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AN APPLICATION OF RELATIVISTIC $SU(6)$ INVARIANCE
TO THE THEORY OF WEAK SEMILEPTONIC INTERACTIONS

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We make use of a relativistic generalization of $SU(6)$ invariance which was studied in two preceding papers ^{1),2)}. This model is based on a higher symmetry group which contains the inhomogeneous Lorentz group and the $SU(3)$ symmetry group as subgroups. Applying the model to the theory of weak semileptonic interactions, we generalize essentially Cabibbo's theory ³⁾. Cabibbo's angle θ remains a phenomenological parameter which cannot be determined by our theory.

Our method treats the leptonic current

$$\ell_\mu = \bar{\Psi}_{e,\mu}(q_1) \gamma_\mu (1 - \gamma_5) \Psi_\nu(q_2)$$

as part of the regular representation of $SU(6)$, where this group $SU(6)$ is defined as the little group belonging to the four-momentum vector p_3 ,

$$p_3 = q_1 + q_2.$$

We ascribe $SU(6)$ transformation properties to the leptonic current in the following manner. With the help of Cabibbo's angle θ we define

$$L^{\mu i}_j = \frac{1}{\sqrt{2}} [\cos \theta (\lambda_1 - i\lambda_2) + \sin \theta (\lambda_4 - i\lambda_5)]^i_j \cdot e^\mu$$

and finally

$$e^{ab} = \sigma_\mu^{\alpha\beta} L^{\mu i}_j.$$

The $SL(6, C)$ indices a and b correspond to $(\alpha, i), (\beta, j)$. Generally we can couple the leptonic current e^{ab} to the current of strongly interacting particles in the four different ways

$$J_{ba} e^{ab} + J^b_a e^a_b + J_{\dot{b}}^{\dot{a}} e_{\dot{a}}^{\dot{b}} + J^{ba} e_{\dot{a}\dot{b}}.$$

2.

where we define

$$\begin{aligned} e^a_b &= \frac{p_3 c b}{\kappa} e^{ac}, \\ e_a^b &= \frac{p_3 a c}{\kappa} e^{cb}, \quad \kappa^2 = p_3^2, \\ e_{ab} &= \frac{p_3 a c}{\kappa} e^{cd} \frac{p_3 d b}{\kappa}. \end{aligned}$$

The denominators κ have been introduced in a way analogous to the treatment of the meson field ²⁾. The meson decay is certainly the most simple case we can discuss. Because p_3 is the meson momentum, all four contributions are equivalent if we make use of the meson field equations. We get

$$M_{ba} e^{ab}$$

where M_{ba} is the meson field. Splitting this into longitudinal and transversal components and using $SL(2, C) \otimes SU(3)$ indices, we get ^{1), 2)}

$$+2 \left[\frac{p_3^\mu}{\kappa} p_i^\mu - T^\mu \right] \left[\frac{1}{12} \cos \Theta (\lambda_1 - i\lambda_2) + \frac{1}{12} \sin \Theta (\lambda_4 - i\lambda_5) \right]^\mu e_\mu$$

We apply now our model to baryon decays. In this case the currents J_{ba}^b , J_a^b , $J_b^{\dot{a}}$, $J^{b\dot{a}}$ have the same form as those known from meson-baryon interaction, with one exception. Parity violation allows for polar and axial contributions simultaneously. We write down all twelve possible couplings :

$$\begin{aligned} d_1^+ &= \bar{\Psi}_1^{\dot{a}}{}_{bc} \Psi_2^c{}_{da} e^{bd}, & d_1^- &= \bar{\Psi}_1^{\dot{a}}{}_{bc} \Psi_2^c{}_{da} e^{bd}, \\ d_2^+ &= \bar{\Psi}_1^{\dot{a}}{}_{bc} \Psi_2^c{}_{da} e^{bd}, & d_2^- &= \bar{\Psi}_1^{\dot{a}}{}_{bc} \Psi_2^c{}_{da} e^{bd}, \\ d_3^+ &= \bar{\Psi}_1^{\dot{a}}{}_{bc} \Psi_2^c{}_{da} e^{bd}, & d_3^- &= \bar{\Psi}_1^{\dot{a}}{}_{bc} \Psi_2^c{}_{da} e^{bd}, \end{aligned}$$

$$\begin{aligned}
m_1^+ &= \bar{\psi}_1^{\dot{a}}{}_{bc} \psi_2^d{}^{bc} \ell^{\dot{a}}{}_{\dot{d}} \quad , \quad m_1^- = \bar{\psi}_1^{\dot{a}}{}_{bc} \psi_2^d{}^{bc} \ell^{\dot{a}}{}_{\dot{d}} \quad , \\
m_2^+ &= \bar{\psi}_1^{\dot{a}}{}_{\dot{b}c} \psi_2^d{}^{bc} \ell^{\dot{a}}{}_{\dot{d}} \quad , \quad m_2^- = \bar{\psi}_1^{\dot{a}}{}_{\dot{b}c} \psi_2^d{}^{bc} \ell^{\dot{a}}{}_{\dot{d}} \quad , \\
m_3^+ &= \bar{\psi}_1^{\dot{a}}{}_{bc} \psi_2^d{}^{bc} \ell^{\dot{a}}{}_{\dot{d}} \quad , \quad m_3^- = \bar{\psi}_1^{\dot{a}}{}_{bc} \psi_2^d{}^{bc} \ell^{\dot{a}}{}_{\dot{d}} \quad .
\end{aligned}$$

Then we introduce twelve parameters $\alpha_i^{\pm}(\kappa^2)$, $\beta_i^{\pm}(\kappa^2)$, $i=1,2,3$, such that

$$\begin{aligned}
\mathcal{L}_{int} &= \frac{G}{12} \left\{ \sum_i (\alpha_i^+ d_i^+ + \alpha_i^- d_i^-) + \frac{2\kappa}{K_1+K_2} \sum_i (\beta_i^+ m_i^+ + \beta_i^- m_i^-) \right\} \\
&\quad + \text{c.c.}
\end{aligned}$$

We consider only the baryon contribution to this Lagrangian and write the results in the form

$$\begin{aligned}
&V_1 \cdot \bar{B}_1 \gamma_\mu B_2 + V_2 \frac{-i}{K_1+K_2} \bar{B}_1 \sigma_{\mu\nu} p_3^\nu B_2 + V_3 \frac{1}{K_1+K_2} \bar{B}_1 B_2 p_{3\mu} \\
&+ A_1 \bar{B}_1 i \gamma_\mu \gamma_5 B_2 + A_2 \frac{1}{K_1+K_2} \bar{B}_1 \sigma_{\mu\nu} p_3^\nu \gamma_5 B_2 + A_3 \frac{i}{K_1+K_2} \bar{B}_1 \gamma_5 B_2 p_{3\mu}
\end{aligned}$$

where B_1, B_2 are Dirac four spinors with four momenta p_1, p_2 and masses K_1, K_2 , respectively. The functions V_i, A_i of κ^2, K_1, K_2 have been calculated. Of course, we have to distinguish F and D type contributions. We use the six Lorentz covariants given above for convenience of comparison with known data. There exists another interesting representation of these Lorentz covariants which shows the difference between the couplings d^+, m^+ and d^-, m^- clearer. Looking at an intermediate step of our calculation, we see that d_i^+ contributes only to the covariants :

$$\gamma_\mu (1 - i \gamma_5), \quad (p_1 \pm p_2)_\mu (1 \pm i \gamma_5)$$

$$d_i^- \quad \text{to} \quad \gamma_\mu (1 + i\gamma_5), \quad (p_1 \pm p_2)_\mu (1 \pm i\gamma_5),$$

$$m_i^+ \quad \text{to} \quad i\sigma_{\mu\nu} (1 - i\gamma_5) p_3^\nu, \quad (p_1 \pm p_2)_\mu (1 \pm i\gamma_5),$$

$$m_i^- \quad \text{to} \quad i\sigma_{\mu\nu} (1 + i\gamma_5) p_3^\nu, \quad (p_1 \pm p_2)_\mu (1 \pm i\gamma_5).$$

Therefore we need only five Lorentz covariants in each case.

As is shown by our final results, reality of all the parameters $\alpha_i^\pm, \beta_i^\pm$ is sufficient for CP invariance ^{*)}.

We discuss the results for the limit $K_1 = K_2$, $\kappa^2 \rightarrow 0$. Assuming that the values of $\alpha_i^\pm, \beta_i^\pm$ for $\kappa^2 = 0$ do not necessarily vanish, the induced pseudoscalar couplings possess a pole for $\kappa^2 = 0$ which is a consequence of our definition of ℓ_{ab} . We get (forgetting about a factor $(18K)^{-1}$):

*) Defining a charge conjugation transformation before reducing the symmetry to $SL(2, C) \otimes SU(3)$ seems to be difficult. For $SL(2, C)$, this transformation consists of the two steps

$$\begin{pmatrix} \psi_\alpha \\ \bar{\psi}_\alpha \end{pmatrix} \rightarrow \begin{pmatrix} \bar{\psi}_\alpha \\ \psi_\alpha \end{pmatrix} \rightarrow \begin{pmatrix} \psi^c_\alpha \\ \bar{\psi}^c_\alpha \end{pmatrix} = \begin{pmatrix} 0 & \varepsilon_{\alpha\beta} \\ \varepsilon^{\alpha\beta} & 0 \end{pmatrix} \begin{pmatrix} \bar{\psi}_\beta \\ \psi_\beta \end{pmatrix},$$

the second one of which cannot be generalized for the group $SL(6, C)$.

$$V_1^D = 0,$$

$$V_1^F = 3(\alpha^+ - \alpha^-), \quad \alpha^\pm = \sum_i \alpha_i^\pm, \quad \beta^\pm = \sum_i \beta_i^\pm,$$

$$V_2^D = 3(\alpha^+ - \alpha^-) + 6(\beta^+ - \beta^-),$$

$$V_2^F = -(\alpha^+ - \alpha^-) + 4(\beta^+ - \beta^-),$$

$$A_1^D = -3(\alpha^+ + \alpha^-),$$

$$A_1^F = -2(\alpha^+ + \alpha^-),$$

$$\left(\frac{\pi^2}{K^2} A_3^D\right) = 24\alpha^-,$$

$$\left(\frac{\pi^2}{K^2} A_3^F\right) = 16\alpha^-,$$

$$V_3^D = -3(\alpha_2^+ - \alpha_3^+) - 3(\alpha_2^- - \alpha_3^-),$$

$$V_3^F = +(\alpha_2^+ - \alpha_3^+) + (\alpha_2^- - \alpha_3^-) + 6(\beta^+ + \beta^-),$$

$$A_2^D = -3(\alpha_2^+ - \alpha_3^+) - 3(\alpha_2^- - \alpha_3^-) - 6(\beta^+ + \beta^-),$$

$$A_2^F = +(\alpha_2^+ - \alpha_3^+) + (\alpha_2^- - \alpha_3^-) - 4(\beta^+ + \beta^-).$$

We remark that the coupling constants for the currents of the first class can all be expressed by the experimentally well-known parameters

$$\mu_p = 2 \frac{\beta^+ - \beta^-}{\alpha^+ - \alpha^-},$$

the anomalous magnetic moment of the proton, if we identify the baryon vector currents and their parameters $\alpha_i^+ - \alpha_i^-$, $\beta_i^+ - \beta_i^-$ for weak leptonic and electromagnetic interactions, and

$$\frac{G_p^A}{G_p^V} = -\frac{5}{3} \frac{\alpha^+ + \alpha^-}{\alpha^+ - \alpha^-},$$

the ratio of beta decay coupling constants for the neutron.

If we want to eliminate the pole in the induced pseudoscalar terms, we have to assume a zero in α_i^- . We insert simply $\alpha_i^- = 0$ everywhere in the results given above, except A_3^D and A_3^F . In these terms we have to account for the next order and get :

$$A_3^D = 24(\alpha^-)' + 3(-\alpha_1^+ + \alpha_2^+ + \alpha_3^+) - 12(\beta_1^+ - \beta_1^-) - 6(\beta_2^+ - \beta_2^-),$$

$$A_3^F = 16(\alpha^-)' - (-\alpha_1^+ + \alpha_2^+ + \alpha_3^+) - 2(\beta_1^+ - \beta_1^-) - 4(\beta_2^+ - \beta_2^-) - 6(\beta_3^+ - \beta_3^-)$$

with

$$(\alpha^-)' = \left(\frac{\partial \alpha^-}{\partial x^2} \right)_{x^2=0} k^2.$$

The ratio of beta decay coupling constants for the neutron is now fixed, equal to $-\frac{5}{3}$.

Without assuming an underlying universality, we get the same ratio $-\frac{5}{3}$ by excluding the pole. This is connected with the fact pointed out above that the couplings d_i^- contribute only to the covariant $\gamma_\mu (1+i\gamma_5)$ if the other Lorentz covariants are suitably chosen.

Second class currents can be made to vanish by inserting $\alpha \frac{+}{2} = \alpha \frac{\pm}{3}$, $\beta_i^+ = -\beta_i^-$ which, as can be seen above, corresponds to a certain conjugation invariance of the baryon currents.

R E F E R E N C E S

- 1) W. Rühl, CERN preprint TH.505, December 1964, to be published in Nuovo Cimento.
- 2) W. Rühl, CERN preprint TH.514, January 1965, to be published in Nuovo Cimento.
- 3) N. Cabibbo, Phys.Rev. Letters 10, 531 (1964).