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CP VIOLATION AND SU_3 SYMMETRY

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A B S T R A C T

A generalization of the Cabibbo rotation for the weak hadronic current is proposed within the SU_3 symmetry. The results of the Cabibbo theory are not modified. In leptonic processes CP violation appears only for $\Delta S=1$. A parameter ω is defined through the difference of generalized rotations of the axial and vector parts, which measures the amount of CP violation. Leptonic and non-leptonic processes attain a maximum of CP violation for $\omega = \pi/2$, the violation being large for the former but only a few times per thousand for the latter as compared to CP conserving processes. The decay of neutral K mesons is studied and other experimental tests of the theory considered.

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The experiment of Christenson et al.¹⁾ has shown the existence of CP violating processes in the non-leptonic decays of neutral K mesons. A number of different schemes have been proposed to incorporate the small observed CP violation into the theory of weak interactions.

In particular^{2),3)}, it has been suggested that the hadronic current may have a mixed CP character, thus being responsible for the CP violation in non-leptonic decays and for possible (as yet unobserved) violations of CP in leptonic decays.

A mixed CP character of the hadronic current can be achieved in a number of different ways. The purpose of this paper is to generate such a current with no assumptions foreign to its well-known SU_3 nature, and to study some of the experimental consequences which would follow from it.

In order to introduce our proposal, let us recall the essential idea in the construction of a universal hadronic current for the ordinary CP conserving leptonic decay Hamiltonian of Cabibbo. It amounts to saying that the natural hadronic weak entity is associated with a privileged direction in SU_3 , obtained by rotating the $\Delta S = 0$ isotopic vector in a charge conserving way. The rotation $e^{2i\theta F_7} = e^{\sqrt{6}\theta(E_3 - E_{-3})}$ does not commute with the strangeness operator, thus providing a $\Delta S = 0$ and $\Delta S = 1$ mixture from a $\Delta S = 0$ object.

Following this same spirit, it would seem natural to build a hadronic current with a mixed character under CP by further rotating the universal Cabibbo current with an SU_3 operator not commuting with CP. Since this assumption does not go outside the frame of the usual SU_3 treatment of weak currents, it would seem a rather conservative way of introducing CP violation, and it seems interesting to compare its implications with those of other proposals.

From an aesthetic point of view, it would seem attractive to postulate just one new parameter associated with the new CP violating rotation for both the vector and axial vector currents. In other words, one could rotate both currents by means of the general charge conserving rotation in SU_3 :

$$R = e^{2i(\theta F_7 + \varphi F_6)} \quad (1)$$

where the rotation associated with θ would be the ordinary Cabibbo rotation and the new rotation associated with φ would give rise to CP non-conserving effects.

However, it so happens that this simplest possibility introduces no observable effects, since it just gives rise to a phase factor which can in turn be absorbed through a redefinition of the relative phases of particles belonging to multiplets with different strangeness.

But one may perhaps capitalize on this failure, arguing first that what it really shows is that the general rotation R is just as good and plausible as Cabibbo's rotation. And second that the equality of Cabibbo's rotation for the axial and vector currents is only a good approximation, since the rotation angles are known to be slightly different. The idea then suggests itself that the weak vector and axial vector currents are indeed rotated by the general rotation R in SU_3 space. In a first approximation, both rotations can be taken to be equal, and the ordinary Cabibbo theory is obtained. In a more refined approach, the rotations are different and as a consequence CP violating effects appear while all the results of the ordinary theory still hold.

Following standard notations, the universal hadronic vector current will be accordingly defined by

$$J_V = R_V^{-1} (J_V^1 + i J_V^2) R_V$$

and the axial current by

$$J_A = R_A^{-1} (J_A^1 + i J_A^2) R_A$$

where $R_{V,A}$ is given by (1) with different parameters for each case.
A simple calculation yields

$$J_{V,A} = \cos \sqrt{\theta_{V,A}^2 + \varphi_{V,A}^2} (J_{V,A}^1 + i J_{V,A}^2) + \frac{\theta_{V,A} + i \varphi_{V,A}}{\sqrt{\theta_{V,A}^2 + \varphi_{V,A}^2}} \sin \sqrt{\theta_{V,A}^2 + \varphi_{V,A}^2} (J_{V,A}^4 + i J_{V,A}^5) \quad (2)$$

The actual universal hadronic current J_U would be

$$J_U = J_V + J_A \quad (3)$$

Since one can always fix the arbitrary phase of the $\Delta S=1$ part of our current, so as to compensate for example the phase of its vector component, we write (3) in the form

$$J_U = a_V (J_V^1 + i J_V^2) + b_V (J_V^4 + i J_V^5) + a_A (J_A^1 + J_A^2) + b_A e^{i\omega} (J_A^4 + i J_A^5) \quad (4)$$

where

$$a_{V,A} = \cos \sqrt{\theta_{V,A}^2 + \varphi_{V,A}^2}$$

$$b_{V,A} = \sin \sqrt{\theta_{V,A}^2 + \varphi_{V,A}^2}$$

and the non-trivial phase ω is :

$$e^{i\omega} = \frac{\theta_A + i \varphi_A}{\theta_V + i \varphi_V} \sqrt{\frac{\theta_V^2 + \varphi_V^2}{\theta_A^2 + \varphi_A^2}} \quad (5)$$

In order to show the transformation properties of J_U under CP, we recall that axial and vector currents behave in such a way under C and P that ⁴⁾ :

$$CP J_{V,A}^i (CP)^{-1} = \eta(i) J_{V,A}^i$$

where

$$\eta(i) = \begin{cases} +1 & \text{for } i = 1, 3, 4, 6, 8 \\ -1 & \text{for } i = 2, 5, 7 \end{cases}$$

It is then clear that because of the self-hermitian character of the operators $J_{V,A}^i$:

$$(J_{V,A}^i)^+ = J_{V,A}^i$$

we have

$$\begin{aligned} CP J_U (CP)^{-1} &= CP (J_U^R + J_U^I) (CP)^{-1} = \\ &= (J_U^R)^+ + (J_U^I)^+ \end{aligned}$$

where we have split the universal current (4) into a "regular part" J_U^R and an "irregular part" J_U^I , the irregular part being given by

$$J_U^I = i \sin \omega \cdot b_A (J_A^4 + i J_A^5) \quad (6)$$

We now write the semi-leptonic Hamiltonian H_{SL} in the usual way

$$\begin{aligned} H_{SL} &= \frac{G}{\sqrt{2}} \left\{ (J_U)^\lambda \ell_\lambda + (J_U)^{\lambda\dagger} \ell_\lambda \right\} = \\ &= \frac{G}{\sqrt{2}} \left\{ (J_U^R)^\lambda \ell_\lambda + (J_U^I)^\lambda \ell_\lambda + (J_U^R)^{\lambda\dagger} \ell_\lambda + (J_U^I)^{\lambda\dagger} \ell_\lambda \right\} \quad (7) \end{aligned}$$

Under the CP operation, H_{SL} goes into

$$CP H_{SL} (CP)^{-1} = \frac{G}{\sqrt{2}} \left\{ (J_U^R)^{\lambda\dagger} \ell_\lambda - (J_U^I)^{\lambda\dagger} \ell_\lambda + (J_U^R)^\lambda \ell_\lambda - (J_U^I)^\lambda \ell_\lambda \right\}$$

which shows its mixed character under CP.

It should be remarked first that (to lowest order in weak interactions) the CP violating term in (7) is strangeness violating. Therefore, $\Delta S=0$ weak processes even at high momentum transfer should show with this theory no CP violation. Thus, a search for transverse muon polarization in neutrino induced interactions ⁵⁾ should provide a test of the theory. $\Delta S=0$ processes ought to give no polarization, while the reverse should be true if $\omega \neq 0$ for $\Delta S=1$ processes.

Secondly, no polarization perpendicular to the decay plane should be observed in the decay muons of K^+ in $K^+ \rightarrow \pi^0 + \mu^+ + \nu$ since the process involves only the vector current and an interference between vector and axial vector current terms is essential in our theory to get CP violation.

Thirdly, the Cabibbo analysis of K and hyperon decays can be repeated throughout. The only inessential difference is a change in the parametrization. Instead of having $\theta_{A,Cab}$ $\theta_{V,Cab}$ as parameters, we would now have, respectively, $(\theta_A^2 + \varphi_A^2)^{\frac{1}{2}}$, $(\theta_V^2 + \varphi_V^2)^{\frac{1}{2}}$. For the actual value of these parameters we will take ^{6),7)} :

$$\sqrt{\theta_A^2 + \varphi_A^2} = 0.264$$

$$\sqrt{\theta_V^2 + \varphi_V^2} = 0.242$$

Next, we construct the non-leptonic Hamiltonian as a product of hadronic currents :

$$H_{NL} = \bar{J}_U \bar{J}_U^+ + \bar{J}_U^+ \bar{J}_U \quad (8)$$

In order to be more specific, we will assume that there is an "octet enhancement" of H_{NL} , so that only that part of H_{NL} which lies on 8-S will be considered.

For any product of currents (axial vector or vector) we have

$$\begin{aligned}\left\{(\bar{J}^1 - i\bar{J}^2)(\bar{J}^1 + i\bar{J}^2)\right\}_{8S} &= \frac{2}{\sqrt{3}} H^8 \\ \left\{(\bar{J}^1 - i\bar{J}^2)(\bar{J}^4 + i\bar{J}^5)\right\}_{8S} &= H^6 + iH^7 \\ \left\{(\bar{J}^4 - i\bar{J}^5)(\bar{J}^1 + i\bar{J}^2)\right\}_{8S} &= H^6 - iH^7 \\ \left\{(\bar{J}^4 - i\bar{J}^5)(\bar{J}^4 + i\bar{J}^5)\right\}_{8S} &= H^3 - \frac{1}{\sqrt{3}} H^8\end{aligned}$$

where the H^i are members of a new octet, whose parity violating part $H_{P=+1}^i$ has $C=-1$, whereas its parity preserving part $H_{P=-1}^i$ has $C=+1$. We are only interested in that part of H_{NL} which carries $|\Delta S| = 1$. The rest of H_{NL} is CP conserving and we omit it for simplicity. We then have :

$$\begin{aligned}(H_{NL})_{|\Delta S|=1} &= (a_V b_V + a_A b_A e^{i\omega}) (H_{P=1}^6 + iH_{P=1}^7) + (a_A b_V + a_V b_A e^{i\omega}) (H_{P=-1}^6 + iH_{P=-1}^7) + \\ &\quad + h.c. \quad (8) \quad (9)\end{aligned}$$

Let us call $g_1 = g(a_V b_V + a_A b_A e^{i\omega})$, $g_2 = g(a_A b_V + a_V b_A e^{i\omega})$, g being an over-all weak coupling constant, and $g_1/g_2 = \rho e^{i\xi}$. Then the phases of g_2 and g_2^* for example are irrelevant, since they can be absorbed into the parity violating terms, but the imaginary part of $\rho e^{i\xi}$ gives rise to a CP violating term in H_{NL} , which is thus split into

$$\begin{aligned}\rho \cos \xi H_{P=1}^6 + H_{P=-1}^6 &\quad (CP = 1 \text{ part}) \\ -\rho \sin \xi H_{P=1}^7 &\quad (CP = -1 \text{ part})\end{aligned} \quad (10)$$

We have

$$\rho \sin \xi = \frac{b_A b_V (a_A^2 - a_V^2) \sin \omega}{b_V^2 (a_A^2 + a_V^2) + [b_A^2 (a_A^2 + a_V^2) + 4 a_V a_A b_V b_A] \cos^2 \omega}$$

$$\rho \cos \xi = \frac{a_A a_V (b_A^2 + b_V^2) + b_A b_V (a_A^2 + a_V^2) \cos \omega}{b_V^2 (a_A^2 + a_V^2) + [b_A^2 (a_A^2 + a_V^2) + 4 a_V a_A b_V b_A] \cos^2 \omega}$$

which gives a measure of the amount of CP violation in non-leptonic processes. It is important to notice that a maximum of CP violation in leptonic transitions, corresponding to $\omega = \pi/2$ also gives a maximum CP violation in non-leptonic processes, but since in this case

$$\rho \sin \xi = \frac{b_A (a_A^2 - a_V^2)}{b_V (a_A^2 + a_V^2)} \cong 5 \times 10^{-3}$$

for our accepted values, the relative amount of CP violating over CP conserving processes cannot exceed a few times per thousand.

We now discuss the decay of the K^0 mesons, utilizing our Hamiltonian (10). We follow closely Refs. ^{9), 10)} where a detailed study is carried out. The K_L^0 and K_S^0 (long and short-lived decaying neutral mesons) are the eigenfunctions of the equation

$$(\Gamma + iM) K^0 = \lambda K^0$$

where Γ is a Hermitian 2×2 matrix depending on the matrix elements of H_{NL} , and iM is an antihermitian matrix giving the mass corrections to the K^0 .

The solutions of this equation are :

$$K_S^0 = \begin{pmatrix} p \\ q \end{pmatrix} \frac{1}{\sqrt{p^2 + q^2}} e^{-i\lambda_S t}, \quad K_L^0 = \begin{pmatrix} p \\ -q \end{pmatrix} \frac{1}{\sqrt{p^2 + q^2}} e^{-i\lambda_L t}$$

where, in the notation of Ref. 10) :

$$p^2 = A_0^2 + x_{3\pi} + iy_{3\pi} + iM_r - M_i$$

$$q^2 = A_0^2 + x_{3\pi} - iy_{3\pi} + iM_r + M_i$$

A_0^2 is the square of the decay amplitude $K \rightarrow 2\pi$, with the 2π system in the $I=0$ state (no other state of the 2π , and no semi-leptonic amplitudes are considered here since they would be forbidden in our Hamiltonian). A_0 is supposed real, which is in agreement with our choice of phase for $K^0, \bar{K}^0$ in (10). $x_{3\pi} + iy_{3\pi}$ is the square of the decay amplitude $K \rightarrow 3\pi$, and $M_{12} = M_r + iM_i$ is the off diagonal element of the mass matrix.

We must now split M_{12} in two parts coming respectively from the parity conserving and parity violating parts of H_{NL}

$$M_{12} = (M_{12})_{P=1} + (M_{12})_{P=-1}$$

Since these terms arise from second order weak interactions, it is difficult to know what is their relative magnitude. On general grounds, even if coming essentially from 3π and 2π virtual intermediate states, the first would not be so depressed with respect to the second as the decay amplitudes are.

An interesting calculation has been performed by Zimmerman, Riazuddin and Okubo ¹¹⁾ which yields a high value for both $(M_{12})_{P=1}$ and $(M_{12})_{P=-1}$ but opposite signs, pointing to a possible accidental cancellation of both contributions in order to have the experimental value $2|M_{12}| \sim 10^{-5}$ eV. Following for example this calculation, one sees that cross terms (products of $P=1$ and $P=-1$ parts of H_{NL}) do not contribute to M_{12} , so that in our case one gets $(M_{12})_{P=1} = -36 \text{ MeV} \times (g_1/G)^2$, where G is the medium strong coupling constant. A crude estimate $\frac{1}{20} < |G|^2 < \frac{1}{10}$, $|g_1|^2 \sim 10^{-13}$ makes M_{12} about one order of magnitude too large.

Coming back to the expressions for p^2 and q^2 , one sees that the term $(M_{12})_{P=-1}$, as calculated from (10), would be real as A_0^2 , while $(M_{12})_{P=1}$ has the phase of $g_1^2/g_2^2 = p^2 e^{2i\xi}$. Therefore

$$\begin{aligned} p^2 &= A_0^2 + x_{3\pi} + iy_{3\pi} + i(M_{12})_{P=-1} - i|(M_{12})_{P=1}| e^{2i\xi} \\ q^2 &= A_0^2 + x_{3\pi} - iy_{3\pi} + i(M_{12})_{P=-1} - i|(M_{12})_{P=1}| e^{-2i\xi} \end{aligned}$$

The amplitude ratio for decays into $\pi^+\pi^-$ is given by

$$\left| \frac{K_L \rightarrow 2\pi}{K_S \rightarrow 2\pi} \right| = \left| \frac{p-q}{p+q} \right| = 2.1 \times 10^{-3}$$

We can drop the $x_{3\pi} + iy_{3\pi}$ term since it is known to be very small compared to A_0^2 and cannot explain by itself the experimental results in the absence of an imaginary term¹⁰⁾.

Then, since the quantity $p-q/p+q$ is very small, $M_1 = -\sin 2\xi \times |(M_{12})_{P=1}|$ must be small and we have in first order

$$\frac{p-q}{p+q} = \frac{1}{2} \frac{\sin 2\xi |(M_{12})_{P=1}|}{A_0^2 + iM_r}$$

where $M_r = (M_{12})_{P=-1} - \cos \xi |(M_{12})_{P=1}|$ is known experimentally to be of the order of magnitude of A_0^2 .

For a high value of $|(M_{12})_{P=1}|$ of the order of M_r as suggested above, we would have $\xi \simeq 3 \times 10^{-3}$. Only for $|(M_{12})_{P=1}| \lesssim 5 \times 10^{-3} M_r$ one would be unable to explain the CP violating decays of K^0 along these lines.

A good experimental determination of the angle ξ would be obtained in our theory by means of the asymmetry parameters α, β in the non-leptonic hyperon decays. Supposing the interaction to be of the type :

$$H_{NL} = \{ \bar{u}_B \gamma_\mu (g_2 + g_1 \gamma^5) u_Y \} \partial_\mu \phi$$

then it can be shown ¹²⁾ that the amplitude rate for final relative s or p wave in the decay $\Lambda \rightarrow p + \pi^-$ for example would be

$$\frac{A_p}{A_s} = - \frac{g_1}{g_2} \frac{m_\Lambda + m_p}{m_\Lambda - m_p} \left(\frac{p_p}{E_p + m_p} \right) \sim .7 \left| \frac{g_1}{g_2} \right| e^{-i\xi}$$

Since the asymmetry parameters are defined

$$\alpha = - \frac{2 \operatorname{Re}(A_s A_p^*)}{|A_s|^2 + |A_p|^2}, \quad \beta = - \frac{2 \operatorname{Im}(A_s A_p^*)}{|A_s|^2 + |A_p|^2}$$

we have

$$\frac{\beta}{\alpha} = -\tan \xi$$

The experimental result for β is 0.19 ± 0.19 , which is compatible with 0 (which is required by CP invariance) if final state interactions are taken into account. However, more refined experiments might show also here a CP violation, and provide a means of measuring the angle ξ . In any event, it would have to be a very small angle, in accordance with what was obtained for the CP violating neutral K decays.

In conclusion, we wish to point out that our suggestion for a possible CP violation in weak interactions introduces a minimum of changes in its present SU_3 structure and makes rather definite predictions both for leptonic and non-leptonic interactions. It is therefore hoped that forthcoming experiments will substantiate it or completely disprove it.

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