1 + \beta_2}{1 - \beta_2} \right] + \right. \\ \left. + 2 \left(x_0 - \frac{m_l^2}{2} \right) \frac{\Delta^2}{(x_1 + x_2)^2} \left[\frac{1 + \beta_1 \beta_2}{\beta_1 + \beta_2} \left(\ln \frac{1 + \beta_1}{1 - \beta_1} + \ln \frac{1 + \beta_2}{1 - \beta_2} \right) - 2 \right] \right\} \quad (2.6)$$

where we have defined

$$\begin{aligned} x_0 &= -l \cdot P = M\varepsilon \\ x_1 &= -K \cdot P = MK \\ x_2 &= -l \cdot K = K\varepsilon(1 - \cos \Theta_K) \end{aligned} \quad (2.7)$$

and

$$\Delta = [2(X_0 - X_1 - X_2) + M^2]^{1/2}$$

$$\beta_1 = \frac{[(\Delta^2 - m_l^2 - M^2)^2 - 4m_l^2 M^2]^{1/2}}{\Delta^2 - M^2 + m_l^2} \quad (2.8)$$

$$\beta_2 = \frac{[(\Delta^2 - m_l^2 - M^2)^2 - 4m_l^2 M^2]^{1/2}}{\Delta^2 + M^2 - m_l^2}$$

In the same approximation, we can write for the elastic cross-section

$$\sigma_{el.} = \frac{4G^2}{\pi} \frac{[(2X_0 - m_l^2)^2 - 4m_l^2 M^2]^{1/2}}{2(M^2 + 2X_0)} \left(X_0 - \frac{m_l^2}{2}\right) \quad (2.9)$$

Therefore we can write for the photon energy spectrum

$$\frac{1}{\sigma_{el.}} \frac{d\sigma_{el.}}{dX_1} = \frac{\alpha}{2\pi} \frac{M^2 + 2X_0}{X_0 \left(X_0 - \frac{m_l^2}{2}\right) [(2X_0 - m_l^2)^2 - 4m_l^2 M^2]^{1/2}} \int_0^{\bar{X}_2} dX_2 \frac{[(\Delta^2 - m_l^2 - M^2)^2 - 4m_l^2 M^2]^{1/2}}{2\Delta^2} \quad (2.10)$$

$$\left\{ \frac{\beta_1 + \beta_2}{\beta_1 \beta_2} \left[\ln \frac{1 + \beta_1}{1 - \beta_1} + \ln \frac{1 + \beta_2}{1 - \beta_2} \right] + 2 \left(X_0 - \frac{m_l^2}{2}\right) \frac{\Delta^2}{(X_1 + X_2)^2} \left[\frac{1 + \beta_1 \beta_2}{\beta_1 + \beta_2} \left(\ln \frac{1 + \beta_1}{1 - \beta_1} + \ln \frac{1 + \beta_2}{1 - \beta_2} \right) - 2 \right] \right\}$$

where

$$\bar{X}_2 = \text{MINIMUM} \begin{cases} \frac{2}{M^2} X_0 X_1 \\ X_0 - X_1 - M_l M - \frac{1}{2} m_l^2 \end{cases}$$

These results are plotted in Figs. 3 and 4 for the case

$$\nu_{\mu} + n \rightarrow \mu^{-} + \gamma + p$$

and incident neutrino energies of 2 GeV and 5 GeV ^{*)}.

One limiting case helps one to get a feeling for the results. We give the result for $M \rightarrow \infty$ and $m_{\ell} \rightarrow 0$

$$\frac{1}{\sigma_{el}} \frac{d\sigma_{\gamma}}{dK} \xrightarrow[M_{\ell} \rightarrow 0]{M = \infty} \frac{\alpha}{\pi} \left\{ \ln \frac{2(\varepsilon - K)}{m_{\ell}} + 2 \frac{\varepsilon(\varepsilon - K)}{K^2} \left[\ln \frac{2(\varepsilon - K)}{m_{\ell}} - 1 \right] \right\} \left(\frac{K}{\varepsilon} \right) \frac{1}{\varepsilon} \quad (2.11)$$

Thus one sees that, in addition to the $1/K$ spectrum coming from the lepton charge, there is an additive term which gives a spectrum which goes as K and thus becomes the most important term for large K . This term arises because of the creation of the charged lepton's magnetic moment. A similar situation exists in K capture of electrons where the electron magnetic moment disappears ³⁾.

B. Form Factors in Weak Interactions

In this Section we include the form factors for the weak interactions by multiplying the amplitudes in Eq. (2.3) by

$$F_V(q^2) = \frac{1}{(1 + 1.25 \frac{q^2}{M^2})^2} \quad (2.12)$$

$$q = P - P'$$

^{*)} Note that as $K \rightarrow 0$ we have an infra-red divergence, and we must combine the bremsstrahlung with the radiative corrections to the elastic process to get finite results in the usual fashion. We are not interested in that region of the photon spectrum in this paper.

Equation (2.12) is a phenomenological approximation for the electromagnetic form factors, which make the elastic neutrino cross-section constant at high energies. This treats graph (A) of Fig. 1 correctly, if we assume $F_A(q^2) = F_V(q^2)$ and neglect the weak magnetism and induced pseudoscalar interactions, and leaves the whole amplitude gauge invariant. The phase space integral in (2.1) was carried out numerically and the results then divided by the elastic cross-section calculated under the same approximations ($F_A = F_V$ and neglect of weak magnetism and induced pseudoscalar terms). These results are plotted in Figs. 3 and 4. One sees that the form factors tend to enhance the high energy photon part of the spectrum. In Fig. 5 we plot as a function of incident neutrino energy the total elastic cross-section computed under the above assumptions, as well as the total radiative cross-section to produce photons with energy greater than 100 MeV and 500 MeV. We shall return to these results in the discussion.

C. Model of the Weak Vertex

In this Section we give the result of computing with graph (A) of Fig. 2 and the convection current parts of the other graphs in Fig. 2. Thus we take

$$\eta_\mu = -i \left[\bar{u}(l') \left\{ Q_\mu + \frac{1}{2l' \cdot K} \frac{1}{(1 + g^2/m^2)} \gamma_\mu K \right\} \gamma_5 (1 + \gamma_5) u(l) \right] \left[\bar{u}(P') \left\{ \gamma_5 (1 + a \gamma_5) + \frac{\lambda_V}{2M} \sigma_{\lambda\rho} g_\rho \right\} u(P) \right] \quad (2.13)$$

where we have used the following definitions :

$$Q_\mu = \frac{1}{1 + g^2/m^2} \left(\frac{l'_\mu}{l' \cdot K} - \frac{Q_\mu}{g \cdot K} \right) + \frac{1}{1 + g'^2/m^2} \left(\frac{g_\mu}{g \cdot K} - \frac{P'_\mu}{P' \cdot K} \right) \quad (2.14)$$

m = mass of intermediate vector and axial vector mesons

$$\begin{aligned}
q' &= \ell' - \ell = P - P' - K \\
\lambda_V &= \lambda_p - \lambda_n = 3.70 \\
a &= -F_A(0)/F_V(0) = 1.15 .
\end{aligned}$$

In this model one obtains a one pole approximation for the form factors which asymptotically makes the elastic cross-section increase with the logarithm of the neutrino energy. We take $m = 550$ MeV which fits reasonably with recent data for the electromagnetic vector form factors ⁴⁾. The amplitude is clearly gauge invariant,

$$K_\mu M_\mu = 0$$

and treats the lepton convection current and magnetization current correctly ^{*)}. We have neglected the induced pseudoscalar interaction because of the extra $1/(q^2 + m_\pi^2)$ denominator [the induced pseudoscalar term makes a negligible contribution to the elastic neutrino cross-section] and because an analysis of the terms that go as $\bar{u}(P') \gamma_5 u(P)$ in Eq. (A.3) of the Appendix shows that these terms tend to cancel each other. [We shall discuss the corrections to Eq. (2.13) in the next Section.] One finds, in the notation of Eq. (2.1)

$$|M|^2 = -2Q^2 S(\ell') + \frac{2}{\ell' \cdot K} \frac{1}{(1 + q^2/m^2)} \left[\frac{1}{(1 + q^2/m^2)} - \ell' \cdot Q \right] S(K) + \frac{2}{(1 + q^2/m^2)} S(Q) \quad (2.15)$$

with

$$\begin{aligned}
S(v) &\equiv \left[\lambda_V (1 + \lambda_V) - \frac{1}{2} \lambda_V^2 \left(1 + \frac{q^2}{4M^2} \right) \right] \left[2(\ell \cdot [P + P']) (v \cdot [P + P']) + (q^2 + 4M^2)(\ell \cdot v) \right] - \\
&\quad - 2M^2(\ell \cdot v) [(1 + \lambda_V)^2 - a^2] - 2(1 + \lambda_V - a)^2 (v \cdot P)(\ell \cdot P') - 2(1 + \lambda_V + a)^2 (v \cdot P')(\ell \cdot P) .
\end{aligned} \quad (2.16)$$

^{*)} In our computation starting from Eq. (2.13) we do not neglect the lepton mass.

The elastic cross-section computed under the same set of assumptions is given by

$$\sigma_{el.} = \frac{G^2}{(2\pi)^2} \frac{1}{EM} \iint \frac{d\vec{l}'}{2\varepsilon'} \frac{d\vec{p}'}{2E'} S^{(4)}(l+p-l'-p') [-2S(l')] \quad (2.17)$$

The resulting photon spectrum is given in Figs. 3 and 4. In Fig. 6 we again plot the elastic cross-section and total radiative cross-section to produce photons with energy greater than 100 MeV and 500 MeV.

3. DISCUSSION

The results of our calculations are presented in Figs. 3-6. In Figs. 3 and 4 we have given the high energy photon spectrum calculated in our three different approximations as discussed in the text :

- i) point V-A weak coupling to the nucleon;
- ii) V-A weak coupling to the nucleon including form factors;
- iii) model of the form factors as due to vector, axial vector, and π meson exchange (magnetic coupling included).

For ii) and iii) we have also put in a correction to take the exclusion principle into account, assuming a Fermi gas model for the nucleus with Fermi momentum equal to 265 MeV/c. We see that as long as we always normalize to the elastic cross-section, there is not a great deal of difference in the results. The form factors in approximation ii) increase the high energy part of the photon spectrum while a more detailed model including the weak magnetism term then decreases it slightly. This can be qualitatively understood as follows : phase space favours equal momenta for the outgoing particles. The form factors distort phase space

and decrease the momentum transferred to the nucleon, thus increasing the photon energy. The weak magnetic coupling, since it goes as q , tends to increase the momentum transferred to the nucleon, thus reversing the process slightly. Because of the creation of the magnetic moment of the lepton, one has more high energy photons than a simple $1/K$ bremsstrahlung spectrum might lead one to expect.

The individual cross-sections for elastic scattering and for production of photons with energies greater than 100 MeV and 500 MeV computed under approximations ii) and iii) are given in Figs. 5 and 6, respectively. The radiative cross-sections grow rapidly with energy while the elastic cross-sections level off at high energy. We have integrated the curves over the CERN neutrino spectrum and find the following ratios :

approximation ii) :

$$\left. \frac{\sigma_\gamma}{\sigma_{el.}} \right)_{\mu^-} = 0.40 \% \quad K_\gamma \geq 500 \text{ MeV}$$

$$= 1.5 \% \quad K_\gamma \geq 100 \text{ MeV} ;$$

approximation iii) :

$$\left. \frac{\sigma_\gamma}{\sigma_{el.}} \right)_{\mu^-} = 0.34 \% \quad K_\gamma \geq 500 \text{ MeV}$$

$$= 1.4 \% \quad K_\gamma \geq 100 \text{ MeV} .$$

The final question is how large are the contributions we have neglected in the above approximations. In answer to this we have computed the squares of the neglected (gauge invariant) contributions of the amplitude given in the Appendix. We have computed :

- a) the squares of the terms involving the coupling of the anomalous moments of the neutron and proton to the photon $\left[M_\mu^{(B)} \text{ and } M_\mu^{(C)} \right];$

b) the square of the term going as

$$\frac{\lambda_V}{2M} \bar{u}(P') \left[\sigma_{\lambda\mu} - \frac{P_\mu'}{P' \cdot K} \sigma_{\lambda\rho} g_{\rho\mu} \right] u(P)$$

which comes from the term dictated by gauge invariance $\left[\bar{M}_\mu^{(D)} \right]$ and what is left over when we extract the convection current part of $M_\mu^{(B)}$ in Eqs. (2.13) and (2.14).

c) the square of the sum of the contributions going as

$$\bar{u}(P') \gamma_5 u(P)$$

$\left[\text{excluding those in the term proportional to } (1+\bar{K}) \text{ in } M_\mu^{(E)} + M_\mu^{(I)} \right]$.

The ratio of these terms (in percent) to the result of approximation iii) is indicated in the Table as a function of $\mathcal{E}$. The contributions are all small giving us some confidence in our results although if all the corrections were to add in phase they could substantially modify our result. The rising fractional contributions of the anomalous moment terms comes from the fact that we have computed them using just the static anomalous moments of the nucleons. At these high energies, $\mathcal{E} = 10$ GeV, this approximation is almost certainly wrong as strong interaction effects will modify this coupling. The contributions of c) are very small because in the approximation of neglecting the lepton mass, the terms involving the pion mass disappear from this amplitude and one is left with a term involving two propagators $\sim 1/(q^2+m^2) 1/(q'^2+m^2)$. For the same reason, the term going as $1+\bar{K}$, which is the only term we have not considered may be expected to be small. Since nothing is known about $\bar{K}$, one can do nothing more with this (it would be interesting to have the whole cross-section as a function of $\bar{K}$). Our above considerations are correct for $\bar{K} = -1$.

T A B L E

Relative magnitude in percent of the squares of the terms
neglected in approximation iii), "vector meson exchange"
 $K_\gamma \geq 500 \text{ MeV.}$

E_γ/GeV	a) Neutron Anomalous Moment	a) Proton Anomalous Moment	b) "Gauge Terms"	c) Pseudoscalar Terms
1	3.4	3.0	3.2	0.05
2	4.5	4.0	3.8	0.17
5	5.9	5.2	4.9	0.35
10	7.6	6.7	6.2	0.48

In summary then, the relative insensitvety of our results for the ratio of cross-sections to the approximations mode and the small size of the correction terms indicate that our numerical results are probably fairly reliable, the accuracy increasing from approximations i) to iii). However, we emphasize again that our results depend on the radiation from the strongly interacting system and therefore a really quantitative calculation is not possible.

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A P P E N D I X

We give the amplitude corresponding to the graphs of Fig. 2. We have set all terms proportional to m_l equal to zero. The differential cross-section in the laboratory system corresponding to these amplitudes is given by

$$d\sigma = \frac{2G^2\alpha}{(2\pi)^4} \frac{1}{\varepsilon M} \delta^{(4)}(P+l-P'-l'-K) |M|^2 \frac{d\vec{K}}{2K} \frac{d\vec{l}'}{2\varepsilon'} \frac{d\vec{P}'}{2E'} \quad (A.1)$$

$$|M|^2 = \sum_{\text{spins}} \sum_{\text{pol.}} (\varepsilon \varepsilon' M \varepsilon') |e_\mu M_\mu|^2 \quad (A.2)$$

where we have

$$M_\mu^{(A)} + M_\mu^{(F)} = \bar{u}(l') \gamma_\mu \frac{1}{i(\not{P} + \not{K}) + m_l} \gamma_\lambda (1 + \gamma_5) u(l) \frac{1}{1 + \not{q}^2/m^2} \bar{u}(P') \left[\gamma_\lambda (1 + a\gamma_5) + \frac{\lambda_\nu}{2M} \sigma_{\lambda\nu} q'_\nu + \frac{2Ma(1 - \frac{m_\pi^2}{m^2})}{\not{q}^2 + m_\pi^2} i\gamma_5 \right] u(P) \quad (A.3)$$

$$M_\mu^{(B)} = -\bar{u}(l') \gamma_\lambda (1 + \gamma_5) u(l) \frac{1}{1 + \not{q}^2/m^2} \bar{u}(P') \left\{ \left(\gamma_\mu + \frac{\lambda_\nu}{2M} \sigma_{\mu\nu} K_\nu \right) \frac{1}{i(\not{P}' + \not{K}) + M} \left[\gamma_\lambda (1 + a\gamma_5) + \frac{\lambda_\nu}{2M} \sigma_{\lambda\nu} q'_\nu \right] \right\} u(P)$$

$$M_\mu^{(C)} = -\bar{u}(l') \gamma_\lambda (1 + \gamma_5) u(l) \frac{1}{1 + \not{q}^2/m^2} \bar{u}(P') \left\{ \left[\gamma_\lambda (1 + a\gamma_5) + \frac{\lambda_\nu}{2M} \sigma_{\lambda\nu} q'_\nu \right] \frac{1}{i(\not{P} - \not{K}) + M} \left(\frac{\lambda_m}{2M} \sigma_{\mu\nu} K_\nu \right) \right\} u(P)$$

$$M_{\mu}^{(D)} = i \bar{u}(l') \gamma_{\lambda} (1 + \gamma_5) u(l) \frac{1}{1 + q'^2/m^2} \bar{u}(p') \left\{ \frac{\lambda_v}{2M} \sigma_{\lambda\mu} \right\} u(p)$$

$$M_{\mu}^{(E)} + M_{\mu}^{(F)} = i \bar{u}(l') \gamma_{\lambda} (1 + \gamma_5) u(l) \frac{1}{1 + q'^2/m^2} \frac{(-2q_{\mu})}{q'^2 + m^2} \bar{u}(p') \left\{ \gamma_{\lambda} (1 + a \gamma_5) + \frac{\lambda_v}{2M} \sigma_{\lambda\rho} q_{\rho} - \frac{2Ma}{m^2} i q_{\lambda} \gamma_5 \right\} u(p) -$$

$$- i \bar{u}(l') \gamma_{\lambda} (1 + \gamma_5) u(l) \frac{1}{1 + q'^2/m^2} \frac{1}{q'^2 + m^2} [(1 + K)(K_{\sigma\mu} S_{\lambda} - K S_{\lambda\mu\sigma})] \bar{u}(p') \left\{ \gamma_{\lambda} (1 + a \gamma_5) + \frac{\lambda_v}{2M} \sigma_{\lambda\rho} q_{\rho} - \frac{2Ma}{m^2} i q_{\lambda} \gamma_5 \right\} u(p) +$$

$$+ i \bar{u}(l') \gamma_{\lambda} (1 + \gamma_5) u(l) \left[\frac{1}{q'^2 + m_{\pi}^2} - \frac{1}{q'^2 + m^2} \right] \bar{u}(p') [2Ma i \delta_{\lambda\mu} \gamma_5] u(p)$$

We use the definitions

m = vector (axial vector) meson mass $\cong 550$ MeV

M = nucleon mass

$\lambda_n = -1.91$

$\lambda_p = 1.79$

$\lambda_v = \lambda_p - \lambda_n = 3.70$

K = anomalous moment of vector and axial vector mesons -
(assumed the same for convenience)

$a = -F_A(0)/F_V(0) = 1.15$

$q = p - p'$

$q' = l' - l = q - K.$

We have used the Goldberger-Treiman relation ⁵⁾ to relate the pion lifetime to the axial vector coupling constant, that is if the pion-lepton vertex is written $iq_\lambda f_\pi$ and the pion-nucleon vertex is $\sqrt{2}g_{\pi N} \gamma_5$ then $\sqrt{2}g_{\pi N} f_\pi \cong 2Ma$.

One easily establishes (to terms proportional to m_ℓ) that

$$K_\mu M_\mu = 0 \quad (\text{A.4})$$

so that our amplitude is gauge invariant.

FIGURE CAPTIONS

- Figure 1 Feynman Diagrams for $\nu_{\ell} + n \rightarrow p + \ell^{-} + \gamma$ in a point interaction model.
- Figure 2 Feynman Diagrams for $\nu_{\ell} + n \rightarrow p + \ell^{-} + \gamma$ with meson exchange to account for nuclear form factors.
- Figure 3 The photon energy spectrum given as a ratio to the elastic cross-section with 2 GeV incident neutrino energy for the three models considered :
- i) point weak interactions
 - ii) point weak interactions multiplied by form factors
 - iii) vector, axial vector and pi meson exchange model of the strong interaction form factors (labeled "vector meson exchange").
- Figure 4 Same as Fig. 3 for 5 GeV incident neutrino energy.
- Figure 5 Elastic and radiative cross-sections for model ii), "pure" V-A weak coupling multiplied by a form factor.
- Figure 6 Elastic and radiative cross-sections for model iii), vector, axial vector and pi meson exchange model of the strong interaction form factors.

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(A)

$$l + P = l' + P' + K$$

(B)

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG.6